

# Policy Stability under Different Electoral Systems \*

MASSIMO MORELLI\* and MICHELE TERTILT\*\*

\* *Ohio State University*

\*\* *University of Minnesota*

OSU Working Paper no. 00-13, October 2000

## Abstract

This paper looks at the stability of policy outcomes under different electoral systems. The political game among parties and party supporters of different ideologies involves pre-election coalition formation as well as after-election parliamentary bargaining. Voters' preferences vary across districts and over time. We show that under proportional representation there exists a unique stable policy, whereas the policy outcomes under plurality voting are unstable. Moreover, we find that under plurality voting there exist strong incentives for pre-election coalition formation, which are absent under proportional representation. The comparative results may be different depending on the interaction of strategic voting and strategic party coordination.

**Keywords:** Electoral Systems, Heterogeneous Districts, Policy Stability, Demand Bargaining.

---

\*We thank David Austen-Smith, Larry Jones, and Roger Myerson for their extensive comments and suggestions. We also thank V.V. Chari and Daniel Diermeier for helpful discussions. The first draft of this paper was written when Massimo Morelli was visiting the University of Minnesota. Their hospitality is gratefully acknowledged. Michele Tertilt acknowledges support from the MacArthur Foundation.

# 1 Introduction

Research on political institutions has been a growing field within economics over the last decade. Institutional details like party systems, separation of powers, decisiveness of the president vs. congress, are analyzed with particular emphasis on the interplay of political and economic variables.<sup>1</sup> Within this field, we believe that the study of electoral systems is of particular importance. The question of which electoral system to adopt can be found in every day political discussions in many countries.<sup>2</sup>

To see how the electoral system can make a difference, consider the following example. Suppose a country is divided into three districts and suppose voters can vote for any one of three parties within their district. Under simple Plurality Voting (like in the UK, for example), each district is represented by its relative majority winner. Thus, in principle, it is possible that a party wins the overall election (that is, two districts out of three) with little more than  $2/9$ th of the seats.<sup>3</sup> A small perturbation of the voting behavior can easily push a different party in the relative majority range and thereby affect the overall policy dramatically. The same could not happen if the electoral rule was proportional, that is, if seats were allocated according to total votes within the country. This suggests that policies in countries with Proportional Representation (henceforth PR) should be relatively stable, while in countries with Plurality Voting (henceforth PV) policies should change more often. However, this example does not capture the possibility of strategic voting or

---

<sup>1</sup>See Persson and Tabellini (2000) for an excellent survey.

<sup>2</sup>New Zealand held a referendum in 1993 on the electoral system, which led to a switch from a plurality voting system to a more proportional system. On the other hand, Italy changed its system in 1992 in the opposite direction, from Proportional Representation to 75% Single Member District Plurality. The discussion on the electoral system is still on in Italy: a referendum in 1999 and a similar one in 2000 resulted in a majority for pure Plurality Voting, but since less than 50% of the population went to vote the referendum was not effective. Spain and Belgium have also discussed changes in the electoral system at various points in the last decades.

<sup>3</sup>This happens if in two districts one party receives  $1/3 + \epsilon$  of the seats whereas the two other parties get  $1/3 - 1/2\epsilon$  each, and the third district is split among the two latter parties. In this case the first party wins the first two districts with only  $2/9 + 2\epsilon$  of the total votes (assuming that the districts have equal size).

strategic coalition formation. Say the three parties are left-wing, center, and right-wing. Then one of the extreme parties may wish to collaborate with the center party and thereby ensure an absolute majority in a given district, which would then eliminate the vulnerability to small perturbations mentioned above. Thus, even in this example it is not obvious whether electoral systems influence policy stability or not. The balance of these forces is what we will try to clarify in this paper.

We will assume that the candidates of each district must be endorsed by at least one of the existing national parties. Agreeing on candidates is a way for parties to form electoral coalitions.<sup>4</sup> This strategic coalition formation helps voters to coordinate their votes. We will assume that at the beginning of an election period, before the parties begin the pre-election bargaining process, there is a realization of some district-specific shocks, which affect the distribution of voters' preferences in each district. These shocks could be caused by party scandals, changes in the economic situation, or alike. This will cause a fraction of voters in each district to make up their minds,<sup>5</sup> whereas the rest of the voters have ideological preferences, which are not affected by scandals or any other news. In any case, once all news are out and candidates are chosen, elections take place. Under PV the relative majority winner of each district is automatically a member of the parliament, whereas under PR overall majorities are decisive. After the members of parliament are elected, they negotiate within the legislature for the policy decisions. If they do not find any agreement, then the status quo is implemented.

We show that under PR the policy outcome advocated by the median party always arises in equilibrium. Under PV there is instead variance of policy outcomes, and there is no convergence to any specific policy outcome over time. The intuition for these results is that under PV a party might have the relative majority of preferences in the majority of districts, and this makes the status quo, as well as the whole parliamentary bargaining stage, irrelevant.<sup>6</sup> In other words, our main

---

<sup>4</sup>Italy is probably one of the countries where electoral coalition formation has been more pervasive. Especially after the electoral reform of 1992, there have been many changes in the number and composition of electoral coalitions supporting candidates to all kinds of elections.

<sup>5</sup>Alternatively, one could think of this fraction as new voters.

<sup>6</sup>Throughout the paper we will use the terminology "party  $i$  has the majority of preferences"

finding is that PR determines a stable policy, whereas under PV small shocks can have considerable effects on policy outcomes. However, we also characterize a set of situations in which the electoral system is irrelevant.

This result is consistent with the findings of a variety of empirical studies. For example, Houser and Freeman (1998) find that labor tax variability has been much higher in the UK (PV) than in Germany (PR) between 1980 and 1995. Grilli et al. (1991) use the average number of years between a significant government change as an index of stability in a sample of 18 democracies between 1960 and 1990.<sup>7</sup> All countries with a stability index above 10 have PR systems.<sup>8</sup> Although some of the countries with low stability have proportional systems, note that these are mainly young democracies (like Greece or Portugal), which may not have been stable because of some adjustment process after establishing a new democracy. A large literature on public debt and political institutions (see for example Alesina and Perotti (1995), Grilli et al. (1991), and Roubini and Sachs (1989)) finds that PR countries tend to have higher debts, which confirms our result in a subtle way. The evidence on public debt is related to policy stability in the following sense: The productivity slowdown in the 70's caused large budget deficits in most industrial countries. Under PR, the news of the slowdown did not affect the chosen policies and government spending remained at the level desired by the median party. The consequence was a large public debt. In PV countries, on the other hand, the policy choice was more sensitive to the economic news and hence the productivity slowdown influenced public policy in the direction of reducing the public debt. This argument about policy stability complements the more familiar argument that larger public debt to indicate that the majority of citizens prefer the policy advocated by party  $i$ . This is obviously different from the "majority of votes," which could well not be given to party  $i$ , either because of an electoral coalition between two other parties, or because their supporters strategically coordinate their vote.

<sup>7</sup>The authors define a government change to be significant if the following conditions are met: there is a change in the party of the prime minister, and – for coalitional governments – a change in the coalition of parties supporting the government.

<sup>8</sup>The only exception is Japan, but Japan uses multi-member district plurality rule, which has been shown to have properties much closer to PR than to single-member district plurality voting. See Cox (1997).

debts are associated with greater *political* instability (see Ozler and Tabellini (1991)). That policy stability and political stability are far from being necessarily concurrent, can be seen from the example Italy. Between World War II and 1992, Italy was an example of high political instability: Governments lasted on average less than a year. However, matching the predictions of our model, Italy is also an example of a country with maximal policy stability. Over that period, Italy has experienced a very stable size of the welfare state and of redistributive policy. A concrete example of policy stability is given by Tsebelis (1999) who finds that Italy passed only one significant labor law between 1981 and 1991, whereas the corresponding number for the UK is six, and France passed eight. In our model, a country like Italy (before 1992) corresponds to the possibility that the same stable equilibrium policy  $\alpha_M$  is supported by various parliamentary coalitions.

