Quality Disclosure and Competition

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Abstract

We analyze costly quality disclosure with horizontally differentiated products under duopoly and monopoly, and characterize the effect of competition on disclosure and welfare. Expected disclosure is higher under monopoly than under duopoly. A duopolist faces external competition, and is tempted to disclose quality to steal customers, but firms also face internal competition, based on the inability of a firm’s types to collude, ex ante. A monopolist faces stiffer internal competition than a duopolist does, and the net effect is more disclosure under monopoly. When the disclosure cost is low, welfare is higher under monopoly than duopoly, but when the disclosure cost is high (but not so high as to eliminate disclosure), welfare is higher under duopoly. Both market structures result in excessive disclosure, as compared to the socially optimal level, and imposing a small tax on disclosure improves welfare. Modifications of the model, allowing for equilibrium under-disclosure, are also discussed.

1 Introduction

A free and efficient flow of information is important for modern, information driven economies. Although prices are the main carriers of information critical for decision making, important information is revealed to consumers directly by firms, either voluntarily or as mandated by law. For example, drug manufacturers must incur substantial costs in order to certify the safety and efficacy of new drugs. A substantial portion of these costs can be attributed to the cost of disclosing the quality,

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as opposed to the cost of learning the quality. In other industries, disclosure is not regulated, yet firms choose to incur expenses in order to disclose quality. Private schools and summer camps produce glossy brochures and glitzy videos, detailing the quality of the services they provide. What determines firms’ disclosure behavior? What is the motive for a firm to reveal its private information? In a market with inefficient disclosure, how can a regulatory policy correct the inefficiency? We are motivated by these questions, and in particular, by how the answers depend on the market structure in the relevant industries.

As far as we know, we are the first to address these questions in a model where (i) quality is privately observed by firms, (ii) after observing quality, firms can credibly disclose it by incurring a positive cost, (iii) products are differentiated in terms of quality and a non-quality characteristic like flavor, (iv) monopoly and duopoly are considered. In particular, we consider a Hotelling model of price competition with differentiated products. Consumers are located along the unit interval, interpreted to be the most preferred product characteristic, and the two firms are located at either endpoint. Each firm’s product is also defined along a second dimension, interpreted to be product quality. Product quality is privately observed by the firm, and the two qualities are independent. In the symmetric duopoly equilibrium we characterize, firms whose quality is below a certain threshold, \( q^*_{D} \), choose not to disclose, and firms whose quality is above the threshold disclose. Prices depend on the perceived or observed difference in the firms’ qualities.

For the analogous monopoly model, we assume that one firm produces both goods, and privately observes both qualities. Thus, the monopoly must decide which of the two qualities to disclose. We find that, if both qualities are below a threshold, \( q^*_{M} \), then nothing is disclosed. If the higher quality is above \( q^*_{M} \) and the lower quality is below a threshold (that depends on the higher quality), then only the higher quality is disclosed. Otherwise, both qualities are disclosed. Prices depend on the two perceived (or disclosed) qualities.

In Section 5, we compute expected levels of disclosure and welfare for the duopoly model and the monopoly model. One might think that there would be more disclosure under duopoly, because a duopolist ignores the losses that disclosing imposes on its rival. Surprisingly, we find that the

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1 After all, firms sometimes learn that drugs approved to treat one condition are effective in treating a completely different condition. Manufacturers often bypass the costly trials necessary to certify safety and efficacy for the new use of the drug, suggesting that the trials are more about certification of quality than learning about quality.
expected amount of disclosure is always higher under monopoly than under duopoly. Another surprise is that welfare does not monotonically decline in $\delta$ for either market structure. As to the comparison across market structures, for small values of $\delta$, welfare is higher under monopoly, but for large values of $\delta$, welfare is higher under duopoly. In Section 6, we introduce the benchmark of a social planner, with the ability to control disclosure cutoffs but not prices. This allows us to compute the socially optimal level of disclosure for the oligopoly model and the monopoly model. In both models, the equilibrium level of disclosure is excessive, and a small tax on disclosure improves social welfare. Although there is excessive disclosure from the perspective of society as a whole, we find that consumer surplus is higher when more disclosure is mandated. Based on the political economy of consumer interests versus societal interests, we can reconcile the typical result in the literature, excessive disclosure, with the “man in the street” view that mandated disclosure is a good thing.

The intuition for this surprising combination of results begins with the familiar “unraveling” argument appearing often in the literature. If disclosure cost were zero, we would have full disclosure in equilibrium. A firm with the highest quality, in the set of qualities that are not disclosed, would increase its perceived quality and equilibrium profits by disclosing. With a positive disclosure cost, a firm whose quality is at the threshold is indifferent between disclosing and not disclosing. However, since this firm acts after observing its quality, it does not take into account the ex ante increase in expected profits that not disclosing would provide, by increasing its perceived quality for realizations below the threshold. Thus, beyond the competition that a firm might face from another firm, there is additional competition that a firm faces from itself, due to the inability of its own types to collude with each other (by eliminating incentive compatibility constraints).

When we consider the internal competition that a firm faces from its other types, an extension of the unraveling idea, it is no longer surprising that there will be more disclosure under monopoly than duopoly. The lack of external competition strengthens the internal competition. After all, if $q^*_D = q^*_M$ were to hold, a firm with the threshold quality could appropriate more of the benefits of disclosure under monopoly than under duopoly; therefore, the threshold must be lower under monopoly, leading to more disclosure. The explanation for our welfare results is similar, but more subtle. Internal competition between a firm’s types leads to disclosure costs that are socially excessive. The welfare loss, due to excessive disclosure costs being incurred, is greater for monopoly than for duopoly, but relatively unimportant when $\delta$ is either small (disclosure is not very costly per
unit) or high (there is not much disclosure). The other source of welfare loss is due to market share expansion. A duopolist will take advantage of a higher quality perception partly by raising its price and partly by expanding its market share. The expansion of market share turns out to be socially excessive, so the allocation of consumers to products is inefficient. Because a monopolist internalizes the impact of an expanding market share of one product on the declining market share of the other product, this misallocation is greater for duopoly than for monopoly.² For low δ, the welfare loss due to misallocation dominates, and monopoly yields higher welfare than duopoly. For moderate δ, the welfare loss due to excessive disclosure costs being incurred dominates, and duopoly yields higher welfare. For large δ, there is no disclosure and welfare is the same under both monopoly and duopoly.

In Section 6, we discuss a modification to the model that allows for the possibility of equilibrium with under-disclosure. By supposing that only a fraction of the disclosure cost represents a true social cost, the welfare calculations change without affecting the equilibrium. For example, part of the disclosure cost could represent surplus received by other industries or individuals (advertising, scientific research). However, if only a relatively small amount of the disclosure cost represents a cost to society, equilibrium disclosure is insufficient. Alternatively, the cost of disclosing a low quality might be the loss of revenue from consumers who do not update their priors unless a disclosure announcement is made (see Hirshleifer, Lim and Teoh (2002) and Fishman and Hagerty (2003)). Under this behavioral approach, we would have equilibrium under-disclosure, but arguments for government intervention would rest on the assumption of bounded rationality.

2 Literature Review

An important result in the early literature is that, if disclosure is costlessly credible, a privately informed seller will voluntarily disclose all information. This is the unraveling result discussed above. See, for example, Grossman and Hart (1980) or Milgrom and Roberts (1986) in the monopoly context, and Okuno-Fujiwara, Postlewaite, and Suzamura (1990) in the oligopoly context. In such

² All consumers are served in equilibrium, so there can be a distortion due to consumers purchasing the wrong product, but not due to some consumers being inefficiently priced out of the market.
environments, mandatory disclosure rules are unnecessary.\textsuperscript{3} However, we often do not observe full disclosure in practice. To reconcile the gap between the theoretical predictions and this empirical observation, other studies utilize one of two approaches.

The first approach assumes that disclosing is costly. Viscusi (1978) and Jovanovic (1982) show that if disclosure is costly, sellers will voluntarily disclose only if their quality exceeds some threshold. Introducing a disclosing cost thus leads some sellers to choose not to disclose. However, the analysis does not really justify the motive for a mandatory disclosure rule. In their models, consumers are equally well off with and without disclosure, since the price they pay equals their expected valuation conditional on their information. Sellers may be even worse off with mandatory disclosure, because it eliminates the option to withhold information and save the disclosure cost. Jovanovic shows that, in equilibrium, there is actually too much disclosure. A small subsidy for sales in which quality is not disclosed improves welfare.

