Ch. 7 Specification: Functional Forms

- **Linear Form**: linear in variables $Y$ and $X$

$$Y = \beta_0 + \beta_1 X_1 + \cdots + \beta_k X_k + \epsilon.$$  

1) The slope of $X$ on $Y$ is a constant, i.e.,

$$\frac{\partial Y}{\partial X_j} = \frac{\Delta Y}{\Delta X_j} = \beta_j.$$  

2) The elasticity of $Y$ with respect to $X$ (the percentage change of $Y$ caused by a 1 percent increase in $X$) is not constant,

$$\frac{\partial \ln Y}{\partial \ln X_j} = \frac{(\Delta Y/Y)}{(\Delta X_j/X_j)} = \beta_j \frac{X_j}{Y}.$$  

--- the linear functional form of $Y$ and $X$ should be used if the slope of the relationship between $Y$ and $X$ is expected to be constant.

- **Double-Log Form**:

$$\ln Y = \beta_0 + \beta_1 \ln X_1 + \cdots + \beta_k \ln X_k + \epsilon.$$  

1) The elasticities of the model are constant, but the slopes are not.

2) It is applicable only if there are no negative or zero observations in the data $Y$ and $X$.

3) The relation between $Y$ and $X$s is

$$Y = e^{\beta_0} X_1^{\beta_1} \cdots X_k^{\beta_k} e^\epsilon.$$  

(Figure 7.2 here)

- **Semilog Form**: Some variables are in logs and some are not

1) $Y = \beta_0 + \beta_1 \ln X_1 + \beta_2 X_2 + \epsilon$

   — The relationship between $X_1$ and $Y$ is hypothesized to have the ‘increasing at a decreasing rate’ form (if $\beta_1 > 0$).

   a) Economic example: Engel Curves - relations between consumption expenditure and income.

2) $\ln Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \epsilon.$

   — The dependent variable $Y$ is adjusted in percentage terms to a unit change in $X$.

   a) Economic example: salary of an individual worker $Y$ may be raised in percentage term with $X$ being his/her experience.

- **Polynomial Form**: some variables are raised to powers (other than one)

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \epsilon.$$
1) The slope of $X$ on $Y$ changes as $X$ changes, i.e.,
\[ \frac{dY}{dX} = \frac{\Delta Y}{\Delta X} = \beta_1 + 2\beta_2 X. \]

2) Economic example: (convex) cost function, $Y$ being the average cost of production and $X$ being the level of output; $\beta_1 < 0$ but $\beta_2 > 0$.

(Figure 7.4 here)

3) Economic example: (concave) earning profile, $Y$ being earning and $X$ being age; $\beta_1 > 0$ but $\beta_2 < 0$.

- **Inverse Form**: $Y$ is a function of the reciprocal (inverse) of $X$, i.e.,
\[ Y = \beta_0 + \beta_1 \frac{1}{X} + \epsilon. \]

→ the impact of $X$ is expected to approach zero as $X$ increases to infinity.

(Figure 7.5 here)

1) \[ \frac{dY}{dX} = \frac{\Delta Y}{\Delta X} = -\frac{\beta_1}{X^2}. \]

2) Economic example: Phillips curve – a nonlinear relationship between the rate of unemployment and the percentage change in wages
\[ W = \beta_0 + \beta_1 \frac{1}{U} + \epsilon \]

where $W$ is the percentage change in wages and $U$ is the rate of unemployment.

- Using Dummy Variables: a dummy variable takes on the values of 0 and 1.

1) **Intercept Dummy**
\[ Y = \beta_0 + \beta_1 X + \beta_2 D + \epsilon, \]
where $D$ is a dummy variable.

→ the intercept dummy (here, $D$) changes the intercept but the slopes (of $X$) remain constant.

(Figure 7.7 here)

2) **Slope Dummy Variables**
\[ Y = \beta_0 + \beta_1 X + \beta_2 D + \beta_3 X D + \epsilon. \]

a) The slope dummy variable $D$ allows the slope of $X$ on $Y$ to be different depending on whether the condition specified by the dummy variable $D$ is met.

b) There really are two equations:
\[ Y = \beta_0 + \beta_1 X + \epsilon, \quad \text{when } D = 0 \]
\[ Y = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X + \epsilon \quad \text{when } D = 1. \]

(Figure 7.8 here)
c) The slope of $Y$ with respect to $X$ changes if $D$ changes:

When $D = 0$, \[ \frac{dY}{dX} = \frac{\Delta Y}{\Delta X} = \beta_1 \]

When $D = 1$, \[ \frac{dY}{dX} = \frac{\Delta Y}{\Delta X} = (\beta_1 + \beta_3). \]

d) Economic example: (structural change) consumption function estimated over a time period that includes a major war.

e) Examples in labor economics: use as a special kind of interaction terms, e.g., interaction of gender and working experience in earnings.