To the best of our knowledge, this paper constitutes the first attempt to study the stability of policies under different institutional systems with a game-theoretic model. Persson and Tabellini (2000) discuss at length some differences across systems in terms of expected outcomes, whereas our comparison concerns the second moment. Baron and Diermeier (1998) analyze a model of parliamentary democracy with proportional representation and an explicit government formation process within the parliament. Contrary to our model, they can generate policy instability even under PR. The main reason for this contrast is that they allow for transfer payments between the parties that form a coalition, while we focus on policy compromises. The nature of shocks considered is also different from ours: they analyze two types of uncertainty, one resulting from the randomization over the agenda-setter, and the other one caused by shocks to the status quo. In this paper we concentrate instead on the effects of exogenous shocks on the preferences of undecided voters (in addition to the randomization over the agenda-setter). Moreover, Baron and Diermeier do not analyze PV systems, focusing exclusively on PR. Merlo (1997) presents an interesting dynamic model in a stochastic environment. However, it takes coalitions as given, and bargaining is only about cake division within the coalition. Austen-Smith (2000) studies the different levels of redistribution chosen in democracies with different electoral systems, but the stability of policies is not

the object of his study. Other static comparisons of electoral systems can be found in Myerson (1993), Lizzeri and Persico (1999), Persson and Tabellini (1999), and Morelli (2000).

The paper is organized as follows. In Section 2 we present the general model. Section 3 contains the main comparative results, analyzing stability as well as coalition formation. Section 4 discusses some important extensions of the model, and Section 5 concludes.

## 2 The Model

Consider a parliamentary democracy where the set of voters is divided in three equal-size voting districts (each district has measure one voters). There are three national parties, each of which is characterized by an ideological position,  $\alpha_i$  ( $i = L, M, H$ ), on a uni-dimensional policy space. One party prefers right-wing policies (for example, low government spending)  $\alpha_L$ , one party prefers left-wing policies (high spending)  $\alpha_H$ , and one party is in between,  $\alpha_M \in (\alpha_L, \alpha_H)$ . Let  $k \equiv \frac{\alpha_H - \alpha_M}{\alpha_M - \alpha_L}$ . Note that  $k < 1$  means that  $M$  is ideologically closer to  $H$  than to  $L$ , and *vice versa* for  $k > 1$ . The utility of policy  $\tau_t$  for party  $i$  is  $u_i(\tau_t) = -(\tau_t - \alpha_i)^2$ , where  $t$  is the time index. Within the population, there are *ideological* voters (or party-supporters) and *undecided* voters. A typical ideological voter prefers one of the three parties in any state of the world. On the other hand, undecided voters form an opinion about their most preferred policy only after the realization of some shock. For example, in any given country some people always prefer less intrusion by the government, and others support a big welfare state as a matter of principle. However, there are voters who prefer low government spending during a boom, and high intervention during a recession. So in this case some news on the economic situation constitutes the shock, after which some voters decide what is their preferred policy. Moreover, it is natural that the undecided voters of different districts may react differently to news about the economy: in a poor district bad economic news can lead the undecided voters to vote left, while in a rich district the reaction may be the opposite, for fear of having to subsidize even more the poorer regions. In particular, we assume that in each district a fraction  $\delta \in (0, 1)$  of the voters is undecided. The shock (or

news) at the beginning of each period makes those voters decide in each district. For simplicity, we assume that each shock affects the opinion of the undecided voters within a given district in the same way, so that within a district all undecided voters will have the same preferences after the shock is realized.

The fraction of ideological voters in favor of party  $i$  ( $i = L, M, H$ ) in district  $l$  ( $l = 1, 2, 3$ ) is denoted by  $v_i^l \in [0, 1 - \delta]$ , so that  $\sum_i v_i^l + \delta = 1 \forall l$ . We will denote by  $V_i \equiv \sum_l v_i^l$  the measure of ideological voters who prefer the policy position of party  $i$  in the whole country. Correspondingly, let  $w_i^l \in [0, 1]$  denote the fraction of voters in favor of party  $i$  in district  $l$  after the realization of the shock:  $w_i^l = v_i^l + \delta$  for some  $i \in \{L, M, H\}$ , and  $w_j^l = v_j^l \forall j \neq i$ . Let  $W_i \equiv \sum_l w_i^l$  denote the measure of voters who prefer party  $i$  in the whole country after the realization of the shocks.

Each party has the option to send one of its supporters into the electoral race in any given district, but can also decide to form a coalition with another party. If parties  $i$  and  $j$  form a coalition, then they will endorse the same candidates in the three districts. Any coalition imposes a small cost  $\epsilon$  on each of the involved parties. Throughout the paper we assume that  $\epsilon$  is small enough to make coalition formation viable.<sup>9</sup> Only citizens that are nominated by a party or coalition run for election.

Under PV, citizens vote for their representative within each district, and the candidate with the most votes of each district is sent to the parliament. So, the three members of the parliament are the relative majority winners of the three voting districts. Under PR, the number of seats accruing to one party depends on the overall vote shares in the country. Following the law in many countries, the seats are assigned according to the Hare Quota, in order to solve the indivisibility problem:<sup>10</sup> A seat is assigned to each party that reaches more than  $1/3$  of the total

---

<sup>9</sup>The main reason for introducing this cost is to ensure that no coalitions form when parties are indifferent. This cost can be interpreted as simple transactions cost of coalition formation. In principle, this cost could be endogenized: for example, it could be derived from some voters abstaining because they do not like compromises; alternatively, one could model the cost as an  $\epsilon$ -probability of one coalition partner defaulting after the election; there could also be uncertainty about what the policy question is, or uncertainty about the quality of the coalitional match. While the above are reasonable interpretations, we do not model them explicitly. We simply assume that coalition formation imposes a utility cost  $\epsilon$  on both coalition partners.

<sup>10</sup>There are two chief families of PR systems, one based on quotas and largest remainders as the

votes, and the remaining seats are distributed according to the largest remainders. It is then the party that chooses the party member who actually occupies the seat.<sup>11</sup>

We now turn to describe the *political game*. Let  $\tau$  be the status quo. The political game can then be denoted by  $\Gamma_{e,v}(\tau)$ , where  $e$  represents the electoral system (i.e,  $e = \text{PR}$  or  $\text{PV}$ ) and  $v$  characterizes the economy by giving the distribution of decided voters' preferences in each district. The time line is as follows.

1. A shock occurs, affecting the preferences of undecided voters (and thereby determining  $\{w^l\}_{l=1,2,3}$ ).
2. The shocks are observed by everybody, so that no uncertainty on voters' preferences is left before the elections.<sup>12</sup>
3. Parties involve in pre-election coalition bargaining. Some parties may agree to have common candidates.
4. After the candidates have been announced, elections are called and voters go to the polls, and vote for the candidate who supports their most preferred policy platform.
5. Election results determine the allocation of seats in the parliament depending on the electoral system as described above.
6. Once the seats in the parliament are determined, the elected members of parliament play an after-election bargaining game to determine the final policy outcome (unless one party has the majority of seats, in which case no bargaining is needed).

---

Hare Quota described in the text, and one based on divisors and largest averages. The best-known system in the second family (invented by Viktor d'Hondt) is based on the calculation of averages that reflect how much each party has paid in votes for each seat. All methods give similar results, the main difference is that the d'Hondt system is the least favorable to small parties. See Cox (1997) for more details on the different PR systems.

