The Effects of Monetary Policy Shocks on Exchange Rates: A Structural Vector Error Correction Model Approach

Kyungho Jang*

Masao Ogaki†

The Ohio State University

March 1, 2001

Ohio State University
Department of Economics
Working Paper No. 01-02

ABSTRACT

This paper investigates the effects of shocks to U.S. monetary policy on the dollar/yen exchange rate, using structural Vector Error Correction Model (VECM) methods. We compare our estimates of the impulse responses with those based on levels Vector Autoregression. We also compare results from short run and long run restrictions imposed on the structural VECM. We find evidence of overshooting behavior of exchange rates with all methods. We find the price puzzle with levels Vector Autoregression and VECM with short-run restrictions. In contrast, we do not find the price puzzle with VECM with long-run restrictions.

Keywords: Vector Error Correction Model, Impulse Response, Monetary Policy Shock, Cointegration, Indentification, Long Run Restriction
JEL Classification: E32, C32

*Ohio State University, Department of Economics, Columbus OH 43210. Tel. (614) 292-9487 E-mail: jang.39@osu.edu.

†Ohio State University, Department of Economics, Columbus OH 43210. Tel: (614) 292-5842, Fax: (614) 292-3906, E-mail: mogaki@econ.ohio-state.edu.

*† We thank Charles Evans for providing us with the data for this study, Steve Cecchetti, Paul Evans, Pok-Sang Lam, Nelson Mark, and seminar participants at the Bank of Japan, International University of Japan, Kobe University, Kyoto University, and Tezukayama University for comments.

1 Introduction

This paper examines the effects of shocks to U.S. monetary policy on the dollar/yen exchange rate, using structural Vector Error Correction Model (VECM) methods. We compare our estimates of the effects with those of Eichenbaum and Evans (1995) based on levels Vector Autoregression (VAR). We also compare results from short-run and long-run restrictions imposed on the structural VECM.

The standard exchange rate model (see, e.g., Dornbusch, 1976) predicts that a contractionary shock to U.S. monetary policy leads to appreciation in U.S. nominal and real exchange rates. However, empirical evidence for two important building blocks of the model is mixed at best. These two building blocks are Uncovered Interest Parity (UIP) and long-run Purchasing Power Parity (PPP). Therefore, it is not obvious whether or not this prediction of the model holds true in the data. Eichenbaum and Evans (1995) directly investigate this prediction by estimating impulse responses to U.S. monetary shocks and find evidence in favor of the prediction, even though their results do not support some aspects of the standard exchange rate model.

In order to investigate the impulse responses to a monetary policy shock, it is necessary to identify the shock by imposing economic restrictions on an econometric model. When economic restrictions are imposed, the econometric model is called a structural model. Both the choice of the econometric model and the choice of the set of restrictions can affect the point estimates and standard errors of impulse responses. For this reason, it is important to study how these choices affect the results.

Most variables used to study exchange rate models are persistent, and usually modeled as series with stochastic trends and cointegration. In such a case, both levels VAR and VECM can be used to estimate impulse responses. Levels VAR is more robust than VECM because it can be used even when the system does not have stochastic trends and cointegration. Perhaps for this reason, it is used in most studies of impulse responses and by Eichenbaum and Evans (1995). However, structural VECM has some important advantages in systems with stochastic trends and cointegration. First, other things being equal, estimators of impulse responses from structural VECM are more precise. For example, levels VAR can lead to exploding impulse response estimates even when the true impulse response is not exploding. This possibility is practically eliminated with structural VECM. Second, it is possible to impose long-run restrictions as well as short-run restrictions to identify shocks.

A method of imposing long-run restrictions on VECM is developed in King, Plosser, Stock and Watson (1991, KPSW for short). This paper employs Jang's (2000) method rather than the KPSW method. Compared to the KPSW method, Jang's method has an advantage in that it requires neither identification nor estimation of cointegrating vectors. This greatly faciliates the impulse response analysis because identification assumptions for cointegrating vectors can be complicated, and may be inconsistent with some long-run restrictions a researcher wishes to impose to identify shocks. Another feature of Jang's method is that it applies block recursive assumptions to structural VECM with long run restrictions. The block recursive system has been well developed in structural VAR and structural VECM with short run restrictions, yet is not studied in the structural VECM with long run restrictions. The identification scheme of one permanent shock in structural VECM with long run restrictions is first developed, to our knowledge, by Jang(2000), and it is applicable with minimal assumptions when impulse responses to only one permanent shock are of interest.

2 Long run restrictions on Error Correction Models

When economic variables are cointegrated $I(1)$ processes, the system has a reduced rank and there exists an error correction model according to the Granger representation theorem (see, Engle and Granger, 1987). Johansen (1988) develops maximum likelihood estimators of cointegrating vectors and provides a rank test to determine the number of cointegrating vectors, r.

The estimation method developed in this paper differs from the Johansen method in the sense that i)long run restrictions are imposed on an error correction model, ii)long run impulse response analysis is of interest, but not estimation of cointegrating vectors.

The paper adopts the standard notation as following: i) x_t is a $n \times 1$ vector of nonstationary variables which are assumed to be cointegrated, ii) r is the number of cointegrating vectors, iii) k is the number of common trends, $k = n - r$, iv) the data generating process is also assumed to be VAR(p) in which p is the lag length, and v) L is the lag operator.

2.1 Error Correction Models

Suppose that x_t has a finite order unrestricted VAR representation:

$$A(L)x_t = \mu + \epsilon_t \tag{2.1}$$

where $A(L) = I_n - \sum_{i=1}^{p} A_i L^i$, $A(0) = I_n$, and ϵ_t is *white noise* with mean zero and variance Σ. From the reduced form VAR, $A(L)$ can be reparameterized as $A(1)L + A^*(L)(1-L)$ where $A(1)$ has a reduced rank, $r < n$. Engle and Granger (1987) show that there exists an error correction representation:

$$A^*(L)\Delta x_t = \mu - A(1)x_{t-1} + \epsilon_t \tag{2.2}$$

where $A^*(L) = I_n - \sum_{i=1}^{p-1} A_i^* L^i$, and $A_i^* = -\sum_{j=i+1}^{p} A_j$. Since x_t is assumed to be cointegrated $I(1)$, Δx_t is $I(0)$, and $-A(1)$ can be decomposed as $\alpha\beta'$ where α and β are $n \times r$ matrices with full column rank r.

2.2 Long run restrictions

As Δx_t is assumed to be stationary, it has a unique Wold representation:

$$\Delta x_t = \delta + C(L)\epsilon_t \tag{2.3}$$

where $\delta = C(1)\mu$, $C(L) = I_n + \sum_{i=1}^{\infty} C_i L^i$. The above reduced form can be represented as a structural form:

$$\begin{aligned} \Delta x_t &= \delta + \Gamma(L)v_t \\ \Gamma(L) &= C(L)\Gamma_0 \\ v_t &= \Gamma_0^{-1}\epsilon_t \end{aligned} \tag{2.4}$$

where $\Gamma(L) = \Gamma_0 + \sum_{i=1}^{\infty} \Gamma_i L^i$, and v_t is a vector of structural disturbances with mean zero and variance Σ_v.

Long run restrictions are imposed on the structural form as in Blanchard and Quah (1989, BQ for short). Stock and Watson (1988) develop a common trend representation showing that it is equivalent to an ECM representation. When cointegrated variables have a reduced rank r, there exist $k = n - r$ common trends. These common trends can be considered to be generated by permanent shocks so that v_t can be decomposed into $(v_t^{k\prime}, v_t^{r\prime})'$, where v_t^k is a k dimensional vector of permanent shocks and v_t^r is an r dimensional vector of transitory shocks. As developed in KPSW, this decomposition ensures that

$$\Gamma(1) = \begin{bmatrix} A & 0 \end{bmatrix}, \tag{2.5}$$

where A is an $n \times k$ matrix and 0 is a $n \times r$ matrix with zeros representing long run effects of permanent shocks and transitory shocks, respectively.

