A Theory of Exchange Rates and 
The Term Structure of Interest Rates

by

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A THEORY OF EXCHANGE RATES AND
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Abstract

This purpose of this paper is to construct a model of the exchange rate
determination which is consistent with stylized facts regarding the
uncovered interest parity for short-term and long-term interest rates. This
task is challenging especially because of the forward premium anomaly found
for short-term interest rates and forward exchange rates. With an
assumption that the investors have a short investment horizon, the model is
consistent with these stylized facts even when the degree of risk aversion
is small. The model predicts a complicated relationship between exchange
rates and the term structure of interest rates.
1. Introduction

This purpose of this paper is to construct a model of the exchange rate
determination which is consistent with stylized facts for short-term and
long-term interest rates. This task is challenging especially because of
the forward premium anomaly found for short-term interest rates and forward
exchange rates.

For short-term interest rates and forward exchange rates, the uncovered
interest parity is typically rejected (see, e.g., Hodrick (1988) and Engel
(1997) for recent surveys). As Engel (1997) emphasizes, one form of the
rejection found in many recent papers is that regressions of the future
depreciation on the current forward premium (which is equal to the short-
term interest rate differential under the covered interest parity) yield
negative estimates of the slope coefficient. This is called the forward
premium anomaly (also see Backus, Foresi, and Telmer (1998) for a recent
discussion). It is difficult to find an economic explanation for the
forward premium anomaly because standard consumption-based asset pricing
model with risk averse investors cannot explain it (see, e.g., Mark and Wu
asset markets which can be consistent with the forward premium anomaly. The
model in the present paper gives an alternative explanation which is not
based on transactions costs. McCallum (1994) and Meredith and Chin (1998)
provide an explanation for the forward premium anomaly based on policy
reactions. Our explanation is complementary to theirs because their models
require an error term which is correlated with the current interest rate
differential in order for the exchange rate to deviate from the level
implied the uncovered interest rate parity.
For long-term bonds, more favorable evidence for the uncovered interest
parity has been found. Direct evidence is found in recent papers by
Meredith and Chinn (1988) and Alexius (1998, 1999). Meredith and Chinn
(1988) and Alexius (1998) find that regressions of the future depreciation
over a long-horizon on the current long-term interest rate differential
typically yield significantly positive estimates of the slope coefficient.
Alexius (1999) finds similar results for returns on long-term bonds over
short investment horizons.

Indirect evidence has been found in the form of more favorable evidence
for standard exchange rate models which assume the uncovered interest parity
and the long-run purchasing power parity (see, e.g., Meese and Rogoff
(1988), Edison and Pauls (1993) and Baxter (1994)) with long-term interest
rate differentials than with short-term interest rate differentials. Similarly,
implications of standard exchange rate models hold better in
long-horizon data than in short-horizon data (see, e.g., Mark (1995)).

An economic model can be consistent with these stylized facts if short-
term domestic and foreign bonds are complements, but long-term domestic and
foreign bonds are substitutes. This paper constructs such a model. Because
the short-term and long-term interest rates tend to move together, a rise in
the domestic short-term interest rate still tends to cause an appreciation
of the domestic currency in the model. However, when a rise in the short-
term domestic interest rate is not associated with a rise in the long-term
interest rate, the complementarity of the short-term domestic and foreign
bonds implies that the rise causes a depreciation of the domestic currency.

The intuition behind this result arises from Ogaki's (1990) concept of
indirect complementarity, which relates to the risk structure of interest
rates under the assumption of risk aversion. In the model, the investors are assumed to have a short investment horizon. This short investment horizon assumption is the key element of the model. Given that many professional traders who actively trade in exchange markets are likely to be assessed by their short-horizon performances by their employers, this assumption is justifiable.

If domestic short-term interest rates unexpectedly rise, holders of domestic long-term bonds suffer a capital loss. If the domestic currency appreciates as a result of this increase, then holders of foreign bonds also suffer a capital loss. Therefore, as long as an increase in short-term interest rates is associated with an appreciation of the domestic currency, risk averse agents have an incentive to avoid holding both domestic long-term bonds and foreign bonds. Hence these two assets are likely to be strong substitutes. Since domestic short-term and long-term bonds are also likely to be strong substitutes, and since a substitute of a substitute is an indirect complement (see Ogaki (1990)), domestic short-term bonds and foreign bonds must be strong indirect complements.