<sup>11</sup>It is almost always the case in reality that under PV the voters get to choose the candidates directly, whereas under PR the vote is for the party, and parties choose the office-holders (typically parties establish the ranking of their members before the election).

<sup>12</sup>In Section 4.2 we will consider the extension of the analysis to the case where shocks are unobservable.

Stage (5) and (6) depend on the choice of electoral system: under PV chances are very high that one party will have the absolute majority in the parliament, which implies that the after-election bargaining game is trivial. On the other hand, PR creates little incentives for pre-election bargaining, which makes stage (3) less likely to be relevant and stage (6) almost always relevant.

The equilibrium concept we use in this paper is subgame perfect Nash equilibrium. As we will see later, the equilibrium outcome may depend on the status quo. Over time, however, the equilibrium policy of one period will logically be the status quo of the next period. It is therefore a crucial question whether a policy  $\tau^*$  exists such that the equilibrium policy when  $\tau^*$  is the status quo is exclusively  $\tau^*$  itself. Such a policy (if one exists) can then be regarded as the long run, or stable, outcome.<sup>13</sup> Formally, denote the set of equilibrium outcomes of the game  $\Gamma_{e,v}(\tau)$  by  $T_{e,v}^*(\tau)$ . Then we define a *stable policy* as follows.

**Definition 1**  $\tau^*$  is a stable policy iff  $\{\tau^*\} = T_{e,v}^*(\tau^*)$ .

In the following two subsections we will describe in greater detail the two bargaining stages, (3) and (6).

## 2.1 Pre-election Coalition Formation

Electoral coalition formation takes place at the national level. National parties try to coordinate votes by *strategically coordinating* on candidates. We model the coalitional negotiation stage as a demand bargaining game, in which parties propose a vector of candidates, one for each district. The bargaining game is sequential, and works as follows. Nature chooses the first mover, and the latter chooses the subsequent order of play, i.e., which party will move second and which party will move third. We denote the probability with which nature chooses party  $i$  as first mover by  $\mu_i$ ,  $\mu_i > 0$  for all  $i$ . Once an order of play  $i, j, k$  is determined, the three parties make demands as follows: The first player,  $i$ , makes a demand of candidate types for all three districts,  $x_i = (x_i^1, x_i^2, x_i^3) \in \{L, M, H\}^3$ . Next, the second mover

---

<sup>13</sup>This concept has first been introduced by Austen-Smith (2000).

observes the demand of the first mover and adds his own demand,  $x_j \in \{L, M, H\}^3$ . Finally, the third player observes both demands and adds a third one.<sup>14</sup>

**Definition 2** *Any two parties  $i$  and  $j$  are members of a coalition  $ij$  if and only if  $x_i = x_j$ .*

## 2.2 Parliamentary Bargaining

In order to decide on a policy, at least two out of the three elected politicians in parliament need to agree. The possible scenarios are as follows. If at least two seats are held by candidates of the same type, then no bargaining is needed, and their most preferred policy is implemented. Otherwise there is another bargaining game within the parliament (stage 6). We model this situation of parliamentary bargaining as a sequential demand game which works as follows. Nature chooses the first mover, and the latter chooses the subsequent order of play, i.e., which party will move second and which party will move third. We denote the probability with which nature chooses party  $i$  as first mover by  $\nu_i$ .<sup>15</sup> Once an order of play  $i, j, k$  is determined, the three parties make demands as follows: The first player ( $i$ ) demands a policy,  $\tau_i$ . The second player observes the first demand and makes her own policy demand. If  $\tau_i = \tau_j$ , then a parliamentary coalition is formed, which implements that policy, and the game ends. If  $\tau_i \neq \tau_j$ , the game reaches the third player, who can agree with the first, or the second, or make a new demand. If no pair of players make the same demand, then the status quo policy is implemented.

The following lemma characterizes the continuation equilibria of the last stage of the game, for the case where all parties have one seat.

**Lemma 1** *Suppose that all parties possess one seat in the parliament. Let  $\tau_0$  be the status quo.*

*(i) If  $\tau_0 = \alpha_M$ , then the equilibrium policy outcome of the post-election bargaining*

<sup>14</sup>The advantages of demand bargaining models with respect to alternate-proposal models are discussed at length in Morelli (1999).

<sup>15</sup>Some empirical work has been done to measure  $\nu_i$ . In particular, Diermeier and Merlo (1999) provide strong empirical evidence showing that the probability of being selected as an agenda-setter is typically proportional to the party's seat share.

subgame is  $\alpha_M$ .

(ii) If  $\tau_0 = \alpha_L$  then there are two possible equilibrium outcomes: (1) if  $k < 1$  and  $H$  is the first mover, then  $\tau^* = \alpha_H$  and (2)  $\tau^* = \alpha_M$  for all other cases.

(iii) If  $\tau_0 = \alpha_H$  then there are two possible equilibrium outcomes: (1) if  $k > 1$  and  $L$  is the first mover, then  $\tau^* = \alpha_L$  and (2)  $\tau^* = \alpha_M$  otherwise.

*Proof.* Let us consider case (i) first. Since party  $M$  knows that the status quo ( $\alpha_M$ ) will be implemented if no agreement is reached, there is no incentive for party  $M$  to make any demand other than  $\alpha_M$ . Since there is no policy that makes both  $L$  and  $H$  better off relative to  $\alpha_M$ , there are no demands that  $L$  and  $H$  would agree upon. Hence, independent of the order of play, the policy outcome will always be  $\alpha_M$ .

Next, consider case (ii) where  $\tau_0 = \alpha_L$ . If the realization of nature is such that  $M$  is the first mover, then  $M$  can always make the demand  $\alpha_M$  and choose  $H$  as second mover, knowing that  $H$  would agree, since  $H$  prefers  $\alpha_M$  over the status quo. Suppose now that  $L$  was chosen as the first mover. Since  $L$  knows that both  $H$  and  $M$  prefer  $\alpha_M$  over  $\alpha_L$ ,  $L$  knows that he would not find a coalition partner for the policy  $\alpha_L$ . Hence, the equilibrium policy is once again  $\alpha_M$  (note that this could either occur with  $L$  demanding  $\alpha_M$ , or with a demand of  $\alpha_L$  letting  $M$  and  $H$  agree upon  $\alpha_M$ ). Finally, suppose  $H$  is the first mover.  $H$  knows that  $L$  would prefer the status quo over any other policy; thus, by choosing the order of play  $(H, M, L)$ ,  $H$  may be able to induce  $M$  to agree on  $\alpha_H$ . Note that this will only work if  $M$  prefers  $\alpha_H$  over  $\alpha_L$ , which is equivalent to  $k < 1$ . On the other hand, if  $k > 1$ , then the only thing  $H$  can do is demand  $\alpha_M$ , choosing  $M$  as the second mover, and thereby assuring the equilibrium policy  $\alpha_M$ , which  $H$  likes better than the status quo.

By the same logic, if the status quo is  $\tau_0 = \alpha_H$  and  $M$  prefers  $\alpha_L$  over  $\alpha_H$  (i.e.,  $k > 1$ ), then if  $L$  happens to be the first mover,  $L$  can choose the order of play  $(L, M, H)$  and demand  $\alpha_L$ , knowing that  $M$  will agree. In all other cases,  $\alpha_M$  is the equilibrium policy. **QED.**

## 2.3 Discussion on the Role of Strategic Coordination

In the model, voters vote sincerely for their most preferred policy platform among the available candidates. Parties, on the other hand, are behaving strategically by forming electoral coalitions. Since strategic coordination by parties can be interpreted as a close substitute of strategic voting, we consider sincere voting a reasonable assumption within the model. Sincere voting is not always a good behavioral assumption.<sup>16</sup> The purpose of this section is therefore to explain why this assumption can be a justifiable one in our context, given all the other modeling choices we made.

