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SAMPLE FINAL EXAM PROBLEMS

These are the problems from the final exam in Spring 2001.

1. Answer True or False. (You do not have to explain your answers for the problems in **1.**).

1.a. For an infinite order moving average process, MA(∞), a sufficient condition for the process to be well defined as a mean square limit is that its moving average coefficients are absolutely summable. This condition is also necessary for the process to be well defined as a mean square limit. [2]

1.b. For a q-th order moving average process, MA(q), a necessary condition for the process to be covariance stationary is that its q-th order moving average polynomial equation to satisfy the invertibility condition. [2]

1.c. If the p-th order polynomial equation for a p-th order autoregressive process satisfies the stationarity condition, then the process is covariance stationary for any initial conditions [2]

1.d. If the p-th order polynomial equation for a p-th order autoregressive process satisfies the stationarity condition, then the autocovariances of the process are absolutely summable. [2]

1.e. If a process is strictly stationary, then it is also covariance stationary. [2]

1.f. The truncated kernel Heteroskedasticity and Autocorrelation Consistent (HAC) Estimator is guaranteed to produce a positive semidefinite estimate of the long-run variance matrix. [2]

1.g. The Bartlett kernel HAC Estimator is guaranteed to produce a positive semidefinite estimate of the long-run variance matrix. [2]

2. Explain what it means to say that "a test over-rejects in small samples" (or "a test is liberal"). Consider a test statistic such that the null hypothesis is rejected when the absolute value of the test statistic is larger than the critical value. When the test is liberal, which is greater, the true critical value or the nominal critical value? [3]

3. Consider the linear model

$$y_i = \mathbf{x}_i'\boldsymbol{\beta} + \epsilon_i. \tag{3.1}$$

where $\mathbf{x}_i$ is a K-dimensional vector. Suppose that this model satisfies the Classical Linear Regression Model assumptions in Chapter 1 of Hayashi for any sample size (n) and the Assumptions 2.1−2.7. Also assume that ϵ_i is normally distributed conditional on $\mathbf{X}$. Let $\mathbf{X}$ be $n \times K$ matrix with $\mathbf{x}_i'$ in its i-th row. Let

$$t_k = \frac{b_k - \overline{\beta}_k}{SE(b_k)} = \frac{b_k - \overline{\beta}_k}{\sqrt{s^2[(\mathbf{X}'\mathbf{X})^{-1}]_{kk}}}$$

be the t statistic for the null hypothesis $\beta_k = \overline{\beta}_k$.

(**3.a**) Prove that t_k converges in distribution to the standard normal distribution as the sample size goes to infinity. You do not have to prove that s^2 is consistent σ^2 for this question. You can assume that s^2 is consistent. [4]

(**3.b**) Based on the asymptotic result in (3.a), suppose that you set the nominal size to be 5 percent and reject the null hypothesis when $|t_k|$ is greater than 1.96. Does this test overreject or underreject. How do you know? Suppose that $K = 4$. Is the

the actual size larger than 10 percent when $n = 5$. What if $n = 8, 9, 10, 11$? Explain. [4]

4. Consider the t-ratio based on a single equation GMM estimator which satisfies Assumptions 3.1 – 3.5:

$$t_\ell = \frac{\sqrt{n}(\hat{\delta}_\ell(\hat{\mathbf{W}}) - \overline{\delta}_\ell)}{\sqrt{\hat{\sigma}^2(\hat{\mathbf{W}})}} \tag{4.1}$$

where $\hat{\sigma}^2(\hat{\mathbf{W}})$ is a consistent estimator of the asymptotic variance of the GMM estimator $\hat{\delta}_\ell(\hat{\mathbf{W}})$. The asymptotic variance is denoted by $\sigma^2(\hat{\mathbf{W}})$. Consider a sequence of local alternatives subject to Pitman drift:

$$\delta_\ell^{(n)} = \overline{\delta}_\ell + \frac{\gamma}{\sqrt{n}} \tag{4.2}$$

for some given $\gamma \neq 0$.