If there are more than one common trend ($k > 2$), permanent shocks can not be identified separately from the above restrictions. Applying long run restrictions in BQ, say $n = 7$ and $k = 5$ as in Section 3.1, long run effects or permanent shocks, A, have a specific structure after re-ordering:

$$x_t = \begin{bmatrix} x_t^1 \\ x_t^2 \\ x_t^3 \\ x_t^4 \\ x_t^5 \\ x_t^6 \\ x_t^7 \end{bmatrix}, \quad v_t^k = \begin{bmatrix} v_t^1 \\ v_t^2 \\ v_t^3 \\ v_t^4 \\ v_t^5 \end{bmatrix}, \quad A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \times & 1 & 0 & 0 & 0 \\ \times & \times & 1 & 0 & 0 \\ \times & \times & \times & 1 & 0 \\ \times & \times & \times & \times & 1 \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \end{bmatrix}.$$

where $\times$ denotes that those parameters are not restricted.

In the above example, long run restrictions are imposed such that a permanent shock, v_t^2, has no long run effects on a variable, x_t^1, and a permanent shock, v_t^3, has no long run effects on variables, x_t^1 and x_t^2, and so on. Note that the causal chains in the sense of Sims (1980) are imposed on permanent shocks, which is the orthogonalizing condition:

$$A = \hat{A}\Pi \tag{2.6}$$

where $\hat{A}$ is an $n \times k$ matrix, and Π is a $k \times k$ lower triangular matrix with ones on the diagonal. Continuing the above example, Π has the following specific form:

$$\Pi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \pi_{21} & 1 & 0 & 0 & 0 \\ \pi_{31} & \pi_{32} & 1 & 0 & 0 \\ \pi_{41} & \pi_{42} & \pi_{43} & 1 & 0 \\ \pi_{51} & \pi_{52} & \pi_{53} & \pi_{54} & 1 \end{bmatrix}.$$

If the purpose of impulse analysis is to examine the effects of only one permanent shock, the recursive assumption on the permanent shocks in (2.6) can be relaxed. A block recursive assumption for permanent shocks, instead, suffices to investigate the impulse responses of economic variables to one permanent shock. Continuing with the example, in order to identify the k_{th} permanent shock, $v_{t,k}^{k}$, the following restrictions are sufficient:

$$A = \hat{A}\Pi = \hat{A} \begin{bmatrix} 1 & \pi_{12} & \pi_{13} & \pi_{14} & 0 \\ \pi_{21} & 1 & \pi_{23} & \pi_{24} & 0 \\ \pi_{31} & \pi_{32} & 1 & \pi_{34} & 0 \\ \pi_{41} & \pi_{42} & \pi_{43} & 1 & 0 \\ \pi_{51} & \pi_{52} & \pi_{53} & \pi_{54} & 1 \end{bmatrix}. \tag{2.7}$$

Thus, four long run restrictions are sufficient to identify the fifth permanent shock. In general, $k-1$ long run restrictions are sufficient to identify the last permanent shock, $v_{t,k}^{k}$.

2.3 Estimation of the model

This section explains how we can construct $\hat{A}$ from estimated cointegrating vectors. Engle and Granger (1987) show:

$$\beta' C(1) = 0 \tag{2.8}$$

which implies that long run effects should lie on cointegrating relations if variables are cointegrated. It follows from $\Gamma(1) = C(1)\Gamma_0$ and (2.5) that

$$\beta' A = 0. \tag{2.9}$$

Let $\beta_\perp$ be a $n \times k$ orthogonal matrix of cointegrating vectors, β, which satisfies $\beta'\beta_\perp = 0$. Johansen (1995) proposes one method for choosing $\beta_\perp$:

$$\beta_\perp = (I_n - S(\beta' S)^{-1}\beta')S_\perp \tag{2.10}$$

where S is an $n \times r$ selection matrix, $(I_r \;\; 0)'$, and $S_\perp$ is an $n \times k$ selection matrix, $(0 \;\; I_k)'$.

In order to maintain BQ-type long run restrictions, one should normalize $\beta_\perp$ so that some parts of the matrix contain a $k \times k$ identity matrix. Let $\hat{\beta}_\perp$ be the normalized orthogonal matrix of cointegrating vectors. From $A = \hat{A}\Pi$, we can choose the matrix:[1]

$$\hat{A} = \hat{\beta}_\perp \tag{2.11}$$

Consider the seven-variable model with long run restrictions in Section 3.1. Let x_t be $(y_t, p_t, y_t^{for}, r_t^{for}, r_t, m_t, e_t^r)'$, in which y_t is output in the U.S., p_t is a price level, y_t^{for} is output in the foreign country, r_t^{for} is an interest rate in the foreign country, r_t is the federal funds rate, m_t is a monetary variable, and e_t^r is a real exchange rate. If it is solely of interest to analyze the responses to a monetary policy shock, one can impose minimal restrictions on the model. Suppose that the monetary shock does not affect the real variables, but affects the level of U.S. price in the long run.[2] These

[1]KPSW, instead, assume that $\hat{A}$ is known *a priori*, which is estimated by dynamic OLS in each cointegrating equation.

[2]Note that we impose four long run restrictions in this example.

restrictions imply that

$$A = \hat{\beta_\perp}\Pi = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \times & \times & \times & \times & \times \\ \times & \times & \times & \times & \times \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & \pi_{12} & \pi_{13} & \pi_{14} & 0 \\ \pi_{21} & 1 & \pi_{23} & \pi_{24} & 0 \\ \pi_{31} & \pi_{32} & 1 & \pi_{34} & 0 \\ \pi_{41} & \pi_{42} & \pi_{43} & 1 & 0 \\ \pi_{51} & \pi_{52} & \pi_{53} & \pi_{54} & 1 \end{bmatrix}. \quad (2.12)$$

where $\times$ denotes that those parameters are not restricted other than $\beta'\hat{\beta}_\perp = 0$.

Now we are ready to construct structural parameters with the long run restrictions given above. The identification scheme follows KPSW, and is well described by Jang (2000). Instead, we focus on the practical estimation steps for empirical studies. The main interest is identification of structural shocks but not of transitory shocks.[3] Therefore, we need to identify the first k columns of Γ_0 and the first k rows of Γ_0^{-1} for the identification of structural shocks only. It is convenient to decompose Γ_0 and Γ_0^{-1} as:

$$\Gamma_0 = \begin{bmatrix} H & J \end{bmatrix}, \quad \Gamma_0^{-1} = \begin{bmatrix} G \\ E \end{bmatrix} \quad (2.13)$$

where the matrices H, J, G and E have dimensions $n \times k$, $n \times r$, $k \times n$, and $r \times n$, respectively. The structural parameters of interest are short run dynamics, $\Gamma(L)^k = C(L)H$, and permanent shocks, $v_t^k = G\epsilon_t$, where $\Gamma(L)^k$ denotes the first k columns of $\Gamma(L)$. The impulse responses to permanent shocks can be identified once H and G are identified.

With conventional assumptions that the variance matrix of permanent shocks is a $k \times k$ diagonal matrix, Λ, structural parameters can be deduced as described

[3]Fisher et al. (1995) consider the identification of transitory shocks imposing causal chains on transitory shocks.

in KPSW.[4] First, permanent shocks are uniquely identified once G is derived. The reduced form ECM in (2.2) is estimated using e.g. Johansen (1988) method. We can derive C(L) in MA representation by inverting (2.2). Let $D = (\hat{\beta}_\perp' \hat{\beta}_\perp)^{-1} \hat{\beta}_\perp' C(1)$ and P be a lower triangular matrix chosen from the Choleski decomposition of $D\Sigma D'$. Then Π is uniquely determined by $\Pi = P\Lambda^{-\frac{1}{2}}$, and permanent shocks are uniquely identified by

$$v_t^k = G\epsilon_t \tag{2.14}$$

where $G = \Pi^{-1}D$. Second, short run dynamics are uniquely identified once H is derived. The dynamic multipliers for v_t^k are given by the first k columns of $\Gamma(L)$, $\Gamma(L)^k$, which are uniquely derived by

$$\Gamma(L)^k = C(L)H \tag{2.15}$$

where $H = \Sigma G' \Lambda^{-1}$.