Consider an example with two slightly different left-wing parties – each of which is the most preferred by 30% of the voters – and one right-wing party – which is the most preferred one by 40% of the electorate. Then, under PV, the right-wing party can win the election if the voters of left-wing views do not coordinate on one of the two left-wing parties. If citizens voted sincerely for one of the three parties, then the equilibrium outcome of the election would indeed be the right-wing policy. Given that for any left-wing voter the right-wing outcome is less desirable than the policy outcome determined by any of the left-wing parties, there is scope for strategic voting *and/or* strategic behavior of parties. Suppose first that parties cannot make coalitional agreements and let us consider voting behavior. Strategic voting typically leads to a large multiplicity of equilibria.<sup>17</sup> Although there is need for voters to coordinate, note that if the left-wing types cannot agree on which candidate to coordinate on, then the outcome can be undesirable to everyone. For exactly this reason, we strongly believe in the role of *parties as coordinators*.<sup>18</sup> Given

---

<sup>16</sup>See Austen-Smith (1989) for an extensive analysis of the legitimacy of sincere voting in electoral competition models.

<sup>17</sup>With a continuum of voters anything is a Nash equilibrium, since no voter can affect the outcome by changing her strategy (and even in finite player games a player is often not pivotal). Farquharson (1969) suggests a concept called *sophisticated voting*, which implies repeated elimination of weakly dominated strategies. While this concept usually does eliminate some multiplicity, it typically still leaves us with a large set of equilibria. In the example above, there are three sophisticated voting equilibrium outcomes (any of the three candidates could win the election).

<sup>18</sup>Alesina and Rosenthal (1996) adopt the concept of coalition-proof Nash equilibrium, which solves the coordination problem in a way that mimics the existence of parties organizing voters'

the multiplicity and coordination problems within strategic voting models, we chose to study explicit coordination among parties instead, keeping the sincere voting assumption. In the example above, the two left-wing parties can avoid all possible risks of having a right-wing outcome by forming an explicit electoral coalition (or a new party), with a common policy platform and common candidates. This approach avoids the multiplicity problem of strategic voting models and, at the same time, gives a more realistic prediction than a sincere voting model where parties are either missing or not acting strategically.

Substituting strategic coordination by parties with strategic behavior by voters, letting the coordination among voters be taken care of some other way, we could generate the same results that we are now going to display. We will not show the results for this alternative substitute framework, which parallel the results of the coming section, but these parallel results are available upon request. Section 4.1 will discuss the implications of having strategic voting and strategic coordination by parties *together* in the model.

### 3 Comparative Results

#### 3.1 Policy Stability

Let us consider proportional representation first. To eliminate uninteresting cases, we assume that no party can win the absolute majority of seats, for any realization of the shocks. Empirically, absolute majorities in PR systems are very rare anyway. Given the definition of shocks and the Hare Quota system, the formal condition that corresponds to this assumption is the following.

**Assumption 1** *For all  $i$ ,  $V_i + 3\delta - 1 < \min\{V_j, V_k\}$ , for  $j, k \neq i$ .*

Intuitively, the left-hand side represents the remainder for party  $i$  in the most favorable case. The condition that the remainder is less than the votes for either of strategies. This is a reasonable solution concept in their two-party model, but it requires a large amount of voter coordination, which we believe is a bit more problematic and questionable in a three-party model.

the two other parties assures that a second seat cannot be assigned to any party. Note that  $\delta$  cannot be larger than  $1/3$ , otherwise Assumption 1 is violated. Hence we will keep  $\delta < \frac{1}{3}$  throughout the paper.

**Proposition 1** *For any  $v$  satisfying Assumption 1,*

$$T_{PR,v}^*(\tau) = \{\alpha_M\} \quad \forall \tau.$$

*Proof.* We need to show that for any status quo  $\alpha_M$  is the unique equilibrium outcome. We will analyze all three potential status quos separately. Consider  $\tau_0 = \alpha_M$  first. We know by Lemma 1 that if the parliamentary bargaining subgame is reached and  $\tau_0 = \alpha_M$ , then the continuation equilibrium outcome is always  $\alpha_M$ . Therefore,  $M$  has no incentive *ex ante* to form any pre-election agreements leading to an equilibrium policy other than  $\alpha_M$ . Indeed, in this case  $M$  has no incentive to be a member of any pre-electoral coalition, since that would mean paying a cost  $\epsilon$  without any policy gains. The other two parties would never find it profitable to form a pre-election agreement between themselves, since their agreement would have to be either weakly to the left or weakly to the right of  $\alpha_M$ , and hence  $H$  or  $L$ , respectively, would prefer  $\alpha_M$ .

Next, consider the case where  $\tau_0 = \alpha_L$ . If  $k > 1$ , then the policy outcome for the subgame where no coalitions are formed is  $\alpha_M$  by lemma 1(ii). Hence, no incentives for pre-electoral coalition formation exist and the equilibrium policy of the overall game is also  $\alpha_M$ . Now suppose that  $k < 1$ . Then, we know by Lemma 1(ii) that the equilibrium policy outcome of the subgame where no coalitions were formed is  $\alpha_H$  with probability  $\nu_H$  and  $\alpha_M$  with probability  $(1 - \nu_H)$ . Note that party  $M$  and party  $L$  would prefer the policy outcome  $\alpha_M$  over a lottery between  $\alpha_M$  and  $\alpha_H$ . Hence, independent of the identity of the first mover of the pre-election stage, there will be a coalition between  $L$  and  $M$  agreeing on candidates of type  $M$  only. (Note that  $L$  cannot demand  $\alpha_L$  since  $k < 1$  implies that  $M$  would never agree.) A similar logic applies for  $\tau_0 = \alpha_H$ . Hence, independent of the status quo,  $\alpha_M$  is always the unique equilibrium policy outcome under PR. **QED.**

Proposition 1 directly implies that  $\alpha_M$  is the unique stable policy under PR. In other words, we can always expect  $\alpha_M$  as the policy outcome in the long run under

PR, independent of the realization of the shocks, and also independent of the initial condition.

In the same parameter space, PV can lead to very different outcomes.

**Proposition 2** *Suppose that the electoral system is PV.*

*The equilibrium policy outcome may differ from  $\alpha_M$  even for  $\tau_0 = \alpha_M$ .*

*Proof.* The proof is by example. Let  $k < 1$  and suppose the distribution of preferences after the realization of the shocks is such that type  $L$  would have the relative majority (but not the absolute) in at least two districts in the absence of coalitional agreements (i.e.,  $w_L^l > w_i^l$  for at least two districts, with  $w_L^l < \frac{1}{2} \forall l$ ). Without loss of generality, suppose this holds for  $l = 1, 2$ .<sup>19</sup> Suppose  $H$  is chosen as the first mover.  $H$  knows that if it cannot secure  $M$  as a coalition partner, then party  $L$  will not join any coalition either, and  $L$  would actually win two districts and implement the policy  $\alpha_L$ . Anticipating this, party  $H$  can obtain  $M$  as a coalition partner supporting two candidates of type  $H$  (recall that  $k < 1$  implies that  $M$  likes  $\alpha_H$  better than  $\alpha_L$ ). In equilibrium  $H$  chooses  $M$  as the second mover and will demand candidates  $(H, H, H)$ . Knowing that not agreeing will lead to  $L$  winning districts 1 and 2,  $M$  will agree to make the same demand. Given that the only available candidates in districts 1 and 2 are of type  $L$  and  $H$ , all  $M$  type voters will vote for  $H$  (since  $k < 1$ ). Since  $L$  does not have the absolute majority in any district, this will lead to at least two politicians of type  $H$  in parliament. Hence, the equilibrium policy is  $\alpha_H$ . Note that a similar situation exists in a country where  $k > 1$ . In this case the realization of shocks has to be such that  $H$  would potentially win two districts, which would then lead to the equilibrium policy outcome  $\alpha_L$  in the subgame where  $L$  is chosen as the first mover. **QED.**

**Corollary 1** *If one party happens to have, after the realization of the shocks, the relative majority in at least two districts, then there is no bearing of the status quo on the equilibrium policy.*

---

<sup>19</sup>Note that this may happen under the same parameter values satisfying Assumption 1.