For example, suppose that the domestic short-term interest rate rises. This will have two effects, each of which will shift the demand for foreign bonds in the opposite directions. The first effect, which arises from the direct substitution between domestic short-term bonds and foreign bonds, reduces the demand for foreign bonds. The second effect, caused by the indirect complementarity, increases the demand for foreign bonds. This is because the strong substitutability between domestic short-term and long-term bonds implies that the demand for domestic long-term bonds decreases. Since domestic long-term bonds and foreign bonds are strong substitutes,
this decrease causes an indirect effect which increases the demand for foreign bonds.\footnote{This intuitive argument is true even when there are more than three goods (or assets); a substitute of a substitute is always an indirect complement (see Ogaki (1990)).} Whether domestic short-term bonds and foreign bonds are substitutes or complements depends on which of these effects is stronger.

When the indirect effect dominates the direct effect, then domestic short-term bonds and foreign bonds are complements. In this case, if the short-term interest rate rises while the long-term interest rate stays the same, then demand for foreign bonds increases if the exchange rate did not move. Because the supply for foreign bonds is essentially fixed in the short-run by the cumulative current balance in the model, this causes the foreign currency to appreciate now, creating expected future depreciation of the currency. When the indirect effect does not dominate the direct effect, then domestic short-term bonds and foreign bonds are substitutes. In this case, if the short-term interest rate rises while the long-term interest rate stays the same, then the foreign currency depreciates as in standard models of exchange rates. However, as long as the indirect effect is quantitatively important, foreign bonds and domestic long-term bonds are stronger substitutes than foreign and domestic short-term bonds. This implies that the long-term interest rate differential is a more important factor than the short-term interest rate differential in determining the exchange rate.

In order to assess the importance of this indirect complementarity, this paper derives a demand function for foreign bonds endogenously by solving a partial equilibrium model of exchange rate determination for a rational expectations equilibrium. The solution shows that this indirect
complementarity is likely to be quantitatively important compared with the

direct substitutability of domestic short-term bonds and foreign bonds. It

shows that the indirect effect can even dominate the direct effect under

reasonable parameter configurations.

In the model, indirect complementarity is determined by the covariance

of the exchange rate and the short-term interest rate (conditional on the

available information). The greater the appreciation of the domestic

currency caused by an increase in the domestic short-term interest rate, the

stronger the substitutability of domestic long-term bonds and foreign bonds.

At the same time, the demand for foreign bonds affects the dynamics of the

exchange rate and this covariance. Demand for foreign bonds is endogenously

derived by solving for the rational expectation of the covariance, so that

the covariance assumed by agents is consistent with the covariance implied

by the demand function.

It is technically difficult to solve for the rational expectations of

the covariance in complicated asset pricing models. For this reason, this

paper adopts a simple partial equilibrium exchange rate model, which is a

three asset version of the models of Driskill and McCafferty (1980) and

Fukao (1983) which use two assets (domestic and foreign short-term bonds) and

two countries. These authors derive the demand function for foreign

bonds endogenously by applying rational expectations to the variance of the

exchange rate.

The result in this paper is in sharp contrast to the conventional view

that short-term capital is more internationally mobile than long-term

capital. The 1960’s Operation Twist, in which the Federal Reserve and the

Treasury attempted to raise short-term interest rates relative to long-term
interest rates, was evidently based on the view. However, empirical work of Fukao and Okubo (1984) suggests that the relationship between domestic long-term interest rates and foreign interest rates is not limited to that which exists as a result of the relationship between domestic short-term interest rates and foreign interest rates. Popper (1989) presents empirical evidence that long-term capital is as internationally mobile as short-term capital.

The model in this paper has policy implications. It implies that the effectiveness of central bank attempts to affect exchange rates through the control of short-term interest rates depends on the responsiveness of long-term interest rates to changes in short-term interest rates.