The second approach employs “behavioral” models (currently fashionable in economics and finance) that assume that not all consumers are fully rational (Hirshleifer et al. (2002), Fishman and Hagerty (2003)). It assumes that there are two types of consumers: fully rational consumers who could correctly infer from firms’ disclosure the relevant quality, and bounded rational consumers who naively neglect to draw inferences from the fact that a firm does not disclose quality, believing instead that the distribution of quality is the prior distribution. The presence of boundedly rational consumers introduces at least some incentive for a seller not to disclose information about quality, even if it is costless. Qualitatively speaking, these behavioral models are similar to the disclosure-cost models. In both types of models, unraveling of the decision not to disclose is only partial. For a firm with sufficiently low quality, either separating oneself from the pool of lowest-quality firms is not worth the cost of disclosure, or separating oneself from the pool of lowest-quality firms is not worth the foregone earnings from naive consumers.

There has been a recent literature addressing the effects of competition on disclosure. However,\textsuperscript{5}

\textsuperscript{3}Matthews and Postlewaite (1985) turn common intuition on its head, by considering a model in which the firm chooses whether to learn the quality of its product, and then chooses whether to disclose the quality. Both learning and disclosing are costless. Since the firm cannot credibly argue that it did not learn its quality, quality is learned and disclosed. However, a mandatory disclosure law might give an incentive for the firm not to observe quality, because now the firm could credibly argue that it did not learn its quality, because then it would have been disclosed. Thus, mandatory disclosure laws can lead to less disclosure.
this literature does not adequately consider what is arguably the most important framework, namely, a framework with “universal private information,” in which products are differentiated in terms of quality and a non-quality characteristic.\(^4\) In Dye and Sridhar (1995), increasing competition may increase disclosure, which is opposite to our result. Rather than measuring the quality of the product sold by the firm, information is about the expected profitability or cash flows of the firm, and firm managers seek to maximize the firm’s stock price. Another distinction is that traders are not sure whether firms observe a quality signal in Dye and Sridhar (1995). Stivers (2004) adapts Dye and Sridhar’s model to oligopolistic competition with vertically differentiated products. He adopts a reduced-form approach, and the structural example presented is inconsistent with our setup of heterogeneous consumer preferences over products. Stivers (2004) finds that increasing competition (either by increasing the number of firms or increasing the intensity of competition among firms) strengthens quality disclosure.

Board (2003) examines a duopoly model with consumers that have heterogeneous preferences for quality (vertically differentiated products), and where disclosing quality is costless. He assumes that both firms know the quality of both products, but consumers cannot observe quality unless it is disclosed. Even though disclosure is costless, full unraveling does not occur, so competition and disclosure can be inversely related. The higher quality is always disclosed, and the lower quality is disclosed if it is neither too high nor too low. The intuition is that, if the two qualities are almost the same, then we are nearly in the situation of pure Bertrand competition, so the lower quality firm is better off pooling with its low quality types. This yields most of the market profits to the higher quality firm, but profitably serves consumers with relatively low demand for quality. Hotz and Xiao (2002) allow for both vertically and horizontally differentiated products in a duopoly model with costless disclosure. Firms observe the qualities of both products, which are perfectly negatively correlated. Hotz and Xiao (2002) avoid full unraveling, again because disclosure leads to fiercer price competition. Cheong and Kim (2004) examine the effect of competition on disclosure, but assume that consumers are homogeneous. As a result, only firms that disclose receive customers in equilibrium. In a supplementary appendix, they briefly consider consumer heterogeneity. However, heterogeneity is over demand for quality, and not the products themselves (vertical but not horizontal

\(^4\)Daughety and Reinganum (2004) define universal private information to mean that firms privately observe the quality of their products, and consumers privately observe their preferences.
differentiation). Furthermore, the comparison between monopoly and duopoly is hampered by the fact that the number of draws from the quality distribution depends on the number of firms.

Our contribution is to introduce what we feel is the most natural and important model for addressing the effects of disclosure on product-market competition. Namely, quality is a firm’s private information unless disclosed, consumers are heterogeneous in their demand for product characteristics other than quality, and disclosure is costly. Our framework allows us to identify two sources of competition that affect equilibrium disclosure: competition from a firm’s rivals and competition from a firm’s other types (due to interim incentive compatibility constraints). A complementary approach is adopted by Daughety and Reinganum (2003, 2004). As in our model, they make the natural assumption that quality is a firm’s private information. Daughety and Reinganum (2003, 2004) model the disclosure choice as occurring before a firm learns its quality. This commitment eliminates competition from a firm’s other types and dramatically changes the impact of competition on disclosure. Because theirs is a multiple period model, pricing in period 1 can signal quality in equilibrium.

3 The Duopoly Model

There are two firms producing a differentiated product, represented by firm 0 and firm 1 located at either endpoint of the unit interval. Products 0 and 1 differ on two dimensions: the “taste” dimension and the quality dimension. Consumers all have the same preference for high quality, but different consumers have different preferences along the taste dimension. We model the taste dimension as the location of a consumer within the unit interval, representing the ideal product characteristic for that consumer. The distance from a consumer’s location to the product location is the loss in utility from consuming a product that is less than ideal. Thus, consumers located near 0 have a strong preference for product 0 over product 1, consumers located near 1 have a strong preference for product 1 over 0, and a consumer located at 1/2 is indifferent between the two products (holding quality constant). A consumer is thus characterized by the distance, $x$, from the location of firm 0 ($x$ can be regarded as the “type” of the consumer). We assume that consumers are uniformly distributed along the unit interval.

For $i = 0, 1$, the quality of the good produced by firm $i$ is $\psi + q_i$, where $\psi$ is a common value com-
ponent exogenously given in our model and $q_i$’s are independent draws from the uniform distribution over $[0, 1]$. A consumer purchases either 0 or 1 unit of output, where the utility of not purchasing is normalized to zero. Given $q_0$ and $q_1$, the values of product 0 and 1 to a type-$x$ consumer are given by $\psi + q_0 - x$ and $\psi + q_1 - (1 - x)$, respectively. Thus, given prices $p_0$ and $p_1$ charged by firms 0 and 1, respectively, a type-$x$ consumer’s utilities obtained from purchasing one unit of products 0 and 1, are given by $\psi + q_0 - p_0 - x$, and $\psi + q_1 - p_1 - (1 - x)$, respectively.

We assume $\psi > \frac{3}{2}$, which guarantees that in equilibrium all consumers will be served in the market. Since the value of $\psi$ is common knowledge to all parties in the model, henceforth we simply refer to $q_i$ as the quality of product $i$, $i = 0, 1$. The consumer will either not purchase, purchase one unit of product 0, or purchase one unit of product 1, depending on which decision induces the highest expected utility conditional on the consumer’s available information.

The time line of the duopoly game is as follows. First, each firm privately observes its quality $q_i$, and decides whether or not to disclose it. Thus, each firm’s disclosure strategy can be represented by a function $D_i : [0, 1] \rightarrow \{0, 1\}$, mapping quality to a disclosure choice, where $D_i = 0$ means “do not disclose” and $D_i = 1$ means “disclose.” We assume that certifying a false quality is impossible. The cost of disclosing is denoted by $\delta$, and the marginal cost of production is normalized to zero. After the disclosure choices have been made and observed by both firms, the firms simultaneously choose prices. Next, consumers observe the disclosure and pricing choices of the two firms, and make their purchasing decisions. Given the firms’ disclosure and pricing choices, let the induced market perceptions be $\tilde{q}_0$ and $\tilde{q}_1$, respectively.

Our solution concept is Perfect Bayesian Equilibrium (PBE), which consists of the equilibrium strategies (firms’ disclosure and pricing strategies and the consumers’ purchasing strategies), and the consumers’ equilibrium beliefs about product qualities. Because quality does not affect production cost, the profit maximizing prices, $(p_0, p_1)$, do not depend on the value of any undisclosed quality. There is no signalling role for prices in our model. Thus, we feel justified in restricting attention

\[5\text{If prices were set simultaneously with the disclosure decision, then a duopolist would have to set prices without knowing what consumers believe about the other product’s quality. Our time line eliminates this discrepancy between duopoly and monopoly, allowing a cleaner comparison between duopoly and monopoly. The other time line leads to similar results, however.}\]

\[6\text{If consumers’ beliefs are represented by probability distributions over the two firms’ qualities, } \tilde{q}_0 \text{ and } \tilde{q}_1 \text{ are the means of the two distributions. We have } \tilde{q}_i = q_i \text{ when product } i \text{ is disclosed, } i = 0, 1.\]
to PBE in which consumers’ beliefs about undisclosed qualities do not depend on either price. We restrict attention to symmetric PBE, in which there is a quality threshold, \( q^D \), such that (1) firm \( i \) discloses its quality \( q_i \) if and only if \( q_i > q_i^D \); and (2) if firm \( i \) does not disclose its quality, beliefs about \( q_i \) are independent of the firm’s pricing choice. We will use the term duopoly equilibrium (or sometimes simply equilibrium) to denote a PBE satisfying these restrictions.