So far we construct all structural parameters for the impulse analysis. However, it is not possible to invert levels VAR to MA representation due to the presence of unit roots. Lütkepohl and Reimers (1992) suggest an algorithm to get impulse responses recursively in a cointegrated system as follows. First, estimate the reduced form ECM in (2.2), then convert ECM to a levels VAR representation in (2.1) using the following relations:

$$A_i = \begin{cases} I - A(1) + A_1^* & i = 1 \\ A_i^* - A_{i-1}^* \quad \text{for} & 2 \le i \le p-1 \\ -A_{p-1}^* & i = p. \end{cases} \tag{2.16}$$

[4]This paper departs from KPSW in the following sense. KPSW assumes $\hat{A}$ is known, which can be chosen from the economic mechanism. Parameters in $\hat{A}$ are chosen by estimates from dynamic OLS in each cointegrating equation. This paper, instead, extends the method of KPSW with minimal assumptions, and our method is applicable even when cointegrating vectors are not identified.

Though a Wold representation does not exist in the presence of a unit root, Lütkepohl and Reimers (1992) address that impulse responses can be recursively computed by

$$\Psi_m = \sum_{l=1}^{p} \Psi_{m-l} A_l, \qquad m = 1, 2, 3, \cdots \tag{2.17}$$

$$\Phi_m = \Psi_m \Gamma_0 \tag{2.18}$$

where $\Psi_0 = I_n$, $\Phi_m = (\phi_{ij,m}) = \Gamma_m$ in (2.4), and $\phi_{ij,m}$ is an m-step response of the i_{th} variable to the j_{th} innovation.[5] It is worth noting that Γ_0 is normalized as ones on diagonal but v_t is unrestricted. The impulse response of x_{t+m} to v_t, $\Phi_m v_t$, can be rewritten as $\Phi_m \Sigma_v^{\frac{1}{2}} v_t^*$, in which v_t^* is a vector of normalized shocks with variance I_n. This implies that Φ_m is an impulse response to a unit shock and $\Phi_m \Sigma_v^{\frac{1}{2}}$ is interpreted as an impulse response to a one standard deviation shock. In particular, the impulse response function of permanent shocks in this paper is calculated by[6]

$$\Phi_m^k = \Psi_m H, \qquad m = 1, 2, 3, \cdots \tag{2.19}$$

Instead, if it is solely of interest to analyze impulse responses to a monetary policy shock, the k_{th} shock, as in Section 3.1, one needs to identify k_{th} column of H, H_k, and k_{th} row of G, G_k. First, the monetary policy shock is identified by

$$v_{t,k}^k = G_k \epsilon_t \tag{2.20}$$

where $G_k = \Lambda_{k,k} H_k' \Sigma^{-1}$ and $\Lambda_{k,k}$ is the variance of the monetary policy shock, which is the $(k,k)_{th}$ component of Λ. Second, the impulse response function of the monetary

[5]This algorithm can be simplified rewriting the VAR in (2.1) as a companion VAR(1) form, and Ψ_m is the same as the first n row and n column submatrix of A_c^m, in which A_c is a companion form coefficient matrix.

[6]One may calculate the impulse response to a one standard deviation permanent shock by $\Psi_m H \Lambda^{\frac{1}{2}}$.

policy shock is uniquely calculated by

$$\Phi_{m,k}^{k} = \Psi_m H_k = \Psi_m \begin{bmatrix} D \\ \alpha'\Sigma^{-1} \end{bmatrix}^{-1} S_k, \qquad m = 1, 2, 3, \cdots \tag{2.21}$$

where H_k is the k_{th} column vector of H, and S_k is an n-dimensional selection vector with value one in the k_{th} row and zeros in other rows, $(0,0,0,0,1,0,0)'$ for this example.[7]

In summary, the estimation procedure of VECM with long run restrictions as in Section 3.1 is described as:

1. Select the lag length of VECM based on some criteria such as AIC and BIC.
2. Estimate cointegrating vectors and determine the rank of cointegrating vectors in (2.2).[8]
3. Impose long run restrictions. For example, this paper adopts restrictions so that monetary shocks do not affect output or the real exchange rate in the long run.[9]
4. Convert VECM to VAR models as in (2.16), and calculate $C(1)$.
5. Calculate the impulse responses to a structural shock by (2.21).
6. Calculate confidence intervals for impulse responses as in Appendix (A).

3 Impulse Response Analysis with Long Run Restrictions

3.1 A Seven-variable VECM with Long Run Restrictions

This section investigates the effects of a contractionary shock to the monetary policy on economic variables including output, price and exchange rates. We begin with the same variables as in Eichenbaum and Evans (1995) for the purpose

[7] $\Phi_{m,k}^{k}$ is equivalent to the k_{th} column of Φ_m^k in (2.19).

[8] The GAUSS code used in this paper provides three possible cases: i) β is unrestricted, ii) β is known or preestimated as KPSW, and iii) β is restricted. In the cases of i) and iii), the program uses the Johansen procedure. The GAUSS code is available at http://economics.sbs.ohio-state.edu/ogaki.

[9] As for identification of one permanent shock, one needs $k-1$ long run zero restrictions.

Table 3.1: Cointegration Rank Tests

Eigen Value	λ_{max}	Trace	Number of Cointegration (r)	k (=n-r)	Critical Value 95% for Trace
0.19	51.30	154.88	0	7	123.04
0.16	42.02	103.58	1	6	93.92
0.08	20.04	61.56	2	5	68.68
0.07	17.08	41.52	3	4	47.21
0.06	14.20	24.44	4	3	29.38
0.03	7.94	10.24	5	2	15.34
0.01	2.31	2.31	6	1	3.84

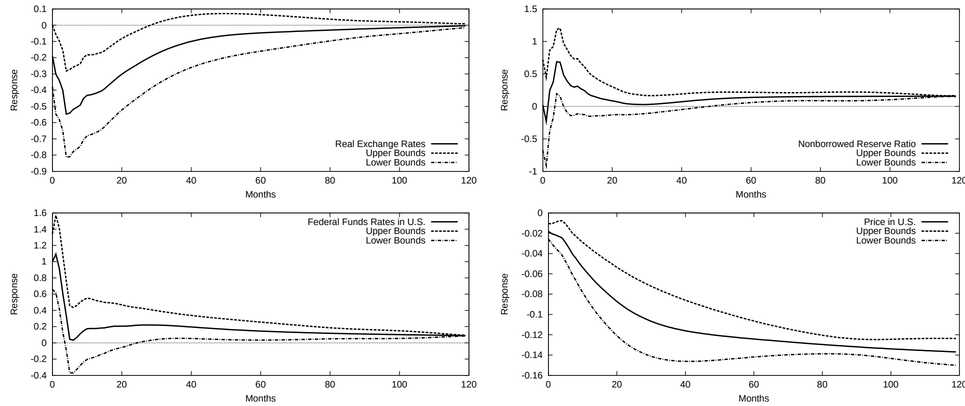
of comparison. The seven variable model includes the federal funds rate (r_{ff}), the nonborrowed reserve ratio (NBRX), output in the U.S. (y_{us}), the price level in the U.S. (P_{us}), output in Japan (y_{jp}), the interest rate in Japan (r_{jp}), and the real exchange rate (e_r, dollar/yen). We select five lags as the lag length of the structural VECM, which is equivalent to six lags in levels VAR.[10] Table 3.1 summarizes the cointegration rank tests for seven variables, which suggest $r = 2$ as the number of cointegrating vectors according to the 95% confidence interval of the trace test developed by Johansen (1988).[11] Therefore, we consider two cointegrating relations and five permanent shocks in the model.[12] This paper examines only the impacts of the monetary policy shock, which enables us to identify only one permanent shock.

[10]Eichenbaum and Evans (1995) chose the lag length so that their inference is robust for higher order lags.

[11]We select the model that satisfies the deterministic cointegration restriction developed in Ogaki and Park (1997).

[12]A unit root test is not necessary in our model, as structural VECM can include stationary series. Instead, determining the number of cointegrating vectors is more relevant. Therefore, we only report the results of rank tests.

Figure 3.1: Impulse Responses to Contractionary Monetary Shocks
(Unit Initial Effects on federal funds rates in U.S.)



Note: VECM uses seven level variables: output in U.S., price in U.S., output in Japan, interest rates in Japan, federal funds rates, nonborrowed reserve ratio, and real exchange rates(dollar/yen). Ordering does not affect the results in our model. The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans(1995). Long run restrictions are imposed that monetary shocks do not affect output in U.S., output in Japan, interest rates in Japan, and real exchange rates in the long run. Monetary shocks are normalized to have unit initial effects on federal funds rates. Upper and lower bounds are calculated by one standard errors.