The rest of the paper is organized as follows. Section II presents the model. Section III derives the rational expectations equilibrium. First, given the covariance and variance assumed by agents, the rational expectation of the mean of the exchange rate is used to solve for the law of motion for the exchange rate. Then the condition for the rational expectation for the covariance is derived. Finally, the unique stable rational expectations equilibrium is found by equating the variance assumed by agents with the variance implied by the demand function. Conclusions are given in the last section.

II. The Model

Consider a partial equilibrium model of exchange rate determination. For simplicity, the price level is assumed to be constant. Alternatively, all variables can be considered to be measured in real terms. Investors are assumed to live for two periods, and the same number of investors are assumed to be born every period. Let $B_{S,t}$ and $B_{L,t}$ denote domestic short-
term and long-term bonds, respectively. The foreign interest rate will be assumed to be constant. Therefore, foreign short-term and long-term bonds will be perfect substitutes, and do not have to be distinguished. Let $B_{f,z}$ denote the foreign bonds.

The long-term and short-term bonds are discount bonds which will pay one unit of the domestic currency after two periods and one period, respectively. Let $q_t$ be the price of long-term bonds during period $t$ and $r_t$ be the domestic short-term interest rate. Then the rate of return on holding long-term bonds for one period is

$$ r_{1,t} = \frac{1}{1 + r_{t+1}} \cdot \frac{q_{t+1}}{q_t}.$$

Let $R_t$ denote the of the long-term interest rate:

$$ (2) \quad q_t = 1/(1+R_t)^2.$$

From (1) and (2), we obtain

$$ (3) \quad r_{L,t} = (1 + R_t)^2/(1 + r_{t+1}) - 1 \equiv 2 R_t - r_{t+1}. $$

Define the risk premium for long-term bonds, $\rho_{L,t}$, to be the difference between the expected rate of return for long-term bonds and that for short-term bonds:

$$ (4) \quad \rho_{L,t} = E_t(r_{L,t}) \cdot r_t = 2 [R_t - \frac{1}{2} \{r_t + E_t(r_{t+1})\}], $$

where $E_t$ is the expectation operator conditional on the information set in period $t$, $\Omega_t$. We assume that $\Omega_t$ includes the current and past values of $r_t$, $K_t$, $r_{t}^{*}$, and $s$, where $r_{t}^{*}$ is the foreign short-term interest rate and $s_t$ is the natural log of the exchange rate expressed in terms of the domestic currency.

Let $r_{t}^{*}$ be the foreign short-term interest rate. Then the one period
holding rate of return on foreign bonds in terms of the domestic currency is

\[ r_{F,t} = r_t^* + s_{t+1} - s_t. \]

Let the risk premium for the foreign bonds, \( \rho_{F,t} \), be defined as the difference between the expected return on foreign bonds and the domestic short-term interest rate:

\[ \rho_{F,t} = r_t^* + E(s_{t+1}) - s_t - r_t. \]

The representative investor in period \( t \) maximizes his expected utility

\[ E_t(w_{t+1}) \]

subject to the budget constraint,

\[ B^d_{S,t} + B^d_{L,t} + B^d_{F,t} = w_t. \]

The superscript \( d \) denotes demand, \( w_t \) is the value of his assets in the beginning of the period \( t \), and \( w_{t+1} \) satisfies

\[ w_{t+1} = B^d_{S,t} (1 + r_t) + B^d_{L,t} (1 + r_{L,t}) + B^d_{F,t} (1 + r_{F,t}). \]

Here \( r_t \) is the short-term interest rate, and \( r_{L,t} \) and \( r_{F,t} \) are one-period holding rates of return for domestic long-term bonds and foreign bonds, respectively.

In the partial equilibrium model, the stochastic processes for the interest rates are exogenously given, and the utility function is parameterized. The equilibrium exchange rate satisfies the condition that \( B^d_{F,t} \) is equal to \( B^s_{F,t} \), where \( B^s_{F,t} \) is the supply of the foreign bonds to the domestic residents. The supply of foreign bonds is equal to the cumulative current account balance, and follows the dynamic equation:

\[ B^s_{F,t} = B^s_{F,t-1} + C_t. \]
where $C_t$ is the current account balance in the period $t$. Neglecting interest received by holders of foreign bonds, $C_t$ is assumed to satisfy

$$
C_t = -\alpha + b s_t + u_t,
$$

where $s_t$ is the log of the exchange rate expressed in terms of the domestic currency, $b$ is a positive number, and $u_t$ is the trade shock which is assumed to be white noise with variance $\sigma_u^2$.