It is easily seen that the firms’ profits only depend on the prices and the market perceptions of the qualities, instead of the true qualities. Since perceived qualities depend on the disclosure choices and not the prices, each firm’s pricing choice can be represented as a function of the two perceived qualities. In equilibrium, there will be a cutoff type of consumer, \( x^* \), where consumers with types \( x < x^* \) purchase from firm 0, consumers with types \( x > x^* \) purchase from firm 1, and consumers with type \( x = x^* \) are indifferent between purchasing from firm 0 and firm 1. Given such cutoff \( x^* \), we simply say that the market share for product 0 is \( x^* \).

Given the prices \((p_0, p_1)\), the perceived qualities \((\tilde{q}_0, \tilde{q}_1)\), and assuming for the moment that all consumers prefer purchasing from one of the firms to not purchasing, \( x^* \) is determined by the following equation:

\[
\psi + \tilde{q}_0 - p_0 - x^* = \psi + \tilde{q}_1 - p_1 - (1 - x^*),
\]

which gives

\[
x^* = \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 - \tilde{q}_1 + p_1 - p_0). \tag{1}
\]

The firms’ profits are given by:

\[
\begin{align*}
p_0(p_0, p_1) &= p_0x^* = p_0 \left[ \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 - \tilde{q}_1 + p_1 - p_0) \right] \quad \text{and} \\
p_1(p_0, p_1) &= p_1(1 - x^*) = p_1 \left[ \frac{1}{2} - \frac{1}{2}(\tilde{q}_0 - \tilde{q}_1 + p_1 - p_0) \right].
\end{align*}
\]

Sequential rationality requires the equilibrium prices to be

\[
\begin{align*}
p_0 &= 1 + \frac{1}{3}(\tilde{q}_0 - \tilde{q}_1) \quad \text{and} \tag{2} \\
p_1 &= 1 + \frac{1}{3}(\tilde{q}_1 - \tilde{q}_0). \tag{3}
\end{align*}
\]
Substituting the prices from (2) and (3) into the expressions for market share and profits, we can express market share and profits as a function of perceived qualities, given by

\[ x^*(\tilde{q}_0, \tilde{q}_1) = \frac{1}{2} + \frac{1}{6}(\tilde{q}_0 - \tilde{q}_1), \]  
\[ \pi_0(\tilde{q}_0, \tilde{q}_1) = \frac{1}{18}(3 + \tilde{q}_0 - \tilde{q}_1)^2, \]  
\[ \pi_1(\tilde{q}_0, \tilde{q}_1) = \frac{1}{18}(3 + \tilde{q}_1 - \tilde{q}_0)^2. \]

From (1)-(3) and our assumption, \( \psi > \frac{3}{2} \), it is straightforward to demonstrate that all consumers are willing to purchase from one of the firms, even in the worst case in which \( \delta \) is close to zero and both firms do not disclose.\(^7\)

**Proposition 1** There is a unique duopoly equilibrium. The disclosure threshold \( q^{*D} \), is given by

\[ q^{*D} = \begin{cases} 
-\frac{5}{3} + \frac{1}{3}\sqrt{25 + 216}\delta & \text{if } \delta \in [0, 13/72] \\
1 & \text{if } \delta > 13/72
\end{cases} \]  

and consumers believe that an undisclosed quality is uniformly distributed over \([0, q^{*D}]\), so that the perceived quality is \( \tilde{q}_i = q^{*D}/2 \). Prices are given by (2) and (3) and consumer behavior is characterized by the market share expression, (4).

**Proof.** Because disclosing a quality of zero is costly and can never raise a firm’s perceived quality, not disclosing must be on the equilibrium path, so the specified consumer beliefs follow immediately from Bayes’ rule. Suppose firm 1 follows a disclosure strategy characterized by a cutoff quality \( q^* \). If firm 0 discloses its quality \( q_0 \), its expected profit from the following duopoly competition is given by

\[ q^* \left[ \frac{1}{18}(3 + q_0 - q^*/2)^2 \right] + \int_{q^*}^{1} \frac{1}{18}(3 + q_0 - q)^2 \, dq - \delta. \]  

If firm 0 does not disclose its quality, its expected profit is given by

\[ q^* \cdot \frac{1}{2} + \int_{q^*}^{1} \frac{1}{18}(3 + q^*/2 - q)^2 \, dq. \]

\(^7\)With \( \psi < 3/2 \), then for some \( \delta \) and some quality realizations, some consumers are not served in the duopoly equilibrium. This is an interesting possibility requiring separate analysis. To keep the paper to a reasonable length, we leave this analysis to future work.
Note that given \( q^* \), profit from disclosing in expression (7) is strictly increasing in \( q_0 \), and profit from not disclosing in expression (8) is independent of \( q_0 \). Therefore, for \( q^* \) to characterize an equilibrium threshold with \( q^* < 1 \), a necessary and sufficient condition is that expressions (7) and (8) be equal at \( q_0 = q^* \). For \( \delta \leq 13/72 \), the unique solution is \( q^{*D} = -\frac{5}{3} + \frac{1}{3}\sqrt{25 + 216\delta} \). There cannot be an equilibrium with \( q^{*D} = 1 \), because firm 0 would receive higher profits by disclosing when \( q_0 = 1 \) occurs.

For \( \delta > 13/72 \), there cannot be an equilibrium with an interior threshold. However, there is an equilibrium with no disclosure, characterized by \( q^{*D} = 1 \). Each firm has a perceived quality of \( \frac{1}{2} \), chooses a price of 1, and receives expected profits of \( \frac{1}{2} \). The disclosure cost is high enough so that a deviation to disclose is not profitable, even with the highest possible quality. If firm \( i \) discloses quality, \( q_i = 1 \), the resulting price competition yields profits of \( \frac{49}{72} - \delta \). Since we have \( \delta > 13/72 \), this deviation is not profitable. All consumers are willing to purchase, so (6) characterizes a duopoly equilibrium. □

4 The Monopoly Model

In order to evaluate how the degree of competition affects disclosure and welfare, we now consider a monopoly model that is otherwise identical to the model of Section 3. The locations and utility functions of consumers is the same, the location of products is the same, and the joint distribution of qualities is the same. The only difference is that now there is a single firm that produces both products and observes both qualities. Our decision to model the monopolist as controlling two products is the only way to have a clean evaluation of the effect of market structure on the levels of disclosure and welfare. If the monopolist were to produce only one product, then moving from monopoly to duopoly would change the number of draws from the quality distribution, entailing a change in technology as well as a change in market structure.

Based on the realizations of both qualities, the monopolist chooses to disclose either none, one, or both of the qualities. Thus the disclosure strategy \( D : [0, 1] \times [0, 1] \rightarrow \{(0, 0), (1, 0), (0, 1), (1, 1)\} \) maps a pair of qualities to a disclosure choice. After the disclosure decision, the firm sets prices for both products. As before, the production cost is normalized to zero, and a disclosure cost, \( \delta \), is incurred for each product whose quality is disclosed.
Unlike in the duopoly case, the monopolist observes both product qualities before making the disclosure decisions. The monopolist potentially can coordinate over both quality disclosures, based on the realizations of both qualities. Let \( q_H = \max\{q_0, q_1\} \) denote the higher of the two qualities and let \( q_L = \min\{q_0, q_1\} \) denote the lower of the two qualities. As in the duopoly model, the profit maximizing prices, \((p_0, p_1)\), do not depend on the value of any undisclosed quality. There is no signalling role for prices in our model. Thus, we feel justified in restricting attention to PBE in which consumers’ beliefs about undisclosed qualities do not depend on either price. On the other hand, beliefs about, say, \( q_0 \), should be allowed to depend on the disclosed value of \( q_1 \). However, we want to avoid undesirable equilibria in which specific high quality values are not disclosed, supported by the belief that the undisclosed quality is zero. To resolve this issue, we restrict attention to the following class of symmetric PBE, in which there is a threshold for \( q_H \) to be disclosed, and there is a threshold for \( q_L \) to be disclosed, which may depend on \( q_H \).

**Definition 1:** A monopoly equilibrium is a PBE for which there is a threshold, \( q^{*M} \), and a function, \( G(\cdot) \), mapping the unit interval into itself, satisfying

1. \( q_H \) is disclosed if and only if we have \( q_H > q^{*M} \),
2. \( q_L \) is disclosed if and only if \( q_H \) is disclosed and we have \( q_L > G(q_H) \),
3. beliefs are independent of prices,
4. if the monopolist discloses a quality \( q < q^{*M} \), then beliefs are that the undisclosed quality is uniformly distributed over \([0, q]\).