Therefore, it is not necessary to identify or to estimate all structural parameters in the model as discussed in Section 2.3. See Jang (2000) for an extended explanation of the identification of one permanent shock in structural VECM with long run restrictions. Four long run restrictions are required to identify the monetary policy shock. We employ the following four restrictions: the monetary policy shock does not affect output in U.S., output in Japan, foreign interest rates, and real exchange rates in the long run. We also impose minimal restrictions on a cointegrating vector that output in the U.S. and in Japan moves in the long run.[13] Figure 3.1 describes the results.[14] We consider a contractionary monetary policy as a shock which affects the federal funds rate to rise in the initial period and satisfies the long run restrictions.

[13]The results are quite robust with the lag length 6, 9, and 12.

[14]The responses of output in both countries and interest rates in Japan are not shown in the figure to save space. These are available upon request.

Upper and lower error bands are calculated by Monte Carlo integration as described in Appendix A.

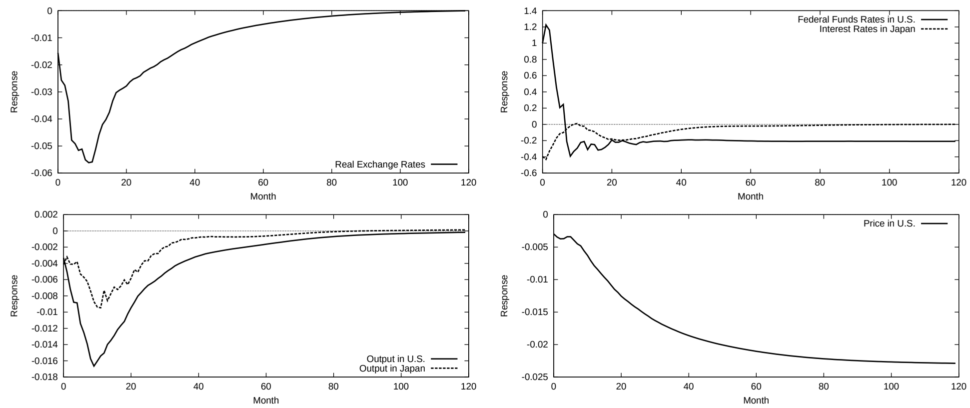
First, a contractionary monetary policy shock leads to an appreciation in the U.S. dollar immediately after the shock. This effect peaks after four months and persists for five years. Therefore, we find evidence for the overshooting behavior of the real exchange rate. This is in contrast with Eichenbaum and Evans (1995), who find such evidence only with a twenty-month delay. Furthermore, our model shows depreciation of the U.S. dollar starting at four months and lasting for five years. This result is consistent with prediction made by UIP. This is in contrast with Eichenbaum and Evans (1995) who find evidence against this prediction.

Second, the federal funds rate increases initially but the effects become relatively small after six months. Third, output in the U.S. and in Japan have similar responses to a contractionary monetary policy shock. Output in the U.S. decreases for three quarters and shows the largest impact after seven months, but it becomes negligible after four years due to the long run neutrality restrictions. Fourth, we find a persistent decrease in the price level in the U.S. after the shock. This may resolve the "price puzzle" addressed by Sims (1992) that a contractionary monetary policy leads to a persistent rise in the price level in structural VAR models.

We, however, find an anomaly that a contractionary monetary policy causes the nonborrowed reserve ratio to rise after two months.[15] This contrasts with the results of Eichenbaum and Evans (1995), in which a decrease in the nonborrowed reserve ratio is interpreted as a liquidity effect. We also find another anomaly that a contractionary monetary policy shock causes the interest rate in Japan to decrease for

[15] This anomaly is, however, not significant according to the error bands calculated by Monte Carlo integration.

Figure 3.2: Impulse Responses to Contractionary Monetary Shocks with Long Run Restrictions on VECM



Note: Contractionary monetary shocks are mesured by positive unit shock to the federal funds rates. VECM uses six variables: federal funds rates, output in U.S., price in U.S., output in Japan, interest rates in Japan, and real exchange rate(dollar/yen). The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans(1995). Long run restrictions are imposed that monetary shocks do not affect output in U.S., output in Japan, interest rates in Japan, and real exchange rates in the long run. Monetary shocks are normalized to have initial unit effects on federal funds rates.

substantial periods. Some of our results were sensitive to the choice of cointegration rank. This often happens in models with long run restrictions as pointed out in Faust and Leaper (1997).

3.2 A Six-variable VECM with Long Run Restrictions

As we find an anomaly of responses in the nonborrowed reserve ratio, we consider a benchmark model with six variables dropping nonborrowed reserve ratio. We select five lags and estimate the model assuming one cointegrating relation to impose the same long run restrictions as the model in Section 3.1, which implies that there are five permanent shocks in the structural VECM and four long run restrictions are required to identify the monetary policy shock as in Section 3.1.

Figure 3.2 describes the impulse responses to a contractionary monetary policy shock. Most results are similar to those of the seven-variable model in Section 3.1.

First, the U.S. dollar exhibits persistent appreciation for seven years compared to the original value. Second, the federal funds rate rises in the initial period but the effects become negative after seven months. Third, output in the U.S. shows persistent negative effects but much longer than those of the previous model. Fourth, the price level decreases after a contractionary monetary policy shock.

4 Impulse Response Analysis in VECM with Short Run Restrictions

4.1 Block Recursive Assumptions in VECM

The reduced form VECM in (2.2) can be represented as a structural form:

$$B^*(L)\Delta x_t = B_0\mu - B(1)x_{t-1} + v_t \tag{4.1}$$

where $B^*(L) = B_0 - \sum_{i=1}^{p-1} B_i^* L^i$, $B^*(L) = B_0 A^*(L), B(1) = B_0 A(1)$, $B_i^* = B_0 A_i^*$, and $v_t = B_0 \epsilon_t$. The short run restrictions are imposed on B_0 to have a block recursive structure. See Christiano et al. (1999) and Keating (1999) for an extended theoretical background. Partitioning x_t into three blocks is convenient to illustrate the block recursive structure:

$$x_t = \begin{bmatrix} x_{1t} \\ s_t \\ x_{2t} \end{bmatrix}, \tag{4.2}$$

where x_t is a vector of $n(= n_1 + 1 + n_2)$ variables of interest, s_t is a monetary policy variable, and x_{1t} includes n_1 variables which are in the information set when the Fed implements a monetary policy while x_{2t} contains n_2 variables which are excluded from the information set. Alternatively, x_{1t} does not respond to a monetary policy shock contemporaneously while x_{2t} does. The block recursive assumption imposes zero restrictions on the following partitioned B_0:

$$B_0 = \begin{bmatrix} \underset{(n_1 \times n_1)}{b_{11}} & \underset{(n_1 \times 1)}{0} & \underset{(n_1 \times n_2)}{0} \\ \underset{(1 \times n_1)}{b_{21}} & \underset{(1 \times 1)}{b_{22}} & \underset{(1 \times n_2)}{0} \\ \underset{(n_2 \times n_1)}{b_{31}} & \underset{(n_2 \times 1)}{b_{32}} & \underset{(n_2 \times n_2)}{b_{33}} \end{bmatrix} \tag{4.3}$$

Two zero restrictions, $b_{12} = b_{13} = 0$, are required for the monetary policy shock to be orthogonal to other structural shocks, while the restriction, $b_{23} = 0$, implies the assumption that the Fed does not have information about variables in x_{2t} when it makes a monetary policy decision.

The block recursive structure gives sufficient conditions to identify a monetary policy shock, and the ordering within x_{1t} and x_{2t} does not affect the results if one is interested in the effects of a monetary policy shock. Instead, the ordering across two groups might affect the results substantially. This is a crucial issue in the structural VECM as well as VAR models, while the ordering does not affect the results in the VECM with long run restrictions.