Suppose that $w_{t+1}$ is normally distributed conditional on $\Omega_t$, and that the measure of the absolute risk aversion of $u_t$, $-\frac{u_t''}{u_t'}$, is a positive constant $\alpha$. As derived in the Appendix, the demand function for foreign bonds and the demand function for long-term bonds are

$$
B_{F,t}^d(E(r_{F,t}), r_t, E(r_{L,t})) = \psi_{F,t} - \psi_{L,t},
$$

$$
B_{L,t}^d(E(r_{F,t}), r_t, E(r_{L,t})) = \psi_{F,t} \frac{\sigma_{L,t}^2}{\sigma_{F,t}^2} - \psi_{F,t},
$$

where

$$
\psi = \frac{1}{\alpha \sigma_s^2 (1 - \text{cor})}
$$

$$
\phi = -\text{cov}/\sigma_s^2
$$

$$
\sigma_s^2 = E\{(s_{t+1} - E(s_{t+1}))^2\}
$$

$$
\sigma_f^2 = E\{(r_{t+1} - E(r_{t+1}))^2\}
$$

$$
\text{cov} = E\{(s_{t+1} - E(s_{t+1}))(r_{t+1} - E(r_{t+1}))\}
$$

$$
\text{cor} = \text{cov} / (\sigma_s^2 \sigma_f^2).
$$

As in (12), the demand function for foreign bonds depends on $\text{cov}$, the
covariance conditional on $\Omega_t$ between the exchange rate and the short-term interest rate, and $\sigma^2_s$ the conditional variance of the exchange rate. At the same time, the stochastic process of the exchange rate and $\text{cov}$ depend on the demand function for foreign bonds. Therefore, it is necessary to solve for a rational expectations equilibrium in which the values of $\text{cov}$ and $\sigma^2_s$ that the investors expect are consistent with the stochastic process of the exchange rate implied by the demand function consistent with these values of $\text{cov}$ and $\sigma^2_s$. Given these second moments, (12) and (13) give the demand functions for foreign bonds and long-term bonds as functions of expected returns. Hence it is possible to define substitution and income effects for changes in expected returns as in Blanchard and Plantes (1977) and Royama and Hamada (1967).

Because absolute risk aversion is assumed to be constant, income effects do not appear in the demand for risky assets, notably foreign bonds and long-term bonds. Therefore, the price effects which appear in (12) and (13) are also the substitution effects. Let $f_{FS} = \frac{\partial B^d_{FS}}{\partial r_i}$ be the substitution effect between foreign bonds and short-term bonds, $f_{TL}$ be the substitution effect between foreign bonds and long-term bonds, and $f_{LS}$ be the substitution effect between long-term and short-term bonds. Then from (12) and (13)

(20) \hspace{1cm} f_{FS} = -\psi(1 - \phi)

(21) \hspace{1cm} f_{TL} = -\psi \phi

(22) \hspace{1cm} f_{LS} = -\psi \left( \frac{\sigma^2_s}{\sigma^2_r} - \phi \right).
According to (21), the sign of \( \phi \) determines whether foreign bonds and long-term bonds are substitutes or complements. From (15), the sign of \( \phi \) is determined by the sign of the conditional covariance between the exchange rate and the short-term interest rate, \( cov \). In the rational expectations equilibrium derived in the next section, \( cov \) is always negative, so \( \phi \) is positive. Therefore, as indicated by (21), foreign bonds and long-term bonds are substitutes. An intuitive explanation of this result is as follows. If the short-term interest rate rises unexpectedly in the next period, the price of long-term bonds falls and long-term bond holders suffer an unexpected capital loss. Because the negative \( cov \) implies that the exchange rate appreciates when the short-term interest rate rises on the average, bond holders will suffer an additional unexpected loss if they hold foreign bonds. Because of this, foreign bonds and long-term bonds are substitutes if \( cov \) is negative: the demand for foreign bonds declines when the expected return for long-term bonds rises.