Because of the disclosure cost, no disclosure is always on the equilibrium path of the game. If both qualities are disclosed, beliefs assign probability one to the disclosed qualities. Thus, the only important beliefs off the equilibrium path arise when a quality below the threshold is disclosed, and part (iv) pins down these beliefs in a compelling way. Without loss of generality, let \( G(q_H) \leq q_H \) hold. Definition 1 implies the following structure of beliefs in a monopoly equilibrium. If nothing is disclosed, consumers believe that both qualities are independently and uniformly distributed over \([0, q^{*M}]\), which follows from Bayes’ rule. If only \( q_i = q < q^{*M} \) is disclosed, consumers beliefs are determined by Definition 1, part (iv). If only \( q_i = q \geq q^{*M} \) is disclosed, consumers believe that the

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The importance of (iv) is to rule out multiple equilibria, based on beliefs that assign a very low mean to the
undisclosed quality is the lower quality and is uniformly distributed over \([0, G(q)]\). Since perceived qualities depend on the disclosure choices and are fixed when prices are chosen, the monopolist’s pricing choice can be represented as a function mapping the perceived qualities into the price of each product.

To characterize the monopoly equilibrium, it only remains to identify the pair \((q^M, G(\cdot))\). Given the disclosure strategy characterized by \((q^M, G(\cdot))\), suppose the induced market perceptions about the qualities are given by \((\tilde{q}_0, \tilde{q}_1)\). Assuming for the moment that all consumers will be served in equilibrium, a monopoly equilibrium must leave the consumer at location \(x^*\) indifferent between purchasing product 0, purchasing product 1, and not purchasing. Therefore, the prices must satisfy

\[
\begin{align*}
p_0 &= \psi + \tilde{q}_0 - x^*, \\
p_1 &= \psi + \tilde{q}_1 - (1 - x^*). 
\end{align*}
\]

The profit maximizing market share is thus given by

\[
x^* \in \arg\max (\psi + \tilde{q}_0 - x) x + (\psi + \tilde{q}_1 - (1 - x))(1 - x),
\]

which results in

\[
x^M = \frac{1}{2} + \frac{1}{4}(\tilde{q}_0 - \tilde{q}_1).
\]

From (9) and (10), we derive the prices and monopoly profits (gross of any possible disclosure cost), as a function of the perceived qualities, given by:

\[
\begin{align*}
p_0 &= \psi - \frac{1}{2} + \frac{3}{4}\tilde{q}_0 + \frac{1}{4}\tilde{q}_1, \\
p_1 &= \psi - \frac{1}{2} + \frac{1}{4}\tilde{q}_0 + \frac{3}{4}\tilde{q}_1, \\
\pi^M(\tilde{q}_0, \tilde{q}_1) &= \psi - \frac{1}{2} + \frac{1}{2}(\tilde{q}_0 + \tilde{q}_1) + \frac{1}{8}(\tilde{q}_0 - \tilde{q}_1)^2
\end{align*}
\]

It is straightforward to show that \(\psi > \frac{3}{2}\) guarantees that the monopolist will want to serve all consumers in equilibrium.\(^9\)

\(^9\) Let \(x_0\) denote the market share of product 0, and let \(x_1\) denote the market share of product 1. If not all consumers are served, so we have \(x_0 + x_1 < 1\), then the necessary first order conditions for profit maximization are \(x_i = (\psi + \tilde{q}_i)/2\) for \(i = 0, 1\). However, no matter what the (nonnegative) perceived qualities are, the first order conditions are inconsistent with \(\psi > \frac{3}{2}\) and \(x_0 + x_1 < 1\).
The monopoly equilibrium is now characterized in Proposition 2 below.

**Proposition 2** There is a unique monopoly equilibrium. The disclosure threshold is

\[
q^{*M} = \begin{cases} 
4\sqrt{1+2\delta} - 4 & \text{for } \delta \in [0,9/32) \\
1 & \text{for } \delta \geq 9/32 
\end{cases} \tag{14}
\]

If \(\delta < \frac{7}{32}\) holds, there is a second threshold for \(q_H\), given by \(\hat{q} = 4 - 4\sqrt{1 - 2\delta}\), at which sufficiently high \(q_L\) is also disclosed, characterized by

\[
G(q_H) = \begin{cases} 
\frac{2q_H - 4}{3} + \frac{2}{3}\sqrt{(q_H)^2 - 4q_H + 4 + 24\delta} & \text{for } q_H \in [\hat{q},1] \\
q_H & \text{for } q_H \in [q^{*M},\hat{q}] 
\end{cases} \tag{15}
\]

If \(\delta \geq \frac{7}{32}\) holds, the lower quality is never disclosed, so (without loss of generality) let \(G(q_H) = q_H\) hold. Beliefs are as implied by Definition 1, and prices are given by (11) and (12).

Proof: See the appendix.

Proposition 2 shows that the disclosure policy in the monopoly equilibrium exhibits some coordination between products, because the decision whether to disclose \(q_L\) can be contingent on the value
of \( q_H \), as seen from the fact that \( G(\cdot) \) varies with \( q_H \). Is this coordination device also beneficial to welfare? This question will be addressed in the next section, where we look at the effect of market structures on welfare.

5 Welfare Analysis

We now consider the impact of market structure on welfare, which is measured as ex ante expected total surplus. There are two potential sources of inefficiency in our model. One is the misallocation of the products to consumers, and the other is the payment of disclosure costs. We first demonstrate that the expected amount of disclosure is higher under monopoly than under duopoly.

**Proposition 3** In equilibrium, the expected number of disclosed products under duopoly is strictly lower than under monopoly for \( \delta \in (0, 9/32) \). Outside this range, either both products are disclosed (\( \delta = 0 \)) or neither product is disclosed (\( \delta \geq 9/32 \)) under both market structures.

Proof: See the appendix.

The comparison is illustrated in Figure 2 below, where \( \Delta \) denotes the expected number of disclosed products.

![Figure 2: Expected Disclosure](attachment:image.png)
Proposition 3 indicates that, on average, equilibrium disclosure costs are higher under monopoly than under duopoly. The intuition for this comparison is based on the higher level of internal competition that the monopolist faces from its other types, as compared to the level of internal competition that a duopolist faces from its other types. Fix \( \delta \), and suppose that the equilibrium thresholds could be the same under duopoly and monopoly, \( q^*_{D} = q^*_{M} \equiv q \). Now suppose that a firm is considering whether to disclose the quality of product 0, where we have \( q_0 = q \). (Since we are comparing \( q^*_{D} \) and \( q^*_{M} \), product 0 is taken to be the higher quality product when discussing the monopoly case.) From (5), the expected benefit from disclosing under duopoly is approximately the increase in perceived quality multiplied by the rate at which perceived quality increases profits, \( \frac{q^2}{2}[3 + q - \tilde{q}] \). From (13), the expected benefit from disclosing under monopoly is approximately \( \frac{q^2}{2}[2 + q - \tilde{q}] \). It is easy to see that the expected benefit is higher under monopoly, contradicting the fact that the threshold equates the expected benefit and the cost, \( \delta \). Instead, we must have \( q^*_{M} < q^*_{D} \). The monopolist is able to coordinate the prices of both products, and appropriates for itself more of the benefit from disclosure than a duopolist can appropriate. Thus, the lack of external competition actually increases the internal competition faced by a monopolist.\(^{10}\)

The following proposition compares welfare under the two market structures. When the disclosure cost is relatively small, welfare is higher under monopoly than duopoly. When the disclosure cost is moderate, then welfare is higher under duopoly than monopoly. When the disclosure cost is high enough, so that nothing is disclosed under either market structure, welfare is the same under monopoly and duopoly.

**Proposition 4** There exists \( \delta^* \in (0, 13/72) \), such that for \( \delta \in [0, \delta^*) \), the expected total surplus is higher under monopoly, and for \( \delta \in (\delta^*, 9/32) \), the expected total surplus is higher under duopoly. For \( \delta \geq 9/32 \), welfare is the same under the two market structures.

Proof: See the appendix.

\(^{10}\)This argument explains why \( q^*_{M} < q^*_{D} \) must hold, so the probability of at least one product’s quality being disclosed is higher under monopoly. However, the comparison of expected disclosure is slightly more complicated, because if \( q_0 \) turns out to be the lower quality, then it might be above the duopoly threshold but below \( G(q_H) \). The overall comparison is unambiguous, as shown in Proposition 3.
Expected total surplus under monopoly and duopoly are depicted in Figure 3. The explanation for the U-shaped patterns is best understood by considering expected payments of the disclosure cost. When $\delta$ is close to zero, both qualities are likely to be disclosed, but the cost is relatively low. As $\delta$ increases, we have less disclosure, but higher payments of the disclosure cost, so welfare goes down. Eventually, we have the surprising result that welfare increases as $\delta$ increases. This result is due to the internal competition within each firm, leading to socially excessive disclosure (demonstrated in the next section). Higher $\delta$ in this range leads to less disclosure, which actually improves welfare.