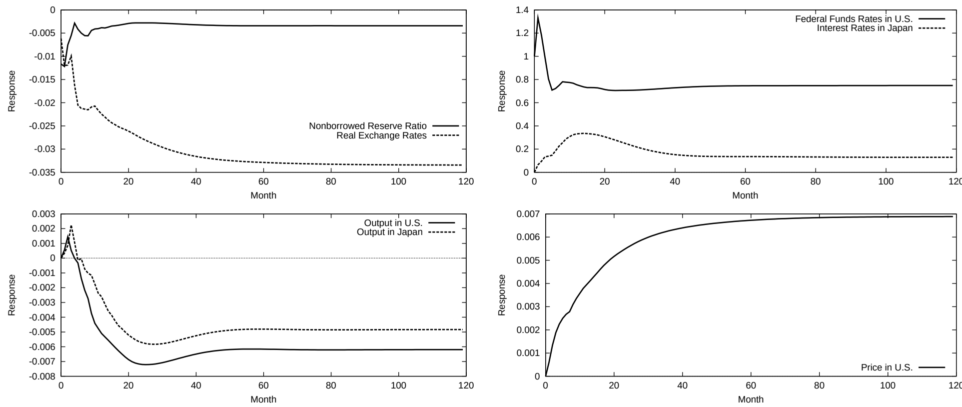
We investigate impulse responses in the structural VECM changing the order of variables and the measure of a monetary policy in the following sections.

4.2 Interpreting federal funds rate as a monetary policy

We begin with VECM when we measure federal funds rate as a monetary policy. As discussed in Section 3.1, we estimate the model using two cointegrating vectors with five lags for all the models in the following sections.

We follow the order in Eichenbaum and Evans (1995). Seven variables are ordered by output in the U.S. (y_{us}), the price level in the U.S. (P_{us}), output in Japan (y_{jp}), the interest rate in Japan (r_{jp}), federal funds rates (r_{ff}), the nonborrowed

Figure 4.1: Impulse Responses to Contractionary Shocks in Federal Funds Rates ($s_t = r_{ff}, NBRX \in x_{2t}$)



Note: VECM uses seven variables, which are ordered by output in U.S., price in U.S., output in Japan, interestest rates in Japan, federal funds rates, nonborrowed reserve ratio, and real exchange rates(dollar/yen). The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans (1995). Two cointegrating vectors are considered according to the rank test.

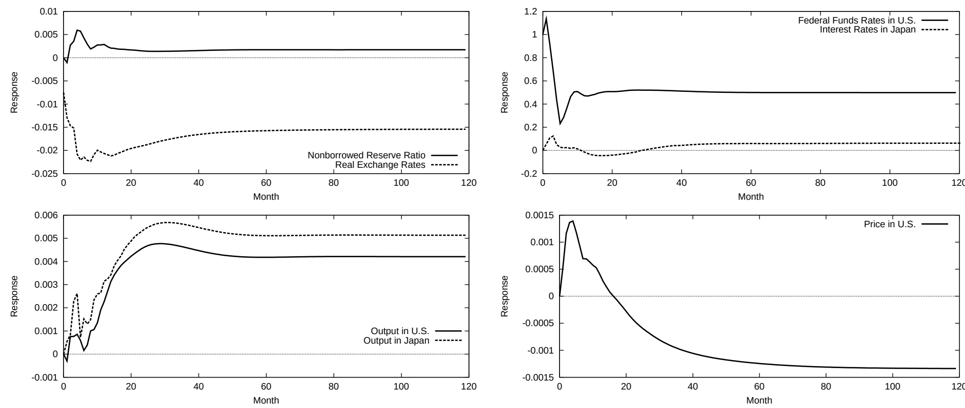
reserve ratio (NBRX), and the real exchange rate (e_r, dollar/yen). Interpreting r_{ff} as a monetary shock, the variables are partitioned as: $x_{1t} = (y_{us}, P_{us}, y_{jp}, r_{jp})'$, $s_t = r_{ff}$, and $x_{2t} = (NBRX, e_r)'$. The Fed has information about y_{us}, P_{us}, y_{jp}, and r_{jp} but not about $NBRX$ and e_r. Christiano et al. (1999) justify the ordering:

> $\cdots$ the Fed does have at its disposal monthly data on aggregate employment, industrial output and $\cdots$ substantial amounts of information regarding the price level. (p.83)

We investigate the impulse responses changing the information set of the Fed, and find that the results are similar whether y_{us}, P_{us}, y_{jp} and r_{jp} are included in x_{1t} or x_{2t}. Therefore, we report only the results of the first.[16] The results are, however, very sensitive to whether $NBRX$ is included in x_{1t} or x_{2t} as examined in the following sections.

[16]This applies to the rest of this paper, as we obtain similar results when we measure the nonborrowed reserve ratio as a monetary policy. The results are also similar in structural VAR regardless of these changes.

Figure 4.2: Impulse Responses to Contractionary Shocks in Federal Funds Rates ($s_t = r_{ff}, NBRX \in x_{1t}$)



Note: VECM uses seven variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, nonborrowed reserve ratio, federal funds rates, and real exchange rates(dollar/yen). The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans(1995). Two cointegrating vectors are considered according to the rank test.

Figure 4.1 shows the effects of a contractionary monetary policy shock when the Fed does not look at the nonborrowed reserve ratio when it makes monetary policy so that $NBRX$ is in x_{2t}. First, we find substantial and persistent appreciation of the U.S. dollar. Contrary to the results in VECM with long run restrictions, the real exchange rates keep decreasing for five years so that the U.S. dollar does not exhibit overshooting behavior. We also find that UIP does not hold even in the long horizon. Second, output in the U.S. increases a little initially but decreases substantially after three months. Output in Japan has similar responses. Third, the federal funds rate and the interest rate in Japan exhibit substantial and persistent increases. Fourth, a contractionary monetary policy shock leads to a decrease in the nonborrowed ratio. Finally, the price level in the U.S. exhibits a persistent increase after a contractionary monetary policy shock, illustrating the "price puzzle".

Figure 4.2 shows the effects of a contractionary monetary policy shock when the Fed has information about the nonborrowed reserve ratio when it makes a monetary policy so that $NBRX$ is in x_{1t}. The results are quite different for output and nonborrowed reserve ratio. First, we find substantial and persistent appreciation in the U.S. dollar. Second, output in the U.S. increases substantially even for the long term, which is inconsistent with common belief. Output in Japan has similar responses. Third, the federal funds rate exhibits substantial and persistent increases, but the interest rate in Japan show relatively small changes. Fourth, a contractionary monetary policy shock leads to an increase in the nonborrowed ratio except for an immediate decrease after the shock. Finally, the price level in the U.S. exhibits a substantial increase for sixteen months showing the "price puzzle", but it shows a persistent decrease in the long run.

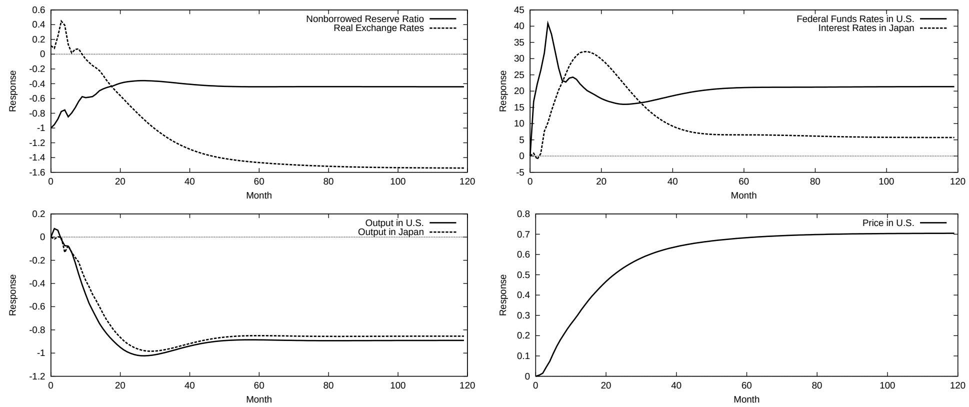
Therefore, the impulse responses of output and the nonborrowed reserve ratio depend on the assumption of whether the Fed looks at the nonborrowed reserve ratio for its policy decision. It may be more plausible that the Fed looks at the nonborrowed reserve ratio when it makes monetary policy. However, this leads to very strange results as illustrated in Figure 4.2.