In (22), it is shown that long-term bonds and short-term bonds are substitutes if \( \sigma^2_s + cov \) is positive. This condition hold if \( \sigma^2_s \) is greater than \( \sigma^2_r \). In the next section, we will focus on parameter configurations for which this condition is satisfied in the rational expectation equilibrium.

In order to examine (20), we decompose the substitution effect into direct and indirect substitution effects, following Ogaki (1990). The direct substitution effect between foreign bonds and short-term bonds, \( j_{FS}^D \), is defined as the substitution effect when investment in long-term bonds cannot be be adjusted. In this case,

\[
(23) \quad j_{FS}^D = -1/(\alpha\sigma^2_r).
\]
is negative. Hence foreign and short-term bonds are direct substitutes. The indirect substitution effect, \( f_{FS}^{I} \), is the total substitution effect minus the direct effect:

\[
(24) \quad f_{FS}^{I} = f_{FS} - f_{FS}^{P} = f_{FL} f_{LS} \alpha \sigma^2 \sigma_r^2 (1 - \sigma_r^2).
\]

Foreign and short-term bonds are defined as indirect complements (substitutes) if \( f_{FS}^{I} \) is positive (negative). As is shown in (24), foreign bonds and short-term bonds will be indirect complements if both foreign and long-term bonds, and long-term and short-term bonds are substitutes.\(^2\) It will be shown in the next section that both groups are substitutes in the rational expectation equilibrium. As a result, foreign and short-term bonds are indirect complements.

The foreign and short-term bonds are complements if the magnitude of the indirect complements relationship, \( f_{FS}^{I} \), exceeds the magnitude of the direct substitute relationship, \( |f_{FS}^{P}| \). This will occur if and only if \( \phi > 1 \). Therefore, \( \phi \) may be refereed to as a measure of the relative magnitude of the indirect complements relationship. If \( \phi > 1 \), the standard gross substitutability assumption is violated. In the next section, it will be shown that \( \phi \) is greater than 1 under certain assumptions.

**III. The Rational Expectations Equilibrium**

In this section, the model presented above will be used to derive the demand for foreign bonds in the rational expectations equilibrium. The stochastic processes of short-term interest rates are assumed to be as follows:

\(^2\)Ogaki (1990) shows that a substitute of a substitute is an indirect complement for a general case.
(25) \[ r_t = \mu + e_t + e_{t-1} + \epsilon_t \]

(26) \[ R_t = \mu + e_t + \frac{1}{2} e_{t-1} \]

(27) \[ r^*_t = \mu \]

where \( e_t \) and \( \epsilon_t \) are an interest rate shock with a duration of two periods and a temporary interest rate shock with a duration of one period, respectively. It is assumed that \( e_t \) and \( \epsilon_t \) are white noise with variance \( \sigma^2_e \) and \( \sigma^2_\epsilon \), respectively, and that they are independent of each other and of \( u_t \).

The conditional expectation is assumed to coincide with the best linear prediction. Because (26) is a fundamental representation in the sense of linear prediction theory (see, e.g., Rozanov (1967)), observing the current and past values of \( R_t \) is equivalent to observing the current and past values of \( e_t \). It follows that

(28) \[ E(r_{t+1}) = \mu + e_t \]

and from (4) and (28),

(29) \[ \rho_{L,t} = -\epsilon_t \]

For the purpose of this paper, we need to assume that the risk premium for long-term bonds, \( \rho_{L,t} \), is nonzero. As shown in (29), the assumption employed here is that only the interest rate shock with the two period duration is transmitted to the long-term interest rate, so that the risk premium is equal to the temporary interest rate shock.

Define \( \eta = \frac{\sigma^2_e}{\sigma^2_\epsilon} \), which may be called the measure of substitution between short-term and long-term bonds. If \( \eta = 0 \), then the risk premium for long-term bonds will always be zero, implying that short-term and long-term
bonds are perfect substitutes. The greater is $\eta$, the smaller the degree of the substitution.