On the other hand, if no party has the relative majority in two districts, then we could have some cases where the agreements *ex ante* depend on the status quo, similar to Lemma 1.

Propositions 1 and 2 show that PV and PR may lead to very different policy outcomes. Suppose, for example, that the initial status quo is  $\tau_0 = \alpha_L$ . Then, by Proposition 1, the equilibrium policy outcome under PR is  $\alpha_M$ . On the other hand, under PV the policy outcome may also be  $\alpha_H$ . Even in the long run the difference remains. This important point is formalized in Proposition 3, which shows that there are no stable policies under PV. Of course, the occurrence of policy fluctuations under PV depends on the distribution of preferences in the country and the magnitude of the shocks. Thus, we now give a precise condition under which policy fluctuations happen.

Let

$$p_i \equiv \text{prob}[(w_i^l > w_j^l \ \forall j \neq i) \cap (w_i^{l' \neq l} > w_j^{l' \neq l} \ \forall j \neq i)]$$

denote the probability that party  $i$  has the (at least relative) majority in at least two districts.

**Assumption 2** *The distribution of preferences and the magnitude of the shocks are such that  $p_i > 0 \ \forall i$ .*

**Proposition 3** *If Assumption 2 is satisfied, then there is no stable policy under PV.*

*Proof.* We need to show that given Assumption 2 there exists no policy  $\tau$  such that  $\{\tau\} = T_{PV,v}^*(\tau)$ . It should be clear from the analysis above that  $\alpha_M$  is always an element of  $T_{PV,v}^*(\tau)$ , for all  $\tau$ . Hence, the only potential candidate to be a stable policy is  $\alpha_M$  itself. So we just need to show that under Assumption 2 there always exists also some  $\tau \neq \alpha_M$  in  $T_{PV,v}^*(\alpha_M)$ . Assumption 2 says that the events where one of the parties has at least the relative majority of preferences in at least two districts occur with positive probability. The following four different scenarios are an exhaustive categorization of what can be implied by this assumption.

1. The probability that  $L$  or  $H$  obtains the absolute majority in at least two districts is zero;

2. The probability for  $L$  to obtain the absolute majority after the realization of the shocks is positive, but not for  $H$ ;
3. The probability for  $H$  to obtain the absolute majority after the realization of the shocks is positive, but not for  $L$ ;
4. For both  $H$  and  $L$  the probability to obtain the absolute majority after the realization of the shocks is positive.

Consider first case 1. Suppose, without loss of generality, that  $k < 1$ . We can then show that  $\alpha_H \in T_{PV,v}^*(\alpha_M)$ . To see this, consider the realization of the shocks such that party  $L$  has the relative majority of preferences in at least two districts. Suppose  $H$  is the first mover in the pre-electoral bargaining game (recall that this happens with probability  $\mu_H > 0$ ). Knowing that  $L$  would win the overall election and implement policy  $\alpha_L$  if no coalitions were formed, and knowing that  $M$  prefers  $\alpha_H$  to  $\alpha_L$  (from  $k < 1$ ),  $H$  can demand  $(H, H, H)$  and  $M$  agrees. Since only candidates of types  $L$  and  $H$  are running, voters of type  $M$  vote for  $H$ , and therefore the coalition wins and the equilibrium policy outcome is  $\alpha_H$ . (With  $k > 1$ , symmetrically, we could show that  $\alpha_L \in T_{PV,v}^*(\alpha_M)$ .)

In case 2, if  $k < 1$  we could replicate the argument above to prove that  $\alpha_H \in T_{PV,v}^*(\alpha_M)$ , and because of the additional possibility of absolute majority even  $\alpha_L$  is an element of  $T_{PV,v}^*(\alpha_M)$ . If  $k > 1$  the latter statement remains true. A symmetric list of possibilities exist in case 3, and in case 4 anything can be an equilibrium (for some realization of the shocks) whatever  $k$  is. **QED.**

Note that the more symmetric the distribution of ideological preferences is across parties, the larger is the range of values of  $\delta$  that satisfy Assumption 2. In particular, note that if the ideological preferences are exactly equally distributed, then any fraction  $\delta \in [0, \frac{1}{3})$  of undecided voters would satisfy Assumptions 1 and 2. As the distribution of  $v$  gets less symmetric, Assumptions 1 and 2 will still be satisfied simultaneously for smaller  $\delta$ s. Hence, the comparison between electoral systems can be done for a large set of parameters. Moreover, notice that Assumption 2 is only

a sufficient condition for Proposition 3, and it could be relaxed.<sup>20</sup>

A situation in which Proposition 3 trivially does not apply is the one where party  $M$  wins at least two districts in every state of the world. Another extreme case is that of regional parties, i.e., when every district is dominated by one preference type. In this case every party would win exactly one seat in the parliament. Therefore, this case is equivalent to PR. Lemma 1 applies. Both of these scenarios involve very extreme distributions of preferences, and are probably not relevant for most countries. In more realistic scenarios the variability of the policy outcome is a general feature of PV. We can then emphasize the main point of this paper with the following remark:

**Remark 1** *Under Assumptions 1 and 2 the variance of policy outcomes is strictly higher under PV than under PR.*

## 3.2 Coalition Formation

So far we have focused on the differences in terms of stability of policy outcomes. We have not yet analyzed incentives to form coalitions. A well known observation in political science is the so called “Duverger’s Law.” This law states that simple plurality rule favors a two-party system, whereas proportional representation favors multipartyism.<sup>21</sup> Duverger’s Law is typically formulated in a context where there are no districts. However, if the distribution of preferences is not too heterogeneous across districts then one should expect similar forces to operate at the national level.<sup>22</sup> It is indeed the case that especially countries that have had a PV system for a long time tend to have a smaller number of parties than countries with a PR system. Our model highlights one key reason for this phenomenon: Coordination

---

<sup>20</sup>For example  $p_L > 0$  when  $k < 1$  and  $p_H > 0$  when  $k > 1$  would be a weaker, but less intuitive, sufficient condition. This milder condition implies that the range of values of  $\delta$  satisfying Assumption 2 is large even if the ideological preferences are far from being symmetrically distributed. It just requires that there exist some states of the world where one of the extreme parties has the relative majority of voters’ preferences.

<sup>21</sup>See for example Cox (1997) for an extensive analysis of Duverger’s original statements.

<sup>22</sup>For an analysis of Duverger’s Law with an explicit modeling of party formation in a multi-district economy see Morelli (2000).

problems among voters are more severe under PV than under PR, which create an incentive to form pre-electoral party alliances. The incentives are reversed under PR. Since coordination is not a problem, there is no incentive to bargain *ex ante*, which leads to more parties running in the election. The negotiation can be postponed to the parliamentary bargaining stage.

Admittedly, a model with exogenous parties like ours misses a large part of party dynamics. We therefore cannot claim that the incentives displayed in our model are the only reason why PV democracies typically have a smaller number of parties than PR countries. But we strongly believe that ours is a large part of the story. The tendency of PV elections to lead to absolute majorities in parliament (and thereby leaving no room for parliamentary bargaining) creates strong incentives to coordinate before the election. We now summarize the above in the following proposition.