Figure 3: Welfare Comparison ($\psi = 2$)

What about the welfare comparison between duopoly and monopoly? Because, as it turns out, internal competition leads to socially excessive disclosure, and this problem is worse for monopoly than duopoly, the loss in welfare due to excessive disclosure costs being incurred favors duopoly over monopoly. The other source of welfare loss is due to the divergence between the actual market share and the optimal market share. Given the perceived qualities of the two products, $\tilde{q}_0$ and $\tilde{q}_1$, the
market share for product 0 that maximizes social surplus is easily seen to be\(^{11}\)

\[
x^{**} = \frac{1}{2} + \frac{1}{2} (\tilde{q}_0 - \tilde{q}_1).
\]

Comparing \(x^{**}, x^{*D}\) from (4), and \(x^{*M}\) from (10), we see that the equilibrium market share is closer to the socially optimal market share under monopoly than under duopoly. When \(\delta\) is near zero, the welfare loss due to excessive disclosure costs being incurred (as a result of internal competition) is also near zero, so the misallocation distortion causes welfare to be higher under monopoly. As \(\delta\) increases, eventually the welfare loss due to excessive disclosure costs being incurred dominates, and welfare is higher under duopoly. When we have \(\delta \geq 9/32\), no disclosure occurs in either market structure. The expected quality and market share of each product is one half under both duopoly and monopoly, so expected total surplus is exactly the same in each market structure.

6 Disclosure Policy

In this section we consider the welfare effects of a tax/subsidy on disclosure. More generally, we consider the socially optimal policy regarding disclosure. This policy is of primary interest as a benchmark to compare with the equilibrium, although the analysis may be of some interest to policy makers. Formulating the appropriate planner’s problem is tricky. The socially optimal level of disclosure should probably take the market structure as given, rather than allowing the planner to choose prices. If the planner could choose prices, then by letting prices be increasing functions of quality, the planner could indirectly disclose quality without incurring any disclosure costs. Therefore, the planner is assumed to be able to choose disclosure cutoffs but not prices.

The new timeline is as follows. First, the social planner picks the disclosure policy (which is \(q^{**D}\) under duopoly and \((q^{**M}, G^{**}(.))\) under monopoly). Then the firms (or the firm) disclose according to the policy chosen by the social planner. Then, after quality disclosure, the firms (or the firm) choose prices. Finally the consumers make their purchase decisions, given the disclosures and prices.

The socially optimal disclosure policy is the policy that maximizes ex ante expected total surplus.

\(^{11}\)Because the price is a transfer from consumers to firms, the transaction of the consumer at location \(x\) purchasing product 0 adds \(\psi + \tilde{q}_0 - x\) to expected total surplus, and the consumer at location \(x\) purchasing product 1 adds \(\psi + \tilde{q}_1 - (1 - x)\) to expected total surplus. To maximize expected total surplus, \(x^{**}\) must equate these two terms.
Proposition 5 Under duopoly, the socially optimal disclosure policy is given by $q^{**D} = 12\sqrt{\delta/5}$ for $\delta \in [0,5/144]$, and $q^{**D} = 1$ for $\delta > 5/144$. Under monopoly, the socially optimal disclosure policy is given by $q^{**M} = G^{**}(q_H) = 8\sqrt{\delta/3}$ for $\delta \in [0,3/64]$, and no disclosure for $\delta > 3/64$. In either market structure, equilibrium disclosure is excessive, as compared to its socially optimal disclosure benchmark.

Proof: See the appendix.

The following proposition shows that, while equilibrium disclosure is excessive from the standpoint of society, consumer surplus is higher with more disclosure. Thus, our model is consistent with consumer advocates, or agencies whose mission is to protect consumer interests, pushing for mandatory disclosure laws. Specifically, we consider a planner who chooses an arbitrary disclosure threshold, $q^D$ or $q^M$. Given the threshold, the firms (or firm) disclose any product whose quality is above the threshold, which determines market perceptions and prices according to (2), (3), and (9). Therefore, we can determine expected consumer surplus as a function of $q^D$ or $q^M$.

Proposition 6 Given the planner’s threshold, $q^D$ or $q^M$, the resulting expected consumer surplus is decreasing in $q^D$ or $q^M$. In other words, more mandated disclosure increases expected consumer surplus.

Proof: See the appendix.

The intuition for our over-disclosure result is based on internal competition (unraveling), and best understood for the case of monopoly. At the threshold chosen by the planner, $q^{**M}$, consumers prefer a lower threshold, by Proposition 6, so the monopolist must be better off ex ante with a higher threshold. Although $q^{**M}$ balances the benefit of more disclosure to consumers and the loss to the monopolist, we see that the monopolist chooses to disclose more in the monopoly equilibrium! It can be readily calculated that the monopolist receives higher ex ante profits by following the planner’s disclosure choice than following its own choice in the monopoly equilibrium. Not only would society be better off by committing to the planner’s disclosure choice, the monopolist would receive higher profits by doing so. The problem is that, conditional on $q_H = q^{**M}$, the monopolist does not take
surprisingly, the socially optimal disclosure policy under monopoly does not involve coordination between the higher and lower qualities. That is, $G^*(q_H)$ does not vary with $q_H$. One might think that the socially optimal cutoff for disclosing the lower quality would depend on the value of the higher quality, and indeed the monopoly equilibrium features such coordination. However, Proposition 5 shows that the planner chooses a product to be disclosed if and only if its quality exceeds $8\sqrt{\delta/3}$. Notice that, although the planner sets a single threshold in either market structure, the threshold for duopoly exceeds the threshold for monopoly. The reason is that the misallocation is larger for duopoly than for monopoly, so the planner is slightly less encouraging of disclosure under duopoly than under monopoly.

Our model does not provide support for a mandatory disclosure policy. In fact, rather than subsidizing disclosure, the following proposition shows that taxing disclosure can be welfare improving. Specifically, we consider the effect on the duopoly equilibrium and monopoly equilibrium of a tax that adds to the cost of disclosure, with the tax revenue being redistributed.

**Proposition 7** Introducing a sufficiently small tax on disclosure improves welfare under duopoly and under monopoly, whenever quality is disclosed with positive probability $[\delta \in (0, 13/64)$ for duopoly and $\delta \in (0, 9/32)$ for monopoly].

Proof: See the appendix.

Our results on excessive disclosure and the benefits of taxation assume that the disclosure cost is sunk. While this result is standard in the literature on disclosure, and is consistent with consumers preferring more disclosure, it merits a discussion of how the model might be modified to generate insufficient disclosure. We now show that the welfare results can be reversed if we reinterpret the disclosure cost. We offer two possible interpretations. The first interpretation relates to the behavioral approach taken by Hirshleifer, Lim, and Teoh (2002). In their model, a fraction of the consumers do not realize that the firm has an opportunity to disclose product quality, unless the

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12 For the duopoly case, a firm does not take into account the negative externality it imposes on the other firm, and on its own lower-quality types. In spite of the positive externality it bestows on consumers, there is too much disclosure in equilibrium.
disclosure occurs. The resource cost of disclosure is zero, but a firm disclosing low quality lowers the market perception of naive consumers. As a result, the equilibrium is characterized by a threshold quality for disclosure, which is a function of the fraction of naive consumers. There is a one-to-one correspondence between our disclosure cost $\delta$, and the fraction of naive consumers that would yield the same disclosure threshold. However, with the new interpretation of $\delta$ as a reduced-form parameter representing bounded rationality, disclosure represents a loss of revenues to the firms but not a cost to society. With this interpretation, we would have underdisclosure in equilibrium. The motivation for mandating or subsidizing disclosure, then, would be to protect naive consumers from their mistakes.