Let $L$ be the lag operator. Then the equilibrium condition in the period $t$ is,

$$E_t(A_0(L)y_t) = \pi + D_0,$$

where

$$A_0(L) = -\psi L^{-1} + (b + \psi)$$

$$D_0 = -\alpha - \psi e_{t-1} - \psi (1-\psi)e_t - B_{t+1}$$

The equilibrium condition for period $t+1$ is, if we take expectations conditional on $\Omega_t$ from both sides,

$$E_t(A(L)y_{t+1}) = \pi + D_{t+1},$$

where

$$A(L) = -\psi L^{-1} + (b + 2\psi) - \psi L$$

$$D_{t+1} = \psi e_{t+1} + \psi e_t + \psi (1-\psi)e_t$$

The equilibrium condition in the period $t+2$ is, if we take expectations conditional on $\Omega_t$ from both sides,

$$E_t(A(L)y_{t+2}) = \pi + D_{t+2}.$$
Solving (30), (33), (37), and (38) as a difference equation system of 
\[ E_i(x_{t+1}), \] with respect to \( \tau \), provides the unique saddle point solution,
\[
(39) \quad s_t = \tilde{\delta} - \left[ 1 - \frac{\lambda}{\mu} \right] u_t \left[ \lambda \phi \right] + \lambda e_{t-1} \left[ \lambda \phi \right] + [\lambda (\phi - 1) e_t \right] - \left[ \frac{1 - \lambda}{b} \right] B_{F, t+1},
\]
where \( \tilde{\delta} = a/b \) is the long-run equilibrium exchange rate which clears the current account, and

\[
(40) \quad \lambda = 1 + \frac{b}{2\psi} - \frac{b}{2\psi} \sqrt{1 + 4\psi/b}.
\]

It is easy to check \( 0 < \lambda < 1, \frac{\partial \lambda}{\partial \psi} > 0, \lim_{\psi \to 0} \lambda = 0, \) and \( \lim_{\psi \to \infty} \lambda = 1. \)

Equation (39) shows that the investors' expected values of \( \text{cov} \) and \( \sigma^2 \)
afface the exchange rate dynamics through \( \lambda \) and \( \phi \). On the other hand, the exchange rate dynamics in (39) imply certain values of \( \text{cov} \) and \( \sigma^2 \), which need to be consistent with the investors' expected values in the rational expectations equilibrium. The equilibrium is analyzed in two steps. First, we solve for the rational expectation of \( \text{cov} \). Second, we show the uniqueness and existence of the rational expectations equilibrium by solving for the rational expectation of \( \text{var} \).

Before solving for the equilibrium, note the nature of (39). The discrepancy between the actual and the long-run equilibrium exchange rate is explained by the trade shock (in the first brackets), the interest rate shock that is transmitted to the long-term interest rate (the second brackets), the interest rate shock that is not transmitted to the long-term interest rate (the third brackets), and the cumulative current account balance (in the fourth brackets). The trade shock, which tends to give rise to current account surplus, makes the domestic currency appreciate. Prolonged increases in the short-term interest rate make the domestic currency appreciate, though the shock which arises during this period, \( e_t \),
has a greater effect than the one from the previous period, $e_{t-1}$. As the cumulative current account balance becomes greater, the domestic currency's appreciation increases: for investors to have incentives to hold more foreign bonds, the domestic currency must appreciate at present, so that investors can anticipate that it will depreciate in the future.

All of these effects are in the expected directions. The interest shock that is not transmitted to the long-term interest rate, $e_t$, however, has a perverse effect if the relative magnitude of the indirect complements relationship, $\phi$, is greater than 1. This is because the indirect complementarity of short-term and foreign bonds will exceed the direct substitutability if $\phi > 1$.

The term $\phi$ may be obtained by solving for the rational expectation of covariance. Calculating $\text{cov} = E_t \{ \sum_{i=1}^{t+1} \{ r_{t+i} - E_t \{ r_{t+i} \} \} \} \}, \{ s_{t+i} - E_t \{ s_{t+i} \} \} \}$ by (25) and taking the one period lead of (39) yields

$$\text{cov} = -(1 + \lambda) \lambda \psi - (\phi - 1) \lambda \psi \sigma_e^2.$$ (41)

Substituting the definition of $\phi$, (15), in (41), and solving for $\text{cov}$, we obtain $\text{cov}$ in the rational expectations equilibrium:

$$\text{cov} = -\lambda (1 + \lambda + \eta)/(1 + \eta) \sigma_e^2/(1 + \eta \lambda) > 0.$$ (42)