4.3 Interpreting nonborrowed reserve ratio as a monetary policy

This section examines the impulse responses when we measure a monetary policy by the nonborrowed reserve ratio, $NBRX$. Following the discussion in Section 4.2, we examine two cases depending on whether r_{ff} is included in x_{1t} or x_{2t}.

Figure 4.3 shows the effects of a contractionary monetary policy shock when the Fed looks at the federal funds rate when it makes monetary policy so that r_{ff} is in x_{1t}.

Figure 4.3: Impulse Responses to Contractionary Shocks in $NBRX$
$(s_t = NBRX, r_{ff} \in x_{1t})$

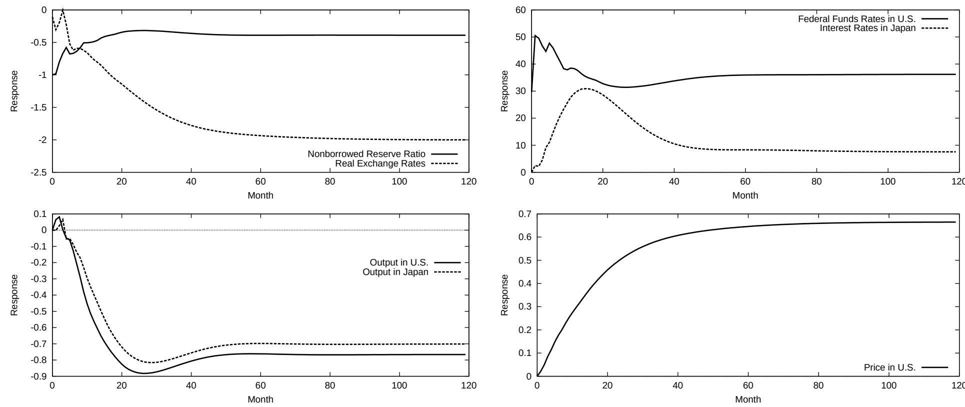


Note: VECM uses seven variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rate in Japan, federal funds rates, nonborrowed reserve ratio, and real exchange rates(dollar/yen). The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans(1995). Two cointegrating vectors are considered according to the rank test.

First, we find that the U.S. dollar depreciates immediately after the shock, and shows persistent appreciation after nine months. We don't find any evidence for overshooting behavior nor prediction of UIP. Second, output in the U.S. shows a substantial and persistent decrease except for the negligible increase in the initial period. Third, the federal funds rate and the interest rate in Japan increase substantially. Fourth, a contractionary monetary policy leads to a decrease in the nonborrowed ratio in the long run. Finally, the price level in the U.S. exhibits a persistent increase after a contractionary monetary policy showing the "price puzzle".

Figure 4.4 shows the effects of a contractionary monetary policy shock when the Fed does not consider the federal funds rate when it makes monetary policy so that r_{ff} is in x_{2t}. Most results are similar to those in the case that r_{ff} is in x_{1t}. One exception is that the U.S. dollar does not show an initial depreciation.

Figure 4.4: Impulse Responses to Contractionary Shocks in $NBRX$
($s_t = NBRX, r_{ff} \in x_{2t}$)



Note: VECM uses seven variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, nonborrowed reserve ratio, federal funds rates, and real exchange rates(dollar/yen). The lag length is five, which is equivalent to six in levels VAR in Eichenbaum and Evans(1995). Two cointegrating vectors are considered according tothe rank test.

Therefore, the responses to a monetary shock do not depend much on the selection of the Fed's information set. Still, however, there remains the "price puzzle".

5 Impulse Response Analysis in VAR with Short Run Restrictions

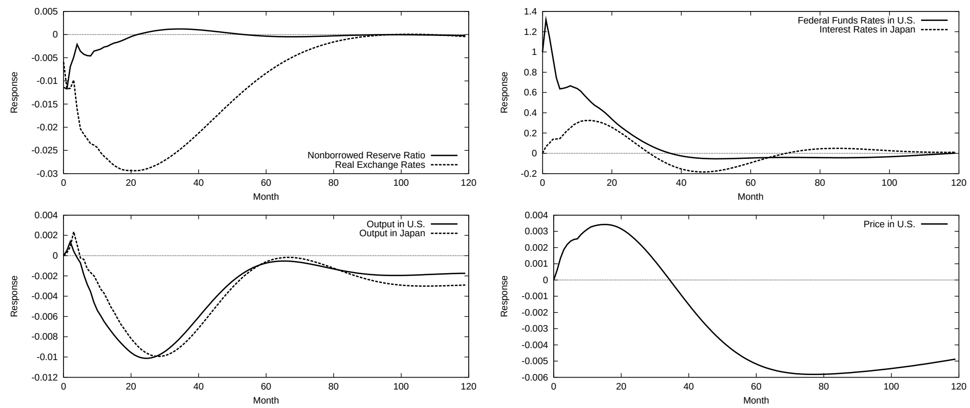
5.1 Block Recursive Assumptions in VAR

The reduced form VAR in (2.1) can be represented in structural form:

$$B(L)x_t = B_0\mu + v_t \tag{5.1}$$

where $B(L) = B_0 A(L), B_i = B_0 A_i$, and $v_t = B_0 \epsilon_t$. The short run restrictions are imposed on B_0 to have a block recursive structure as discussed in Section 4.1.

Figure 5.1: Impulse Responses to Contractionary Shocks in Federal Funds Rates ($s_t = r_{ff}, NBRX \in x_{2t}$)



Note: VAR uses seven level variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, federal funds rates, nonborrowed reserve ratio, and real exchange rates(dollar/yen). The lag length is six, which is chosen in Eichenbaum and Evans(1995).

5.2 Interpreting federal funds rate as a monetary policy

We begin with a VAR model when we measure federal funds rate as the monetary policy. As discussed in Section 3.1, we estimate the levels VAR with six lags for all the models in the following sections.

First, we follow the order in Eichenbaum and Evans (1995) as described in Section 4.2. Interpreting r_{ff} as a monetary shock, the variables are partitioned as: $x_{1t} = (y_{us}, P_{us}, y_{jp}, r_{jp})'$, $s_t = r_{ff}$, and $x_{2t} = (NBRX, e_r)'$. Therefore, the Fed has information about y_{us}, P_{us}, y_{jp}, and r_{jp} but not about $NBRX$ and e_r.

Figure 5.1 shows the effects of a contractionary monetary policy shock when the Fed does not look at the nonborrowed reserve ratio when it makes monetary policy so that $NBRX$ is in x_{2t}. This is the second model used in Eichenbaum and Evans (1995).[17] First, the U.S. dollar exhibits substantial and persistent appreciation in

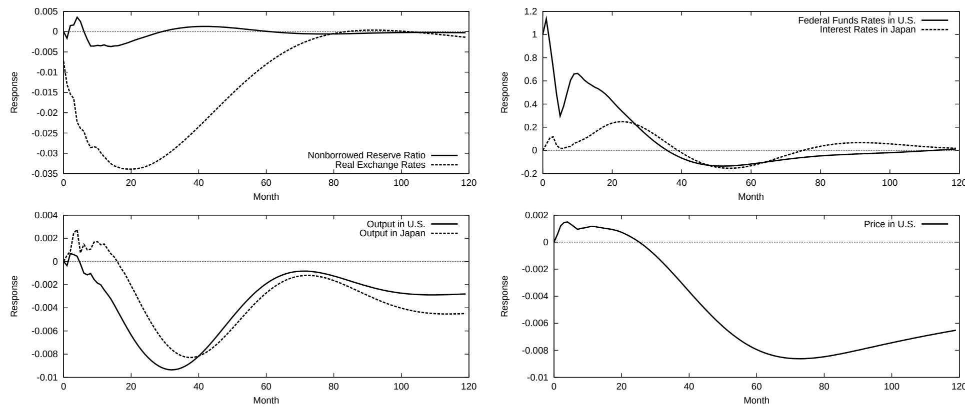
[17]Eichenbaum and Evans (1995) use different ordering in two models. In the first model, they measure the nonborrowed reserve ratio as monetary policy, in which variables are ordered by

real value. In contrast to the results in VECM with long run restrictions, the real exchange rate keeps decreasing for twenty months and exhibits delayed overshooting behavior. There is no evidence of the UIP prediction, as the exchange rate does not bounce back for the initial twenty months. Second, output in the U.S. increases somewhat initially but decreases substantially after three months. Output in the U.S., however, returns to the original level after five years, and decreases a little after then. Output in Japan has similar responses. Third, the federal funds rate rises in the initial period but returns to its original level after three years. The interest rate in Japan also shows a rise in levels for three years. Fourth, a contractionary monetary policy shock leads to a decrease in the nonborrowed ratio, but it returns to the initial level after twenty months. Finally, the price level in the U.S. exhibits a substantial increase for three years showing the "price puzzle", but decreases in the long run.