The second interpretation of the disclosure cost is as a payment to individuals or industries outside the model, part of which does not represent a net cost to society. For example, the disclosure cost could represent rents received by the scientific community or advertising firms. Suppose that the social cost of disclosure is given by $\gamma \delta$, where $\gamma \in [0, 1]$ is a parameter representing the portion of the cost that is sunk. Thus, $\gamma = 1$ corresponds exactly to our main model, and $\gamma = 0$ corresponds to the case in which $\delta$ is a transfer from the disclosing firm to other segments of society. With this interpretation, $\gamma$ affects the welfare calculation but not the equilibrium disclosure choices, pricing choices, or purchasing choices. Therefore, expected total surplus under duopoly, given as (28) for our main model, becomes

$$ETS^D = \psi + \frac{59}{216} - \frac{5}{216}(q^*_{D})^3 - 2(1 - q^*_{D})\gamma \delta. \quad (17)$$

Maximizing $ETS^D$ over $q^*_{D}$, we have the socially optimal disclosure threshold:

$$q^*_{D} = \begin{cases} \frac{12}{5} \sqrt{\frac{\gamma \delta}{5}} & \text{for } \gamma \delta \in [0, \frac{5}{144}] \\ 1 & \text{for } \gamma \delta > \frac{5}{144} \end{cases} \quad (18)$$


In Figure 4, the area below the curve indicates the set of parameters for which the duopoly equilibrium exhibits underdisclosure. The kink occurs at $\delta = 13/72$, where we have no disclosure in the duopoly equilibrium. Above the curve to the left of the kink, we have excessive disclosure in the duopoly equilibrium, and above the curve to the right of the kink, the duopoly equilibrium has no disclosure, which is optimal. What strikes us about Figure 4 is that the underdisclosure region is quite small. Overturning the standard result about excessive disclosure seems to require an extreme assumption about the fraction of disclosure costs that represents a net loss to society.\textsuperscript{13}

\section{Conclusion}

Our framework has several advantages over the previous literature on disclosure. Much of that literature fails to treat quality as private information, or only considers the case of monopoly. The remaining papers complicate the comparison of duopoly and monopoly, by assuming that each firm produces one product. Thus, increasing the number of firms changes the market structure, but it

\textsuperscript{13}The monopoly case is similar, but complicated by the fact that disclosure in the monopoly equilibrium cannot be characterized by a single threshold.
also changes the technology by affecting the number of draws from the quality distribution. Putting the duopoly and monopoly models together, while holding the joint distribution of qualities fixed, allows for a sensible welfare comparison and generates fresh insights. We are able to perform a clean analysis of the effect of market structure on equilibrium disclosure and welfare. We identify a second source of competition, internal competition by a firm’s quality realizations, which is an extension of the familiar unraveling argument but has not been pointed out in the literature.

A disadvantage of our framework is that generalizing the model beyond duopoly would be difficult, as would introducing asymmetries in the distribution of consumer preferences over the unit interval. We believe that considering more general distributions of product quality would be tractable and worthwhile. Our independent, uniform assumption allows us to compute solutions and identify regularities that would have been difficult to discover otherwise. However, our goal in future work is to explore the implications of correlated quality across products.

Appendix

Proof of Proposition 2:

We first show that \((q^{*M}, G(\cdot))\), as specified in the statement of the proposition, characterizes a monopoly equilibrium. Given the monopolist’s strategy and given beliefs, it is clear that consumer behavior, as determined by \(x^*\) in (10), is sequentially rational.\(^{14}\) Given the disclosure choices and consumer behavior, the prices given in (11) and (12) are constructed to uniquely satisfy sequential rationality. As for the disclosure choices, the gross profit expression in (13) is increasing in perceived qualities, so it is never rational for the monopolist to disclose \(q_L\) but not \(q_H\).\(^{15}\) Consider the following three remaining possibilities.

1. Neither product is disclosed.

\(^{14}\)Equation (8) is based upon the monopolist choosing prices that serve the entire market and extract all the surplus from the marginal consumer. If the monopolist chooses prices that leave all consumers with positive surplus, or prices that leave some consumers strictly preferring not to purchase, the unit interval should be partitioned in the obvious way to ensure consumer rationality off the equilibrium path.

\(^{15}\)Without Definition 1, part (iv), the monopolist might benefit by disclosing the lower quality, if that could increase the market perception of the undisclosed product. However, part (iv) and the fact that \(G\) is increasing implies that the perceived quality of the undisclosed product is increasing in the quality of the disclosed product.
By (13), the monopoly profit, as a function of the two qualities, is given by

$$\pi_1(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}q^*M. \quad (19)$$

2. Only $q_H$ is disclosed.

By (13), the monopoly profit is given by

$$\pi_2(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}(q_H + G(q_H)/2) + \frac{1}{8}(q_H - G(q_H)/2)^2 - \delta. \quad (20)$$

3. Both products are disclosed.

By (13), the monopoly profit is given by

$$\pi_3(q_H, q_L) = \psi - \frac{1}{2} + \frac{1}{2}(q_H + q_L) + \frac{1}{8}(q_H - q_L)^2 - 2\delta. \quad (21)$$

The threshold, $q^{*M} = 4\sqrt{1 + 2\delta} - 4$, guarantees that, when $q_H = q^{*M}$ holds, the monopolist is indifferent between disclosing neither product and disclosing $q_H$. To see this, substitute $q_H = G(q_H) = q^{*M}$ into (20) and (19), equate $\pi_2$ with $\pi_1$, and solve for $q^{*M}$. If $\delta < \frac{9}{32}$ holds, then we have an interior threshold with $q^{*M} < 1$. If $\delta \geq \frac{9}{32}$ holds, then the monopolist always prefers not to disclose, and $q^{*M} = 1$ ensures that nothing is disclosed. Because $\pi_2$ is strictly increasing in $q_H$, the monopolist strictly prefers not to disclose anything when $q_H$ is below the threshold, and strictly prefers to disclose $q_H$ (and possibly $q_L$ as well) when $q_H$ is above the threshold.

It remains to show that the decision whether to disclose $q_L$, based on $G(\cdot)$, is also consistent with sequential rationality. Suppose $\delta < \frac{7}{32}$ holds. If $q_H > q^{*M}$ is disclosed, and we have $\hat{q} \leq q_H \leq 1$ and

$$q_L = \frac{2q_H - 4}{3} + \frac{2}{3}(q_H)^2 - 4q_H + 4 + 24\delta,$$

then the monopolist is indifferent between disclosing $q_L$ and not disclosing it. To see this, substitute $q_L = G(q_H)$ in (21), equate $\pi_3$ with $\pi_2$, and solve for $G(q_H)$. We specified $\hat{q}$ to solve

$$q_H = \frac{2q_H - 4}{3} + \frac{2}{3}(q_H)^2 - 4q_H + 4 + 24\delta,$$

so $\hat{q} \leq q_H \leq 1$ implies there is a range of $q_L$ that will be disclosed. The condition, $\delta < \frac{7}{32}$, guarantees that $\hat{q} < 1$ holds, so the interval, $\hat{q} \leq q_H \leq 1$, is well defined and nondegenerate. It is easily verified that $\pi_3$ is strictly increasing in $q_L$, so the monopolist strictly prefers not to disclose $q_L$ when $q_L < G(q_H)$ holds, and strictly prefers to disclose $q_L$ when $q_L > G(q_H)$ holds.
If $q_H > q^*_M$ is disclosed, and we have $q^*_M \leq q_H \leq \hat{q}$, then the monopolist strictly prefers not to disclose $q_L$ for all $q_L \leq q_H$, so not disclosing $q_L$ is sequentially rational.

Now suppose $\delta \geq \frac{7}{32}$ holds. Since $\hat{q} = 4 - 4\sqrt{1 - 2\delta} \geq 1$ must hold, then for all $q_H \in [q^*_M, 1]$, monopolist strictly prefers not to disclose $q_L$ for all $q_L \leq q_H$. Again, not disclosing $q_L$ is sequentially rational. This establishes that the candidate is a monopoly equilibrium.

To show that there is a unique PBE consistent with Definition 1, suppose that $(q^*_M, G(\cdot))$ is different from that specified in Proposition 2. We first show that $G(q^*_M) < q^*_M$ is impossible, so as $q_H$ crosses the threshold, disclosing both qualities cannot be optimal. If $G(q^*_M) < q^*_M$ were to hold, then if the quality realizations are given by $q_H = q^*_M$ and $q_L = G(q^*_M)$, the monopolist must be indifferent between disclosing nothing and disclosing both qualities. From (19) and (21), we have

$$2\delta = \frac{q_L}{2} + \frac{1}{8}(q_H - q_L)^2. \quad (22)$$

For this to be consistent with equilibrium, the monopolist cannot be better off disclosing only $q_H$. From (19) and (20), we have

$$\delta \geq \frac{q_L}{4} + \frac{1}{8}(q_H - q_L)^2. \quad (23)$$

By using (23) to substitute for $\delta$ in (22), we derive

$$q_H(1 - \sqrt{2}) \geq q_L(1 - \frac{\sqrt{2}}{2}),$$

which is clearly impossible. Since disclosing both qualities cannot be optimal as $q_H$ crosses the threshold, $q^*_M$ must be such that the monopolist is indifferent between disclosing neither product and disclosing $q_H$ when $q_H = q^*_M$ holds and beliefs are that the undisclosed quality is uniformly distributed over $[0, q^*_M]$. From the argument above equating $\pi_2$ with $\pi_1$, $q^*_M = 4\sqrt{1 + 2\delta} - 4$ is the unique threshold consistent with a monopoly equilibrium. Similarly, $G(q_H)$ is uniquely determined by either indifference between disclosing $q_L$ and not disclosing it (when $\hat{q} \leq q_H \leq 1$ holds), or strict preference not to disclose $q_L$ (when $q^*_M \leq q_H \leq \hat{q}$ holds). Thus, sequentially rational disclosure choices are uniquely determined. As argued above, given the disclosure choices, beliefs are uniquely determined, and sequentially rational pricing decisions are uniquely determined, given those beliefs.

\[16\] Without our convention, $G(q_H) \leq q_H$, there would be many $G(\cdot)$ functions consistent with a monopoly equilibrium, since the behavior of $G(\cdot)$ above the 45 degree line is arbitrary. However, all such functions lead to the same disclosure choices.
Finally, consumer behavior is uniquely determined as well.