**Proposition 4** *Given  $\epsilon > 0$  and Assumption 1, under PR no electoral coalitions ever form in any period  $t > 0$ . Under PV on the other hand, as long as Assumption 2 is satisfied, there are strong incentives to form electoral coalitions.*

*Proof.* Proposition 1 shows that the policy outcome under PR is always  $\alpha_M$ . Hence, we know that in any period  $t > 0$  the status quo is  $\alpha_M$ . Given Assumption 1 and given the status quo  $\alpha_M$ , if coalition formation was costless ( $\epsilon = 0$ ) party  $M$  would be indifferent between forming a coalition with the policy goal  $\alpha_M$  or entering the election without any coalitions formed. Hence, as soon as  $\epsilon$  is strictly positive,  $M$  will never agree to enter a coalition in equilibrium. Note that coalition formation could potentially occur in the very first period even under PR.<sup>23</sup> Under PV, on the other hand, there is generally no stable policy, as seen in Proposition 3. By an argument similar to the one used in the proof of Proposition 3, coalitions do form under PV. This happens any time when one of the extreme parties has the

---

<sup>23</sup>Suppose  $\tau_0 = \alpha_H$  and  $k > 1$ . Then by Lemma 1 the expected policy outcome in the subgame where no coalitions are formed is between  $\alpha_L$  and  $\alpha_M$ . Knowing this,  $H$  prefers to form a coalition with  $M$  where only candidates of type  $M$  run for election, leading to the policy outcome  $\alpha_M$ . Since in this case such an agreement leads to a strictly higher expected pay-off for both  $H$  and  $M$ , they will form a coalition, independent of the order of play. But this can happen only in period 0.

(relative) preference majority in at least two districts. In these cases, the other two parties will agree to form a coalition. For example, if  $L$  would win two districts in the absence of coalitions, then  $M$  and  $H$  will form a coalition sending either only candidates of type  $H$  (for  $k < 1$ ) or only of type  $M$  (for  $k > 1$ ). **QED.**

Of course, in our model coalition formation can vary from period to period. A strict interpretation of Duverger's Law would be that only two parties exist. In other words, coalitions form once and for all. But this strict interpretation is often a misconception. For example, since 1992 the Italian electoral system is 75% PV, and this led to the formation of two large electoral coalitions, but the composition of such coalitions keeps changing from election to election. Even in one of the prototype PV examples, the UK, a careful reading of history shows that parties do break up and reform from time to time.<sup>24</sup>

## 4 Strategic Voting, Observability, and Commitment

In this section we study the robustness of our main findings to some important modifications of the model. Even though we believe that in the real world the role of coordinating voters is played by the parties, as argued in Section 2.3, it is important to check what happens when and if voters are strategic and parties can anticipate this strategic behavior. We show in Section 4.1 that if voters are strategic and no realization of the shocks can ever give any party the absolute majority of preferences in two or more districts, then the electoral system is irrelevant, at least when considering symmetric equilibria. This result restricts, with respect to the sincere voting model, the set of parameter values that yield instability under PV.

Our second extension deals with the possibility that the relevant pre-election shocks (scandals and economic news) occur right before the election, without giving time to the relevant parties to react. In other words, the pre-electoral coalitions are

---

<sup>24</sup>Just to give one example: The liberal party split in 1916. The national liberals (part of the former liberal party) then formed a coalition with the conservatives. By 1950, the national party had been completely integrated within the conservative party. The labour party, on the other hand, was not established until 1893, but gained more and more influence thereafter. See Ingle (2000) for an extensive discussion of the British party system.

often decided upon without knowing the realization of the shocks. We show that our main results extend to this unobservable shocks case, but, once again, for a smaller set of parameter values.

Finally, we briefly discuss the implications of allowing parties to form long-run commitments, rather than making just a one-period electoral agreement.

#### 4.1 Strategic Voting

Consider a modified game where voters are strategic in their decision about which party to vote for. In other words, the voters who prefer policy  $\alpha_i$  vote for candidates of the party that is expected to bring the policy outcome the closest possible to  $\alpha_i$ , even if that means voting for a party different from party  $i$ . In order to simplify the analysis of this modified game, we assume that all the voters with the same ideology can “perfectly coordinate” their decisions. Everything else remains the same as in the original game form.

The reason for adding the “perfect coordination” assumption (or “symmetry” assumption) when introducing strategic voting is that such an assumption allows us to eliminate the multiplicity of equilibria that would otherwise generally ensue in strategic voting models. The focus on symmetric behavior constitutes an assumption about equilibrium selection.

**Assumption 3** *A voting equilibrium is a Nash equilibrium where the voters with the same preference vote the same way.*

Assumption 3 allows to solve the strategic voting stage as if there were just three players. Now, to introduce a sufficient condition for our irrelevance result, define

$$p_i^a \equiv \text{prob}[(w_i^l > \frac{1}{2}) \cap (w_i^{l'} > \frac{1}{2}), l \neq l']$$

as the probability that party  $i$  obtains the absolute majority of preferences in at least two districts after the realization of the shocks. Correspondingly, define

$$p_i^r \equiv \text{prob}[(w_i^l > w_j^l \forall j \neq i) \cap (w_i^{l'} > w_j^{l'} \forall j \neq i) \cap (w_i^k < \frac{1}{2} \forall k)]$$

as the probability that party  $i$ , after the realization of the shocks, has the relative majority of preferences in at least two districts but never the absolute one.

**Assumption 4**  $p_i^a = 0$  for  $i = L, H$ .

**Proposition 5** *If Assumptions 3 and 4 are satisfied, then the unique stable policy outcome of the modified game where strategic voting is allowed is  $\alpha_M$  under both electoral systems, i.e., the electoral system is irrelevant.*

*Proof.* Under PR strategic voting does not alter the result of proposition 1. Moreover, as noted before, the only candidate for a stable policy under PV is  $\alpha_M$ . In the modified game considered here we can indeed show that  $\alpha_M$  is the unique stable policy under PV as well. To see this, let us consider the status quo  $\alpha_M$ , and let us solve the game backwards.

Lemma 1 guarantees that if in the parliament there is one player of each type then the outcome is always  $\alpha_M$ , whatever the order of play is. Given this, let's go back to the voting stage, which now is a strategic stage of the overall game. Consider the node where no party coalitions were made, so that in each district voters can choose between three candidates of three different types. There are 10 different types of situation that can characterize such a node (ties can be ignored): (1)  $H$  has the relative majority of preferences in every district; (2)  $L$  has it in every district; (3)  $M$  has it in all districts; (4)  $H$  has it in two districts,  $M$  in the third; (5)  $H$  has it in two and  $L$  in the third; (6)  $L$  in two and  $M$  in one; (7)  $L$  in two and  $H$  in one; (8)  $M$  in two,  $H$  in one; (9)  $M$  in two,  $L$  in one; (10) every party has the relative majority of preferences in one district. Because absolute majorities are ruled out by 4, the general feature of these situations is that *any* two groups of voters in any district can provide the absolute majority of votes in that district by voting for the same candidate, no matter what party has the relative majority of preferences. Cases 3,8,9 would all see the median party win, since  $H$  and  $L$  supporters cannot all benefit from coordinating on one of their candidates. In case 10, Lemma 1 implies that all continuation equilibria have  $\alpha_M$  as the equilibrium outcome. Hence,  $M$  supporters do not need to coordinate with anybody, knowing that the other two types cannot both benefit from coordinating on something else. In case 1,  $\alpha_M$  is the only continuation equilibrium outcome if  $k < 1$ , but if  $k > 1$  then there also exist continuation equilibria where  $M$  and  $L$  coordinate on  $L$  candidates, determining  $\alpha_L$ . Symmetrically, in case 2 the unique continuation equilibrium outcome is  $\alpha_M$  if

$k > 1$ , while if  $k < 1$  then there exist continuation equilibria yielding  $\alpha_H$ . Case 4 and 5 are like case 1, and cases 6,7 are like case 2.