Therefore, by (15),

$$\phi = \lambda (1 + \lambda + \eta)/(1 + \eta \lambda) > 0.$$ (43)

In the rational expectations equilibrium, the conditional covariance between the exchange rate and the short-term interest rate, $\text{cov}$, is negative. Hence, foreign bonds and long-term bonds are substitutes as explained in the previous section. The measure of the the relative magnitude of the indirect
complements relationship, $\phi$, is positive. The main issue for the purpose of this paper is whether $\phi$ is greater or less than one. In order to determine this, we will investigate the sign of

$$
\phi - 1 = \{(\lambda^2 + \lambda - 1) - \eta\}/(1 + \eta + \eta \lambda).
$$

In order to examine the sign of (44), we need to know how $\lambda$ depends on the underlying parameters of the model. For this purpose, the existence and the uniqueness of the rational expectations equilibrium will be shown by solving for the rational expectation of the conditional variance of the exchange rate, $\sigma^2 = E_t\{[s_{t+1} - E_t s_{t+1}]^2\}$.

Taking one period lead of (39), we obtain

$$
\sigma^2 = \sigma^2(1 - \lambda)^2/b^2 + \{(1 + \lambda)^2 + (\phi - 1)^2 \eta\} \lambda^2 \sigma^2.
$$

By (42) and the definition of $\text{cor}$, (19),

$$
cor = \lambda(1 + \lambda + \eta)/(1 + \eta + \eta \lambda)\sqrt{(1 + \eta)\sigma^2\sigma^2}.
$$

Using the definition of $\lambda$, (40), we obtain

$$
\psi = b\lambda/(1 - \lambda)^2.
$$

Substituting the definition of $\psi$, (14), into (47), we obtain

$$
\frac{1}{\alpha} = \sigma^2(1 - \text{cor}^2) b\lambda/(1 - \lambda)^2.
$$

The condition for the rational expectations equilibrium value for $\text{var}$ is obtained by substituting (45) and (46) into (48):

$$
\frac{1}{\alpha} = g(\lambda),
$$

where

$$
g(\lambda) = \frac{\lambda \sigma^2}{b} + \frac{b \lambda^2 (2 + \lambda)^2 \eta (1 + \eta) \sigma^2}{(1 - \lambda)^2 (1 + \lambda + \eta) \sigma^2}.
$$
Let $\lambda^*$ be the value of $\lambda$ that satisfies (49). Any such $\lambda^*$ corresponds to a rational expectations equilibrium. It can easily be checked that $g'(\lambda) > 0$, $\lim_{\lambda \to \infty} g'(\lambda) = 0$, and $\lim_{\lambda \to \infty} g(\lambda) = \infty$. Hence there exists a unique rational expectations equilibrium. Moreover, when $\alpha$ is smaller, $\lambda^*$ is larger. The value of $\psi$ can be obtained by substituting $\lambda^*$ for $\lambda$ in (47). The value of $\psi$ is increased by a reduction in the variances $\sigma^2_u$ and $\sigma^2_e$ and by an increase in the measure of constant risk aversion $\alpha$, which in turn diminishes $g(\lambda)$. It is apparent that $\lim_{\alpha \to \infty} \psi = \infty$ and $\lim_{\alpha \to \infty} \lambda = 1$.

Equation (44) shows that $\phi$ can be either greater or less than one, depending on the parameter values. One interesting case arises when the investors are close to being risk neutral. For a very small $\alpha$, an approximate formula for (44) with $\lambda=1$ is

\begin{equation}
\phi - 1 \approx (1-\eta)/(1 + 2\eta).
\end{equation}

Hence the measure of the relative magnitude of the indirect complementarity, $\phi$, is greater than one if the measure of the degree of substitution between short-term and long-term bonds, $\eta$, is smaller than one. If the investors are close to being risk neutral and $\alpha$ is close to zero, the degree of substitution between short-term and long-term bonds must be high, and consequently, $\eta$ should be very small. Therefore, $\phi$ is greater than one, and the indirect complementarity dominates the direct substitutability for reasonable parameter configurations of the model.