Figure 5.2 shows the effects of a contractionary monetary policy shock when the Fed has information about the nonborrowed reserve ratio when it makes monetary policy so that $NBRX$ is in x_{1t}. The results are similar to those in Figure 5.1 except that the nonborrowed reserve ratio shows a small increase in the initial five months.

$y_{us}, P_{us}, y_{jp}, r_{jp}, NBRX, r_{ff}$, and e_r. From the block recursive assumption, the Fed does not look at current level of the federal funds rate when it makes monetary policy decision or, alternatively, the federal funds rate responds to monetary policy contemporaneously. See Figure II, p.990. In the second model, they choose the federal funds rate as a monetary policy measure, in which variables are ordered by $y_{us}, P_{us}, y_{jp}, r_{jp}, r_{ff}, NBRX$, and e_r. The corresponding assumption is that the Fed does not look at current value of the nonborrowed reserve ratio or, alternatively, the nonborrowed reserve ratio responds to monetary policy contemporaneously. See Figure III, p.995.

Figure 5.2: Impulse Responses to Contractionary Shocks in Federal Funds Rates ($s_t = r_{ff}, NBRX \in x_{1t}$)



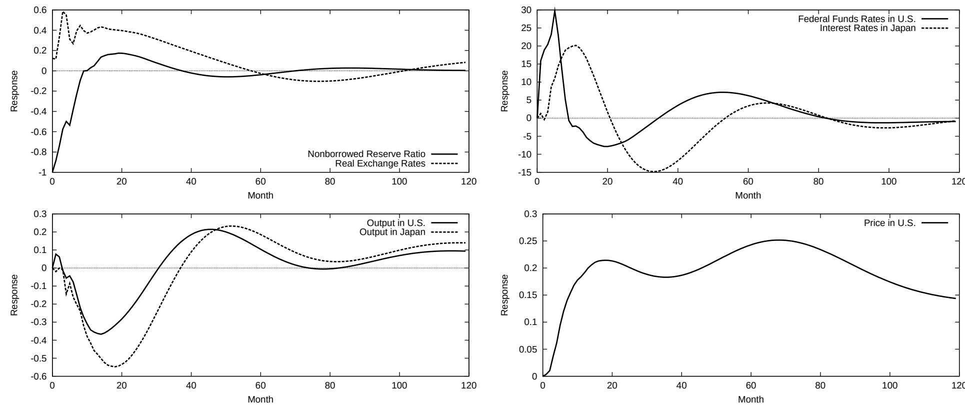
Note: VAR uses seven level variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, nonborrowed reserve ratio, federal funds rates, and real exchange rates(dollar/yen). The lag length is six, which is chosen in Eichenbaum and Evans(1995).

5.3 Interpreting nonborrowed reserve ratio as a monetary policy

This section examines the impulse responses when we measure monetary policy by the nonborrowed reserve ratio, $NBRX$. Following the discussion in Section 5.2, we examine two cases depending on whether r_{ff} is included in x_{1t} or x_{2t}.

Figure 5.3 shows the effects of a contractionary monetary policy shock when the Fed looks at the federal funds rate when it makes monetary policy so that r_{ff} is in x_{1t}. First, we find that the U.S. dollar depreciates for five years compared to the original level, and fails to exhibit either the overshooting behavior or the prediction of UIP. Second, output in the U.S. shows a decrease for three years except for the negligible increase in the initial period. Third, the federal funds rate increases in the initial period and fluctuates thereafter. The interest rate in Japan responds similarly. Fourth, a contractionary monetary policy shock leads to a decrease in the

Figure 5.3: Impulse Responses to Contractionary Shocks in $NBRX$
$(s_t = NBRX, r_{ff} \in x_{1t})$

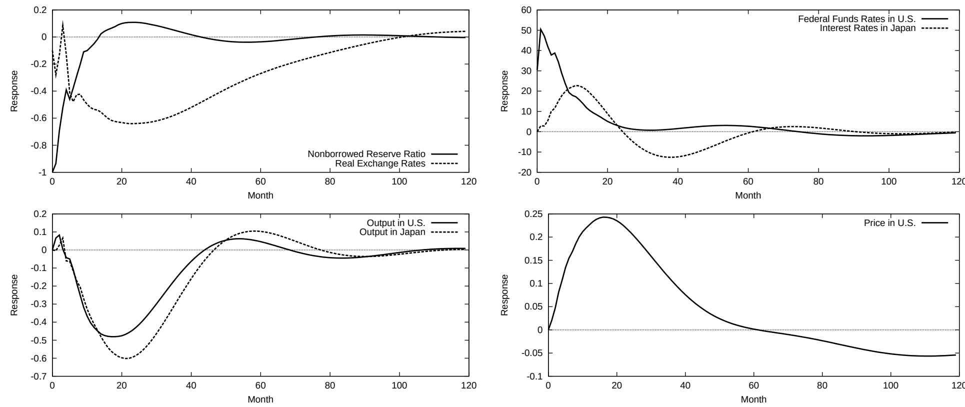


Note: VAR uses seven level variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, federal funds rates, nonborrowed reserve ratio, and real exchange rates(dollar/yen). The lag length is six, which is chosen in Eichenbaum and Evans(1995).

nonborrowed ratio for the initial fourteen months. Finally, the price level in the U.S. exhibits a persistent increase after a contractionary monetary policy showing the "price puzzle".

Figure 5.4 shows the effects of a contractionary monetary policy shock when the Fed does not consider the federal funds rate when it sets monetary policy so that r_{ff} is in x_{2t}. Alternatively, it is assumed that the federal funds rate responds to monetary policy contemporaneously. This is the first model used in Eichenbaum and Evans (1995). The results of output in the U.S., the federal funds rate, the interest rate in Japan, and the nonborrowed reserve ratio are similar to Figure 5.3, while the real exchange rate exhibits delayed overshooting behavior. The U.S. dollar does not show the prediction of UIP, as it continues to appreciate for twenty months. The price level in the U.S. exhibits a substantial increase for four years after a contractionary

Figure 5.4: Impulse Responses to Contractionary Shocks in $NBRX$
$(s_t = NBRX, r_{ff} \in x_{2t})$



Note: VAR uses seven level variables, which are ordered by output in U.S., price in U.S., output in Japan, interest rates in Japan, nonborrowed reserve ratio, federal funds rates, and real exchange rates(dollar/yen). The lag length is six, which is chosen in Eichenbaum and Evans(1995).

monetary policy shock showing the "price puzzle", but its level decreases in the long run.

Therefore, the responses to a monetary shock depends on the selection of the Fed's information set, while the "price puzzle" remains regardless of the information set. These two models used in Eichenbaum and Evans give robust results as long as a monetary policy shock affects both the federal funds rate and the nonborrowed reserve ratio contemporaneously, which is described in Figure 5.1 and Figure 5.4.

6 Concluding Remarks

This paper examined the effects of shocks to U.S. monetary policy on the dollar/yen exchange rate and other economic variables, using structural Vector Error Correction Model (VECM) methods. We compared results from short run and long run restrictions imposed on the structural VECM, and compared our estimates of the impulse responses with those based on levels Vector Autoregression.

From the empirical studies in this paper, we found that estimates of the impulse responses are sensitive to the choice of restrictions. We found evidence for immediate appreciation followed by gradual depreciation in the U.S. dollar with long run restrictions in VECM, but failed to find such evidence with short run restrictions in VECM or in levels VAR. We also resolve the “price puzzle” in VECM with long run restrictions, which is often found in levels VAR.