Having solved for the equilibrium starting from the voting stage at all possible subgames, let us now consider the electoral coalition formation stage. It is obvious (given the  $\epsilon$  cost) that in cases 3,8,9,10  $M$  would not enter any electoral coalition, knowing that the ensuing continuation equilibrium outcome would anyway be  $\alpha_M$ . Let us then consider cases 1,4,5. If  $k < 1$ , then  $\alpha_M$  is once again the only possibility, hence no coalitions can form *ex ante*. If  $k > 1$ , then  $H$  knows that there exist equilibria in the continuation game where  $\alpha_L$  prevails; hence, assuming that all equilibria of the voting game can occur with positive probability,  $H$  is better off demanding  $(M, M, M)$  and forming an electoral coalition with  $M$ , yielding  $\alpha_M$ . A symmetric argument applies to the remaining three cases, and hence we have the complete proof that in this setting  $\{\alpha_M\} = T_{PV,v}^*(\alpha_M)$ . **QED.**

As soon as  $p_i^a > 0$  for  $i = L$  or  $H$ , fluctuation of policy outcomes can of course still occur over time. Thus, since  $p_i^a > 0$  is compatible with Assumption 1, the conclusion of Remark 1 remains valid even with strategic voting.<sup>25</sup> However, since the space of parameters where fluctuations occur is restricted under strategic voting with respect to the sincere voting benchmark, the relative empirical relevance of the results in this versus the previous section may depend crucially on the degree of strategic voting in the various countries that use a PV system. Considering an intermediate case, where there are some sincere and some strategic voters in the population, it is possible to show (details available upon request) that the space of parameters  $(v, \delta)$  generating fluctuations under PV is always non-empty and gets larger and larger as the fraction of sincere voters increases.

Another remark about Proposition 5 is that if the policy space was continuous, i.e., if pre-election agreements could stipulate that the coalition of  $i$  and  $j$  pursues a policy platform in the interval  $(\alpha_i, \alpha_j)$ , then the analysis of cases 1,2,4,5,6,7 in the proof of Proposition 5 would change, and center-left and center-right agreements

---

<sup>25</sup>After the realization of the shocks one party could have a slight absolute majority in two districts and almost nothing in the third one, and under PR this would not allow such a party to obtain two seats if the other two parties share more or less equally the remaining  $2/3 - \epsilon$  of the national electorate's preferences.

would appear in  $T_{PV,v}^*(\alpha_M)$ , destroying the stability of  $\alpha_M$  and the irrelevance result.

## 4.2 Unobservable Shocks

So far we have assumed that the uncertainty about shocks is resolved by the time parties engage in pre-electoral coalitional bargaining. We believe that this is a realistic assumption since opinion polls typically represent voters' preferences pretty well. However, in some countries opinion polls may be less well-developed, or even known to be inaccurate. Therefore, the reader may ask how robust our results are to a reversal of the timing, where the realizations of the shocks become known only after pre-election bargaining has taken place, or are not known at all. Intuitively, risk averse extreme parties will prefer a moderate policy outcome with certainty to a lottery over its own and the other extreme party's bliss points. Thus, one may be inclined to think that the policy instability under PV (as shown in Proposition 3) vanishes when parties are uncertain about undecided voters' preferences during the pre-electoral bargaining phase. However, this is not generally true. In this section we will give the intuition and some examples for why this is not the case.

Note first that shocks do not play any role under PR: As long as Assumption 1 is satisfied, shocks do not affect the relative majorities in parliament. Therefore, Proposition 1 still applies. In the analysis of PV, we can now focus on the case where  $\tau_0 = \alpha_M$ , since if we can find long run variability of the equilibrium policy outcomes under PV even for  $\tau_0 = \alpha_M$ , then we have shown that the difference between PR and PV does not generally disappear in the presence of uncertainty.

One class of situations where a policy outcome different from  $\alpha_M$  always exists in  $T_{PV,v}^*(\alpha_M)$  is characterized by  $p_i^a > 0$  for  $i = L, H$ . The logic is simple. While one of the extreme parties may form an ex-ante coalition with  $M$  agreeing on one of their ideal points, there is always the possibility that the other extreme party obtains the absolute majority of seats in the legislature and therefore implements its ideal point (i.e.  $\alpha_L$  or  $\alpha_H$ ). Note that in the model with perfectly observable shocks (even with strategic voting)  $p_L^a > 0$  or  $p_H^a > 0$  is sufficient for the non-existence of a stable policy under PV. This is not necessarily true when shocks are unobservable. To see this, assume that  $p_L^a > 0$  and  $p_H^a = 0$ . If  $p_L^a$  is small relative to  $p_H^r$ , then  $L$  may still

be willing to form a coalition with  $M$ , leading to the policy  $\alpha_M$ , to prevent  $H$  from winning. If  $p_H^a = 0$ , then  $\alpha_M$  would be the unique stable policy.

The above shows that the unobservability of shocks reduces the class of parameters for which PV leads to policy instability. However, the condition  $p_i^a > 0$  for  $i = L, H$  is not necessary. In the following we provide an example in which even though  $p_i^a = 0$  for all  $i$ , instability prevails: Let  $k = 1/2$  (i.e.  $M$  prefers  $\alpha_H$  over  $\alpha_L$ ),  $p_L^r = 0.5$ ,  $p_H^r = 0.1$ ,  $p_i^a = 0$  for all  $i$  and suppose  $H$  is the first mover in the pre-electoral coalition formation game. Then, it can be shown that  $L$  prefers no coalition over agreeing to  $\alpha_M$ . Moreover, it can be shown that  $M$  prefers  $\alpha_H$  over a gamble with a high probability (0.5 in this example) of  $\alpha_L$  being the equilibrium policy. Therefore, as long as  $H$  is the first mover,  $H$  and  $M$  will form a coalition that sends only candidates of type  $H$  into the race. The coalition will win the election for sure and therefore  $\alpha_H \in T_{PV,v}^*(\alpha_M)$ , and hence no stable policy exists. The above example shows that a certain degree of asymmetry is needed for instability when shocks are unobservable. As long as chances are similar for both extreme parties (e.g.  $p_L^r = p_H^r$ , or  $k$  approaching 1), it is always strictly better for both extreme parties to form a coalition that implements policy  $\alpha_M$ .<sup>26</sup>

To sum up, the above discussion shows that the spirit of remark 1 is still true under uncertainty: There are many cases in which the variance of policy outcomes is strictly higher under PV than under PR.

### 4.3 Long-Run Commitments

In this paper we have assumed that every election period the three parties can renegotiate everything. What if long-run agreements are possible, so that two parties can become one and commit to remain united? Even if two parties merge forever, determining a two-party system, there can still be switching of power between the two parties over time. This is certainly the case in reality, both in the US and the

---

<sup>26</sup>The precise condition identifying a subset of the parameter space of 2 where instability prevails even though  $p_i^a = 0$  for  $i = L, H$  is given by  $k < 1$  and  $[(k+1)^2 - (k^2+1)]p_H^r < 2p_L^r - k^2$ , or  $k > 1$  and  $[(k+1)^2 - (k^2+1)]p_L^r < 2p_H^r k^2 - 1$ . Details available upon request.