4.a. Derive the asymptotic distribution of the $t_\ell$statistic under the Pitman drift alternatives. Explain. [7]

4.b. Imagine that you employ a one-tailed t test with the nominal size of 5 percent, so that you reject the null hypothesis when $t_\ell$is greater than 1.65. Imagine that $\sigma^2(\hat{\mathbf{W}}^*)$ is 36 for a GMM estimator with a weighting matrix $\mathbf{W}^*$ (which is not equal to the optimal $\hat{\mathbf{S}}^{-1}$), and that $\sigma^2(\hat{\mathbf{S}})$ is 16 for the efficient GMM estimator. Let $\gamma = 9$. Use a standard normal distribution table to obtain the asymptotic power of the t test based on $\hat{\delta}(\hat{\mathbf{W}}^*)$, and the asymptotic power of the t test based on $\hat{\delta}(\hat{\mathbf{S}}^{-1})$. [8]

5. Consider the linear model

$$y_t = \mathbf{x}_t'\boldsymbol{\beta} + e_t. \tag{5.1}$$

where $\mathbf{x}_t$ is a K-dimensional vector.

Let $\mathbf{z}_t$ be a $K \times 1$ vector of instrumental variables. We will adopt the following set of assumptions:

(A1) $(e_t, \mathbf{x}_t, \mathbf{z}_t)_{t=1}^{\infty}$is a stationary and ergodic stochastic process.

(A2) $\mathbf{z}_t e_t$ have finite second moments.

(A3) $E(e_t|I_t) = 0$ for a sequence of information sets $(I_t)_{t=1}^{\infty}$ which is increasing (i.e., $I_t \subset I_{t+1}$), $\mathbf{z}_t$ and $\mathbf{x}_t$are in I_t, and y_t is in I_{t+3}.

(A4) $E(\mathbf{z}_t\mathbf{x}_t')$ is nonsingular.

For the following problems, prove asymptotic properties of the instrumental variable (IV) estimator, $\mathbf{b}_{IV}$, for $\boldsymbol{\beta}$ under (A1)-(A4). Use a central limit theorem and a strong law of large numbers given above, and indicate which ones you are using and where you are using them in your proof.

(**5.a**) Express the IV estimator $\mathbf{b}_{IV}$ in terms of $\mathbf{z}_t, \mathbf{x}_t$, and y_t $(t = 1, \ldots, T)$ when $\Sigma_{t=1}^{T}\mathbf{z}_t\mathbf{x}_t'$ is nonsingular. [3]

(**5.b**) Let $\mathbf{g}_t = \mathbf{z}_t e_t$. Prove that $\mathbf{g}_t$ satisfies Gordin's condition. [7]

(**5.c**) Prove that the IV estimator is consistent under (A1)-(A4). [7]

(**5.d**) Prove that the IV estimator is asymptotically normally distributed. Derive the formula of the covariance matrix of the asymptotic distribution. Simplify the formula if possible. [9]

(**5.e**) For the model in this question, imagine that you have added two more variables as instruments, so that $\mathbf{z}_t$ is now a $(K + 2)$ dimensional vector. Suppose that you have applied the efficient GMM, and Hansen's J statistic is 3.457. Do you reject the overidentifying restrictions at the 5% level? Explain. [5]

6. Prove Hayashi's Proposition 3.5 in the summary section above. [14]

7. Consider a system of two equations

$$y_i = \beta_0 + \beta_1 z_i + \epsilon_i \tag{7.1}$$

$$y_i = \beta_2 + \beta_3 z_i + \beta_4 f_i + \beta_5 h_i + e_i. \tag{7.2}$$

where z_i and f_i are endogenous and h_i is predetermined: i.e., $E(h_i \epsilon_i) = 0$ and $E(h_i e_i) = 0$. Let $\mathbf{q}_i$ be an additional k-dimensional vector of random variables which satisfies $E(\mathbf{q}_i \epsilon_i) = 0$ and $E(\mathbf{q}_i e_i) = 0$, so that $(1, h_i, \mathbf{q}_i)$ can be used as instrumental variables for each equation.

(**7.a**) Suppose that $k = 1$. Is it possible for both (7.1) and (7.2) to be identified in the multiple equation GMM? If not, is it possible for either (7.1) or (7.2) to be identified? Explain. [5]

(**7.b**) Suppose $k = 2$, is it possible for both (7.1) and (7.2) to be identified in the multiple equation GMM? If not, is it possible for either (7.1) or (7.2) to be identified? Explain. [5]

(**7.c**) Suppose $k = 3$ and that ϵ_i and e_i satisfy the conditional homoskedasticity assumption (Assumption 4.7). Assume that $E(\epsilon_i e_i) = 0$. In this case, is the 3SLS estimator (which coincides with the multiple GMM estimator in this case) more efficient than the 2SLS estimator asymptotically? Explain. [5]