REFERENCES

Christiano, L. J., M. Eichenbaum, and C. L. Evans (1999): “Monetary Policy Shocks: What Have We Learned and to What End?”, in *Handbook of Macroeconomics*, ed. by J. Tayor, and M. Woodford, vol. 1, chap. 2, pp. 65–148. Elsevier Science.

Doan, T. A. (1992): *RATS User's Manual, Version 4*. Estima, Evanston, IL.

Dornbusch, R. (1976): “Expectations and Exchange Rate Dynamics”, *Journal Political Economy*, 84(6), 1161–76.

Eichenbaum, M., and C. L. Evans (1995): “Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates”, *Quarterly Journal of Economics*, 110, 975–1009.

Engle, R. F., and C. Granger (1987): “Co-Integration and Error Correction: Representation, Estimation and Testing”, *Econometrica*, 55(2), 251–76.

Faust, J., and E. M. Leeper (1997): “When Do Long-Run Identifying Restrictions Give Reliable Results?”, *Journal of Business and Economic Statistics*, 15(3), 345–53.

Fisher, L. A., P. L. Fackler, and D. Orden (1995): “Long-run Identifying Restrictions for an Error-Correction Model of New Zealand Mondy, Prices and Output”, *Journal of International Money and Finance*, 14(1), 127–47.

Jang, K. (2000): “Impulse Response Analysis with Long Run Restrictions on Error Correction Models”, Mimeo., Ohio State University.

Johansen, S. (1988): “Statistical Analysis of Cointegration Vectors”, *Journal of Economic Dynamics and Control*, 12, 231–54.

——— (1995): *Likelihood-Based Inference in Cointegrated Vector Autoregressive Models*. Oxford University Press.

Keating, J. W. (1999): "Structural Inference With Long-Run Recursive Empirical Models", Working Paper No. 1999-3, University of Kansas, forthcoming in *Macroeconomic Dynamics*.

Kilian, L. (1998b): "Small-Sample Confidence Intervals for Impulse Response Functions", *Review of Economics and Statistics*, 80(2), 218–30.

King, R. G., C. I. Plosser, J. H. Stock, and M. W. Watson (1991): "Stochastic Trends and Economic Fluctuations", *American Economic Review*, 81(4), 810–40.

Lütkepohl, H. (1990): "Asymptotic Distributions of Impulse Response Functions and Forecast Error Variance Decompositions of Vector Autoregresssive Models", *Review of Economics and Statistics*, 72(1), 116–25.

Lütkepohl, H., and H.-E. Reimers (1992): "Impulse response analysis of cointegrated systems", *Journal of Economic Dynamics and Control*, 16, 53–78.

Ogaki, M., and J. Y. Park (1997): "A Cointegration Approach to Estimating Preference Parameters", *Journal of Econometrics*, 82, 107–34.

Runkle, D. E. (1987): "Vector Autoregression and Reality", *Journal of Business and Economic Statistics*, 5, 437–42.

Sims, C. A. (1980): "Macroeconomics and Reality", *Econometrica*, 48, 1–48.

——— (1992): "Interpreting the Macroeconomic Time Series Facts: The Effects of Monetary Policy", *European Economic Review*, 36, 975–1000.

Sims, C. A., and T. Zha (1995): "Error Bands for Impulse Responses", Working Paper Series 95-6, Federal Reserve Bank of Atlanta.

Stock, J. H., and M. W. Watson (1988): "Testing for Common Trends", *Journal of the American Statistical Association*, 83(404), 1097–1107.

Zellner, A. (1971): *An Introduction to Bayesian Inference in Econometrics*. Wiley, New York.

APPENDIX A

Monte Carlo Integration

The literature on confidence intervals for impulse response estimates is well explained by Kilian (1998), which can be categorized by following three traditional methods: the asymptotic interval method (see Lütkepohl (1990)), the parametric Monte Carlo integration method (see Doan (1992) and Sims and Zha (1995)), and the nonparametric bootstrap interval method (see Runkle (1987)). This paper uses the Monte Carlo integration method that is used in KPSW. [18]

It is convenient to rewrite the reduced form VECM in (2.2) as:

$$\begin{aligned} \Delta x_t' &= \mu' + x_{t-1}'\beta\alpha' + \sum_{i=1}^{p-1} \Delta x_{t-i}' A_i^{*\prime} + \epsilon_t' \\ &= X_t'\theta + \epsilon_t' \qquad \text{(A.1)} \end{aligned}$$

where $X_t' = (1, x_{t-1}'\beta, \Delta x_{t-1}', \cdots, \Delta x_{t-p+1}')$, and $\theta' = (\mu, \alpha, A_1^*, \cdots, A_{p-1}^*)$. Stacking (A.1) for $t = 1, \cdots, T$, the model becomes a matrix form:

$$Y = X\theta + U \qquad \text{(A.2)}$$

[18]Kilian (1998) examines the accuracy of these confidence intervals in the small samples, and proposes the bootstrap-after-bootstrap method. He finds from Monte Carlo simulations that his method is the best, the Monte Carlo integration method is the second best, the asymptotic interval is the third, and the standard bootstrap interval method is the worst.

Assuming that u_t is i.i.d. and normally distributed, Zellner (1971) finds that Σ follows the Normal-inverse Wishart posterior distribution, with the prior, $f(vec(\theta), \Sigma) \sim |\Sigma|^{-\frac{n+1}{2}}$:

$$\Sigma^{-1} \sim Wishart((T\Sigma_0))^{-1}, T) \tag{A.3}$$

$$\theta \sim N(\theta_0, \Sigma \otimes (X'X)^{-1}) \quad \text{with given } \Sigma \tag{A.4}$$

where θ_0 and Σ_0 are the estimates of θ and Σ, respectively, from OLS or MLE.

The algorithm for estimating confidence intervals of impulse responses is as follows:

1. Estimate (2.2) and let β_0, θ_0 and Σ_0 be these estimates.

2. Let A be a lower triangular matrix of the Choleski decomposition of $(X'X)^{-1}$

3. Let S^{-1} be a lower triangular matrix of the Choleski decomposition of Σ_0^{-1}

4. Generate $n \times T$ random numbers, w_b, from the normal distribution, $N(0, \frac{1}{T})$.

5. Generate $(n(p-1)+r+1) \times n$ random numbers, u_b, from the standard normal distribution, $N(0,1)$.

6. Let $r_b = w_b' S^{-1}$, and get $\Sigma_b^{-1} = r_b' r_b$

7. Let S_b be a lower triangular matrix of the Choleski decomposition of Σ_b

8. Let $\theta = \theta_0 + e_b$, in which $e_b = A u_b S_b'$. Then, $\theta \sim N(\theta_0, \Sigma_b \otimes (X'X)^{-1})$. [19]

9. Draw impulse responses, ir_b, as described in Section 2.3.

[19] Note that $var(e_b) = var(vec(e_b)) = var((S_b \otimes A)vec(u_b)) = S_b S_b' \otimes AA' = \Sigma_b \otimes (X'X)^{-1}$. RATS uses $vec(e_b) = (S_b \otimes I_{n(p-1)+r+1})vec(Au_b)$, which is the same as what I use. Note that $(S_b \otimes A)vec(u_b) = vec(Au_bS_b') = (S_b \otimes I)vec(Au_b)$, in which $vec(ABC) = (C' \otimes A)vec(B)$ is used for transformation.

10. Repeat 4 $\sim$ 9, B times, and calculate one standard error upper and lower error bands of the impulse responses using[20]

$$Upper = \frac{1}{B}\sum_{b=1}^{B} ir_b + (\frac{1}{B}\sum_{b=1}^{B} ir_b^2 - (\frac{1}{B}\sum_{b=1}^{B} ir_b)^2))^{\frac{1}{2}} \tag{A.5}$$

$$Lower = \frac{1}{B}\sum_{b=1}^{B} ir_b - (\frac{1}{B}\sum_{b=1}^{B} ir_b^2 - (\frac{1}{B}\sum_{b=1}^{B} ir_b)^2))^{\frac{1}{2}} \tag{A.6}$$

[20] Note that we fix cointegrating vectors, β, and generate parameters from a normal distribution, $N(\theta_0, \Sigma_b \otimes (X'X)^{-1})$. Note also that we do not update S.