Econ 741 (Spring 2003)

Suggested Answer for Homework 4

Since $\sqrt{n}(\mathbf{x}_n - \beta) \xrightarrow{d} N(0, \Sigma)$, *plim* $\mathbf{x}_n = \beta$.

a) $(x_{n,3})^{\frac{1}{2}}$ is a consistent estimator for $(\beta_3)^{\frac{1}{2}}$. A point estimate is 1.2649. To compute its asymptotic variance, let $a(\mathbf{x}) = (x_{n,3})^{\frac{1}{2}}$. Then, $A(\mathbf{x}) = \frac{\partial a}{\partial \mathbf{x}'} = (0, 0, \frac{1}{2\sqrt{x_{n,3}}})$. Applying the delta method, we obtain $\sqrt{n}((x_{n,3})^{1/2} - (\beta_3)^{1/2}) \xrightarrow{d} N(0, C)$, where $C = (0, 0, \frac{1}{2\sqrt{\beta_3}})\ \Sigma\ (0, 0, \frac{1}{2\sqrt{\beta_3}})'$.
A consistent estimator for C is $(0, 0, \frac{1}{2\sqrt{x_{n,3}}})\ \Sigma_n\ (0, 0, \frac{1}{2\sqrt{x_{n,3}}})' = \frac{1}{4x_{n,3}}(\Sigma_n)_{33} = \frac{1}{4}\frac{1}{1.6}$ x $(0.6$ x $686) \cong 64.3125$. Hence, an estimate of the asymptotic variance is $\frac{64.3125}{223} \cong 0.2884$

b) $\{(x_{n,1})^2 + (\frac{1}{x_{n,2}})\}$ is a consistent estimator for $\{(\beta_1)^2 + (\frac{1}{\beta_2})\}$. A point estimate is 13.678. To compute its asymptotic variance, let $a(\mathbf{x}) = (x_{n,1})^2 + (\frac{1}{x_{n,2}})$. Then, $A(\mathbf{x}) = \frac{\partial a}{\partial \mathbf{x}'} = (2x_{n,1}, \frac{-1}{(x_{n,2})^2}, 0)$. Applying the delta method, we obtain $\sqrt{n}(\{(x_{n,1})^2 + (\frac{1}{x_{n,2}})\} - \{(\beta_1)^2 + (\frac{1}{\beta_2})\}) \xrightarrow{d} N(0, C)$, where $C = (2\beta_1, \frac{-1}{(\beta_2)^2}, 0)\ \Sigma\ (2\beta_1, \frac{-1}{(\beta_2)^2}, 0)'$.
A consistent estimator for C is $(2x_{n,1}, \frac{-1}{(x_{n,2})^2}, 0)\ \Sigma_n\ (2x_{n,1}, \frac{-1}{(x_{n,2})^2}, 0)' =$

$$0.6(7, \frac{-1}{0.49}, 0) \begin{pmatrix} 427 & 0 & 12 \\ 0 & 245 & 25 \\ 12 & 25 & 686 \end{pmatrix} (7, \frac{-1}{0.49}, 0)' \cong 13,166.$$

Hence, an estimate of the asymptotic variance is $\frac{13{,}166}{223} \cong 59.041$