UK.<sup>27</sup>

Formally, suppose that at time 0 parties can form coalitions (or new parties) that will remain united forever. For a convenient comparison, let's assume that the rules of such a coalition formation game are the same as those described in our model. Concretely, if party  $L$  ( $H$ ) announces the same vector of candidate types as  $M$ , then such a vector of candidate types will be kept forever after in every election by the new party  $LM$  ( $MH$ ).<sup>28</sup> In this modified model, the incentives for two parties to form a long run coalition are very similar to the incentives discussed in Section 4.2: Parties do not know which shocks will be realized in the future, and therefore may seek insurance by forming a long run coalition. The results with the possibility of long-run commitments are then as follows: (1) If the cost of forming coalitions (relative to the other parameters) are such that no long-run coalition forms, then the instability analysis of PV is obviously the same as in the previous sections; (2) if party  $LM$  ( $MH$ ) forms, then instability exists if and only if  $p_H^a$  ( $p_L^a$ ) is greater than 0. Hence,  $p_i^a > 0$  for  $i = L, H$  is a sufficient condition for the non-existence of stability under PV, like in the case where shocks are unobservable.

Besides the fact illustrated above that instability is still a relevant phenomenon even when long-run commitment is possible, there are at least two important caveats to the existence of long-run commitments. First, while parties may live forever, party members do not, and new members may have an incentive to renegotiate commitments made by previous party members. Secondly, we believe that there is a difference between countries where PV was adopted from the start and countries where PV was chosen after a long period of PR experience. Consider the example Italy. Even though Italy has now a 75% PV system, the number of parties remains large, and the electoral coalitions keep changing, depending on the most important shocks before any new election. So, the instability property should be an especially robust finding for “young” PV systems.

---

<sup>27</sup>Especially in the UK the presence of an effective two-party system seems to have made the variance of policy outcome actually greater than in other countries, judging from the very different policies of Mrs. Thatcher and the Labour Party.

<sup>28</sup>We limit attention to the potential formation of  $LM$  and  $MH$ , since the formation of  $LH$  can be ruled out with very similar arguments to the ones used in the previous sections.

## 5 Concluding Remarks and Research Agenda

In this paper we have shown that policy outcomes in countries that have a plurality voting system can be very different from the policy outcome in a country with proportional representation, even when distributions of preferences within the countries are identical. While under a plurality voting system the policy outcome is very sensitive to the realization of exogenous shocks, the median party is decisive under proportional representation, no matter what the shocks are. This implies that PR leads to a stable policy that is implemented every period, whereas PV leads to frequent changes of policies. A second difference between the two electoral systems has been analyzed: While plurality voting provides strong incentives for pre-electoral party alliances, such incentives are absent under proportional representation.

The paper proposes a positive method of comparing different electoral systems. A normative analysis of these issues is still missing. At this point it is unclear whether more stability is better or worse. On the one hand, stability ensures security and reduces uncertainty. On the other hand, stability also implies a lack of flexibility, and it is possible that necessary policy changes are not implemented. This discussion is beyond the scope of this paper, but a normative comparative analysis is certainly desirable. We will study the welfare properties of electoral systems in a separate project, together with the role of interest groups in each system.

Another important direction for future research would be the extension to a multi-dimensional policy space. For example, if local public goods are part of the political decision, then the differences between the two electoral systems may be even more profound, since districts are directly represented only under PV. District accountability leads to budgetary externalities, whereas one would expect the free rider problem to be internalized under PR.<sup>29</sup>

The potential extension of this paper that is most interesting to us is related to our findings on the role of strategic versus sincere voting for the comparative results. We have shown that if (1) all voters are strategic, if (2) they can perfectly coordinate within the same ideological group, and if (3) no party can ever have the

---

<sup>29</sup>See Chari, Jones and Marimon (1997) for a model of parliamentary democracy where a budgetary externality leads to over-provision of local public goods.

absolute majority of preferences, then the electoral system becomes irrelevant. The greater variability of policy outcomes over time under PV remains a true statement as long as one of those three conditions is violated. This characterization could be useful in future research for some important behavioral questions, like what is the degree of strategic voting in a given democracy that uses plurality voting. Observed policy instability over time is compatible with our model if one (or more) of the three conditions above is violated. Focusing on countries where the states of the world where one party has the absolute majority of preferences in at least two districts are not frequent, and controlling for the prevalence of party alliances (or using some other measures of voters' coordination), we could interpret high variability of policy outcomes as an empirical indication of the existence of sincere voters.

## References

- [1] ALESINA, A. and R. PEROTTI (1995), "Fiscal Expansions and Adjustments in OECD Countries," *Economic Policy* **21**, 207-248.
- [2] ALESINA, A. and R. ROSENTHAL (1996), "A Theory of Divided Government," *Econometrica* **64-6**, 1311-41.
- [3] AUSTEN-SMITH, D. (1989), "Sincere Voting in Models of Legislative Elections," *Social Choice and Welfare* **6**, 287-299.
- [4] AUSTEN-SMITH, D. (2000), "Redistributing Income under Proportional Representation," *Discussion Paper No. 1270*, Northwestern University.
- [5] BARON, D. and D. Diermeier (1998), "Dynamics of Parliamentary Systems: Selections, Governments, and Parliaments," Northwestern University, Discussion Paper No. 1215.
- [6] CHARI, V.V., JONES, L. and R. MARIMON (1997), "The Economics of Split-Ticket Voting in Representative Democracies," *American Economic Review* **87-5**, 957-976.
- [7] COX, G.W. (1997), *Making Votes Count: Strategic Coordination in the World's Electoral Systems*, Political Economy of Institutions and Decisions series, Cambridge; New York and Melbourne: Cambridge University Press.
- [8] DIERMEIER, D. and A. MERLO (1999), "An Empirical Investigation of Coalitional Bargaining Procedures," Discussion Paper No. 1267, Northwestern University.
- [9] FARQUHARSON, R. (1969), "Theory of Voting," New Haven: Yale University Press.
- [10] GRILLI, V. , MASCIANDARO, D. and G. TABELLINI (1991), "Political and Monetary Institutions and Public Financial Policies in the Industrial Countries," *Economic Policy* **13**, 342-392.

- [11] HOUSER, D. and J. FREEMAN (1998), "Economics Consequences of Political Approval Management in Comparative Perspective," mimeo, University of Minnesota.
- [12] INGLE, S. (2000), *The British Party System*, Pinter, London and New York.
- [13] LIZZERI, A. and N. PERSICO (1998), "The Provision of Public Goods under Alternative Electoral Incentives," University of Toulouse. Mimeo.
- [14] MERLO, A. (1997), "Bargaining over Governments in a Stochastic Environment," *Journal of Political Economy* **105-1**, 101-131.
- [15] MORELLI, M. (1999), "Demand Competition and Policy Compromise in Legislative Bargaining," *American Political Science Review* **93-4**, 809-820.
- [16] MORELLI, M. (2000), "Party Formation and Policy Outcomes under Different Electoral Systems," mimeo.
- [17] MYERSON, R. (1993), "Incentives to Cultivate Favored Minorities under Alternative Electoral Systems," *American Political Science Review* **87**, 856-869.
- [18] MYERSON, R. and R. WEBER (1993), "A Theory of Voting Equilibria." *American Political Science Review* **87**, 102-114.
- [19] OZLER, S. and G. TABELLINI (1991), "External Debt and Political Instability," NBER working paper no. 3772.
- [20] PERSSON, T. and G. TABELLINI (1999), "Marshall Lecture. The Size and Scope of Governments: Comparative Politics with Rational Politicians," *European Economic Review* **43**, 699-735.
- [21] PERSSON, T. and G. TABELLINI (2000), *Political Economics: Explaining Economic Policy*, MIT Press, Cambridge Massachusetts.
- [22] ROUBINI, N. and J. SACHS (1989), "Political and Economic Determinants of Budget Deficits in the Industrial Democracies," *European Economic Review* **33**, 903-938.

- [23] TSEBELIS, G. (1999), "Veto Players and Law Production in Parliamentary Democracies: An Empirical Analysis," *American Political Science Review*, **93**-3, 591-608.