Notation for Welfare Computations.

Under monopoly, the expression for welfare is extremely messy. To save on notation, we define the following:

\[ A = \sqrt{5 - 2\delta - 4\sqrt{1 - 2\delta}}, \quad B = \sqrt{1 - 2\delta}, \]
\[ C = \sqrt{1 + 2\delta}, \quad D = \sqrt{1 + 24\delta}. \quad (24) \]

Proof of Proposition 3:

1. Duopoly Case

For \( \delta \in [0, 13/72] \), the probabilities of zero, one, and two products being disclosed are, respectively, \( (q^*D)^2, 2q^*D(1 - q^*D), \) and \( (1 - q^*D)^2 \), where \( q^*D = -\frac{5}{3} + \frac{1}{3}\sqrt{25 + 216\delta} \).

The expected number of disclosed products under duopoly, \( \Delta^D \), is therefore given by
\[ \Delta^D = \frac{16}{3} - \frac{2}{3}\sqrt{25 + 216\delta}. \quad (25) \]

For \( \delta \geq 13/72 \), we have no disclosure with probability one, and \( \Delta^D = 0 \).

2. Monopoly Case

For \( \delta \in [0, 7/32] \), the probabilities of zero, one, and two products being disclosed are, respectively,

\[ (q^*M)^2 = (4\sqrt{1 + 2\delta} - 4)^2, \]
\[ 2 \int_{q^*M}^{1} \int_{0}^{G(q_H)} dq_L dq_H = \frac{1}{3} [-38 - 128\delta - 8A + 16AB + 96C - 2D] + 16\delta \ln \left( \frac{D - 1}{2(1 + A - 2B)} \right), \]
\[ 2 \int_{q^*M}^{1} \int_{G(q_H)}^{q^H} dq_L dq_H = \frac{1}{3} [-55 + 32\delta + 8A + 64B - 16AB] - 16\delta \frac{D - 1}{2(1 + A - 2B)}. \]

The expected number of disclosed products, \( \Delta^M \), can be computed, as
\[ \Delta^M = \frac{1}{3} [-148 - 64\delta + 8A + 64B - 16AB + 96C + 2D] - 16\delta \ln \left( \frac{D - 1}{2(1 + A - 2B)} \right). \]

For \( \delta \in (7/32, 9/32] \), the monopolist never discloses the lower quality. The probabilities of zero and one product being disclosed are, respectively, \( (4\sqrt{1 + 2\delta} - 4)^2 \), and \( 1 - (4\sqrt{1 + 2\delta} - 4)^2 \).
Therefore, we have $\Delta^M = 1 - (-4 + 4\sqrt{1+2\delta})^2$. For $\delta > 9/32$, there is no disclosure, and we have $\Delta^M = 0$.

The comparison shows that $\Delta^D > \Delta^M$ holds for all $\delta \in (0, 9/32)$, which is illustrated in Figure 2. ■

Proof of Proposition 4:
We first derive the expected total surplus functions under both market structures.

1. Duopoly Case

Following the equilibrium characterization in Proposition 1, we consider four regions in the space of quality realizations:

(a) $q_0, q_1 < q^{*D}$, neither quality is disclosed.

By (4), in this case the marginal consumer type, who is indifferent between product 0 and product 1, is given by $x^* = \frac{1}{2}$. For a type-$x$ consumer, the surplus from purchasing product 0 is given by $\psi + q_0 - x$, and similarly, the surplus from purchasing product 1 is given by $\psi + q_1 - (1 - x)$. (The price paid is a transfer of surplus from consumers to firms.) Integrating over all consumers, we have the total surplus:

$$ts_1(q_0, q_1) = \int_0^{1/2} (\psi + q_0 - x) \, dx + \int_0^{1/2} (\psi + q_1 - x) \, dx$$

Integrating $ts_1(q_0, q_1)$ over the set of qualities with $q_0, q_1 < q^{*D}$, we have the contribution of this region to expected surplus:

$$ETS_1 = \int_0^{q^{*D}} \int_0^{q^{*D}} ts_1(q_0, q_1) \, dq_1 \, dq_0.$$

(b) $q_0 > q^{*D} > q_1$. Only $q_0$ is disclosed.

By (4), the marginal consumer type is given by

$$x^* = \frac{1}{2} + \frac{1}{6}(q_0 - q^{*D}/2). \quad (26)$$

Given $(q_0, q_1)$, the total surplus generated is given by

$$ts_2(q_0, q_1) = \int_0^{x^*} (\psi + q_0 - x) \, dx + \int_0^{x^*} (\psi + q_1 - x) \, dx - \delta.$$
where $x^*$ is given by (26). Integrating again to get the contribution of this region to expected total surplus, we have

$$ETS_2 = \int_{q^D}^{1} \int_{0}^{q^D} ts_2(q_0, q_1) \, dq_1 \, dq_0.$$  

(c) $q_1 > q^D > q_0$, so $q_1$ is disclosed but $q_0$ is not.

By symmetry, we have $ETS_3 = ETS_2$.

(d) $q_0, q_1 > q^D$. Both $q_0$ and $q_1$ are disclosed.

By (4), the marginal consumer type is given by

$$x^* = \frac{1}{2} + \frac{1}{6}(q_0 - q_1). \quad (27)$$

Given $(q_0, q_1)$, the total surplus generated is given by

$$ts_4(q_0, q_1) = \int_{0}^{x^*} (\psi + q_0 - x) \, dx + \int_{0}^{x^*} (\psi + q_1 - x) \, dx - 2\delta,$$

where $x^*$ is given by (27). Integrating again to get the expected total surplus in this region, we have

$$ETS_4 = \int_{q^D}^{1} \int_{q^D}^{1} ts_2(q_0, q_1) \, dq_1 \, dq_0.$$  

Putting all these expressions together, the expected total surplus in equilibrium is given by:

$$ETS^D = ETS_1 + 2ETS_2 + ETS_4 = \psi + \frac{59}{216} - \frac{5}{216}(q^D)^3 - 2(1 - q^D)\delta. \quad (28)$$

Substituting the value of $q^D$, we have for $\delta < 13/72$,

$$ETS^D = \psi + \frac{4093}{5832} - \frac{23}{9}\delta - \left(\frac{125}{1458} - \frac{13}{27}\delta\right)\sqrt{25 + 216\delta}. \quad (29)$$

For $\delta \geq 13/72$, we have $q^D = 1$, and hence $ETS^D = \psi + 1/4$.

2. Monopoly Case

Following the equilibrium characterization in Proposition 2, we consider three regions in the space of quality realizations:

(a) $q_L \leq q_H < q^M$, neither quality is disclosed.
By (10), the market share is given by \( x^* = 1/2 \), and the total surplus given \((q_L, q_H)\) is
\[
ts_1(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x) \, dx + \int_0^{1-x^*} (\psi + q_L - x) \, dx.
\]
Integrating over this region, we have the contribution to expected total surplus, given by
\[
ETS_1 = 2 \int_0^{q^*_M} \int_0^{q_H} ts_1(q_H, q_L) \, dq_L \, dq_H. \tag{30}
\]

(b) \( q_H > q^{*M} \) but \( q_L < G(q_H) \) hold, so only \( q_H \) is disclosed.

By (10), the market share is given by \( x^* = 1/2 + (q_H - G(q_H))/4 \), and the total surplus given \((q_L, q_H)\) is
\[
ts_2(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x) \, dx + \int_0^{1-x^*} (\psi + q_L - x) \, dx - \delta.
\]
Integrating over this region, we have the contribution to expected total surplus, given by
\[
ETS_2 = 2 \int_{q^*_M}^{1} \int_0^{G(q_H)} ts_2(q_H, q_L) \, dq_L \, dq_H. \tag{31}
\]

(c) \( q_H > q^{*M} \) and \( q_L > G(q_H) \) hold, so both \( q_H \) and \( q_L \) are disclosed.