IV. Conclusions

This paper derives demand functions for foreign bonds endogenously by solving for the rational expectations equilibrium. It shows that the indirect complementarity between domestic short-term bonds and foreign bonds
is likely to be quantitatively important. In the model, domestic long-term bonds and foreign bonds are strong substitutes because an unexpected increase in the short-term interest rate causes a capital loss for both foreign and domestic long-term bonds holders. Because domestic short-term and long-term bonds are assumed to be strong substitutes, foreign bonds and domestic long-term bonds are strong indirect complements. The result in the previous section demonstrates that the indirect complementarity can even dominate direct substitutability, allowing foreign bonds and domestic short-term bonds to be complements when the substitutability between foreign and domestic long-term bonds and between domestic short-term and long-term bonds is very strong.

The model implies that the long-term interest rate differential has the effect that standard exchange rate models imply on the exchange rate. However, the short-term interest rate differential has the opposite effect on the exchange rate after controlling for the long-term interest rate differential. Byeon and Ogaki (1999) find such effects for many of the G7 countries with cointegrating regressions of real exchange rates onto the short-term and long-term interest rate differentials. Ogaki and Santaella (1999) obtain similar results for Mexico.

If the indirect complementarity between domestic and foreign short-term bonds is quantitatively important, then the effectiveness of central bank attempts to affect exchange rates by controlling short-term interest rates depends on whether long term interest rates respond to changes in short-term interest rates. Anecdotal evidence suggests that further empirical investigation is warranted. For example, from the middle of March 1982 to the end of November 1982, the Bank of Japan adopted a policy to increase
domestic short term interest rates in order to cause an appreciation of the yen (see, e.g., Komiya and Suda [1983, pp. 347-354]). Short term interest rates in Japan increased but the yen tended to depreciate, rather than appreciate, against the U.S. dollar during this period. One remarkable fact was that long term interest rates did not increase when the Bank of Japan began to increase short term interest rates (Komiya and Suda [1983, p.349]).

The model in this paper suggests that a much more complicated relationship may exist between short-term and long-term interest rates and exchange rates than is implied by exchange rate models with risk neutral agents. The model also applies to the relationship between exchange rates and the term structure of various short-term interest rates if the investment horizon is very short (e.g., 1 month or shorter). In this sense, the model could help explain Clarida and Taylor's (1997) finding that the information in term structure of 1-month to 12-month forward premiums is useful in predicting future exchange rates.

There has been little empirical work on the interaction between the exchange rate and the term structure of interest rates relative to the large volume of empirical work on the exchange rate. Further empirical investigation is warranted.
References


Baxter, M., 1994, "Real exchange rates and real interest differentials: Have we missed the business-cycle relationship?" Journal of Monetary Economics 33, 5-37.


Appendix

Under the assumptions given in the text, the utility maximization of
the representative investor at $t$ is equivalent to maximizing

\[(A-1) \quad E_t(w_{t+1}) - \frac{\alpha}{2} Var_t(w_{t+1})\]

where $Var_t$ is the variance conditional on the information at $t$. Here,

\[(A-2) \quad E_t(w_{t+1}) = B_{S,t}^d (1 + r_t) + B_{L,t}^d (1 + 2 R_t - E_t(r_{t+1})) + B_{F,t}^d (1 + r_t^s) + E_t(s_{t+1}) - s_t.\]

\[(A-3) \quad Var_t(w_{t+1}) = \left( B_{L,t}^d \right)^2 Var_t(r_{t+1}) + \left( B_{F,t}^d \right)^2 Var_t(s_{t+1}) - 2 Cov_t(r_{t+1}, s_{t+1}).\]

Substituting (A-2) and (A-3) into (A-1), and maximizing (A-1) subject to the wealth budget constraint (8), yield the first order conditions:

\[1 + r_t = 0\]

\[1 + 2 R_t - \alpha B_{L,t}^d Var_t(r_{t+1}) + \alpha B_{F,t}^d Cov - k = 0\]

\[1 + E_t(s_{t+1}) - \alpha B_{F,t}^d Var_t(s_{t+1}) + \alpha B_{L,t}^d Cov - k = 0\]

\[B_{S,t}^d + B_{L,t}^d + B_{F,t}^d = w_t.\]

Solving these first order conditions for $B_{F,t}^d$ and $B_{L,t}^d$ gives Equations (12) and (13).