By (10), the market share is given by \( x^* = 1/2 + (q_H - q_L)/4 \), and the total surplus given \((q_L, q_H)\) is
\[
ts_3(q_H, q_L) = \int_0^{x^*} (\psi + q_H - x) \, dx + \int_0^{1-x^*} (\psi + q_L - x) \, dx - 2\delta.
\]
Integrating over this region, we have the contribution to expected total surplus, given by
\[
ETS_3 = 2 \int_{q^*_M}^{1} \int_{G(q_H)}^{q_H} ts_3(q_H, q_L) \, dq_L \, dq_H. \tag{32}
\]

The expected total surplus, \( ETS^M \), is given by \( ETS^M = ETS_1 + ETS_2 + ETS_3 \).

Substituting the values of \( q^{*M} \) and the appropriate expression for \( G(q_H) \) into (30), (31) and (32), we can obtain the expected total surplus under monopoly.

For \( \delta \in [0, 7/32] \), we have
\[
ETS^M = \psi - \frac{49813}{864} - \frac{673}{27} \delta - \frac{320}{27} + 16 \ln 2) \delta^2 + \frac{-56 + 160\delta}{27} AB + \left( \frac{52}{27} - \frac{16}{3} \delta \right) A
\]
\[
+ \left( \frac{272 - 896\delta}{27} \right) B + 16(3 - \delta) C + \left( \frac{1}{108} - \frac{4}{9} \delta \right) D + 16\delta^2 \ln \left[ \frac{D - 1}{1 + A - 2B} \right].
\]
For $\delta \in (7/32, 9/32]$, we have
\[
ETS^M = \psi - \frac{6109}{128} - 65\delta + 8\delta^2 + 16(3 + \delta)\sqrt{1 + 2\delta}.
\]
For $\delta > 9/32$, we have
\[
ETS^M = \psi + \frac{1}{4}.
\]

Straightforward (but tedious) algebra shows that $ETS^D - ETS^M$ is strictly increasing in $\delta$ over $\delta \in [0, 13/72]$. Since $ETS^D - ETS^M$ is negative at $\delta = 0$ and positive at $\delta = 13/72$, by the mean value theorem, there exists a unique $\delta^* \in (0, 13/72)$ such that $ETS^D = ETS^M$ holds. It is also easily verified that we have $ETS^D - ETS^M > 0$ for $\delta \in [13/72, 9/32)$. For $\delta \geq 13/72$, we have $ETS^D = ETS^M = \psi + 1/4$. \[\blacksquare\]

**Proof of Proposition 5:**

1. **Duopoly Case**

The social planner announces and enforces the disclosure policy characterized by $q^*$. By (28) the expected total surplus is given by
\[
ETS = \psi + \frac{59}{216} - 2\delta + 2\delta q^* - \frac{5}{216}q^*^3.
\]
Therefore the socially optimal cutoff is given by
\[
q^{**D} = \begin{cases} 
12\sqrt{\delta/5} & \text{for } \delta \in [0, \frac{5}{144}], \\
1 & \text{for } \delta \geq \frac{5}{144}.
\end{cases}
\]

It is easy to see that we have $q^{**D} > q^{*D}$ for all $\delta \in (0, 13/72)$ and $q^{**D} = q^{*D}$ for $\delta = 0$ and $\delta > 13/72$. Therefore, in the duopoly equilibrium, there is too much disclosure compared to the socially optimal disclosure.

2. **Monopoly Case**

The social planner announces and enforces the disclosure policy characterized by $(q^*, G(\cdot))$. Let $(\tilde{q}_H, \tilde{q}_L)$ be the market perceptions about $(q_H, q_L)$ induced by the monopolist’s disclosure.
announcement. Also let \( x^* \) denote the consumer type that is indifferent between purchasing the higher quality product and the lower quality product. Then we have

\[
x^* = \frac{1}{2} + \frac{1}{4}(\bar{q}_H - \bar{q}_L)
\]

Following the same steps as in the proof of Proposition 4, we can write the expected total surplus, given the disclosure rule \((q^*, G(\cdot))\), as follows.

\[
ETS = 2 \int_0^{q^*} \int_0^{q_H} (ts_1(q_H, q_L) dq_L, dq_H) + 2 \int_{q^*}^1 \int_0^{G(q_H)} ts_2(q_H, q_L) dq_L + \int_{q^*}^{q_H} ts_3(q_H, q_L) dq_L \overset{=\Omega(G)}{=} \Omega(G)
\]

where \( ts_1, ts_2 \) and \( ts_3 \) are all given in the proof of Proposition 4 for the monopoly case.

The optimal \( G(\cdot) \) maximizes \( \Omega(G) \) for any given \( q_H \). Differentiating yields

\[
G^{**}(q_H) = \begin{cases} q_H & \text{for } q_H < 8\sqrt{\delta/3} \\ 8\sqrt{\delta/3} & \text{for } q_H \geq 8\sqrt{\delta/3} \end{cases}
\]

Substituting \( G(\cdot) = G^{**}(\cdot) \) into (33), and then differentiating with respect to \( q^* \), we have

\[
\frac{dETS}{dq^*} = -\frac{3}{32}(q^*)^3 + 2\delta q^*.
\]

The optimal \( q^* \) is thus given by

\[
q^{**M} = \begin{cases} 8\sqrt{\delta/3} & \text{for } \delta \in [0, 3/64] \\ 1 & \text{for } \delta > 3/64. \end{cases}
\]

Combining (34) and (35), the social planner’s disclosure policy is given below:

\[
q^{**M} = \begin{cases} 8\sqrt{\delta/3} & \text{if } \delta \in [0, 3/64] \\ 1 & \text{if } \delta > 3/64 \end{cases}
\]

and \( G^{**}(q_H) = 8\sqrt{\delta/3} \) for \( q_H \in (q^{**M}, 1] \).

Since \( G^{**}(q_H) \) does not vary with \( q_H \), in effect the socially planner sets a uniform threshold for both quality disclosures. In other words, the potential coordination instrument provided by \( G(\cdot) \) does not help to improve social welfare.
We have $q^{*M} > q^M$ for all $\delta \in (0, 9/32)$ and $q^{*M} = q^M$ for $\delta = 0$ and $\delta > 9/32$. It can also be verified that $G^{**}(q_H) \geq G(q_H)$ holds for any $q_H$. Therefore, in the monopoly equilibrium, there is too much disclosure as compared to the socially optimal disclosure.

Proof of Proposition 6:

Under duopoly, given $q^D$, a product with quality $q_i < q^D$ has market perception $\bar{q}_i = q^D/2$. Equations (2) and (3) determine prices for each pair of quality realizations, and (4) determines which consumers buy which product. This allows us to write consumer surplus as a function of the realized qualities. Integrating over the quality distribution, we have expected consumer surplus,

$$ECS^D = \psi - \frac{1}{216}(q^D)^3 - \frac{1}{161},$$

from which the result follows.

Under monopoly, given $q^M$, a product with quality $q_i < q^M$ has market perception $\bar{q}_i = q^M/2$. Equation (9) determines prices for each pair of quality realizations, and (10) determines which consumers buy which product. This allows us to write consumer surplus as a function of the realized qualities. Integrating over the quality distribution, we have expected consumer surplus,

$$ECS^M = -\frac{1}{96}(q^M)^3 + \frac{25}{96},$$

from which the result follows.

Proof of Proposition 7:

Under duopoly, suppose a tax $t$ is imposed for each product whose quality is disclosed. Then by (29) and (25), the expected total surplus for $\delta \in (0, 13/72)$ is given by:

$$ETS^D(\delta + t) = \psi + \frac{4093}{5832} - \frac{23}{9}(\delta + t) - \left(\frac{125}{1458} - \frac{13}{27}(\delta + t)\right) \cdot \sqrt{25 + 216(\delta + t)} + E N^D(\delta + t) \cdot t$$

$$= \psi + \frac{4093}{5832} - \frac{23}{9} \delta + \frac{25}{9} t + \left[-\frac{125}{1458} + \frac{13}{27} \delta - \frac{5}{27} t\right] \sqrt{25 + 216(\delta + t)}$$

Differentiating with respect to $t$ and evaluating at $t = 0$, we have

$$\frac{dETS^D}{dt}|_{t=0} = -\frac{125 + 25\sqrt{25 + 216\delta} + 108\delta}{9\sqrt{25 + 216\delta}} > 0$$

17 The expressions in each of the four regions of quality space are omitted to save on space.
for all $\delta \in (0, 13/72)$, which means that introducing a small tax can always improve welfare.

For the monopoly case, the computation is tedious but the result remains the same. We omit the details. ■
References


