Chapter 6: The Risk and Term Structure of Interest Rates

1 Risk Structure of Interest Rates

1.1 Default Risk

- default: The issuer of the bond is unable or unwilling to make interest payments when promised or pay off the face value when the bond matures.

- default-free bonds: Bonds that are considered to have no default risk. U.S. Treasury bonds have usually been considered default-free.

- risk premium: The spread between the interest rates on bonds with default risk and default-free bonds. See Figure 1.

- Response to an increase in default risk on corporate bonds: See Figure 2.

- risk premium: Credit-rating agencies. See Table 1.

- junk bonds: Bonds with rating below Baa (or BBB)

1.2 Liquidity

1.3 Income Tax Consideration

- Municipal bonds are not default-free.
- Municipal bonds are less liquid than U.S. Treasury bonds.
- Puzzle: Why have municipal bonds had lower interest rates than U.S. Treasury bonds for at least 40 years as in Figure 1?

2 Term Structure of Interest Rates

2.1 Yield Curve

A yield curve is a plot of the yields on bonds with differing terms to maturity at a point in time (cf. A plot over time in Figure 4).

Yield curves can be classified as upward-sloping, flat, and downward-sloping (inverted yield curve).

Three stylized facts:

1. Interest rates on bonds of different maturities tend to move together over time.

2. When upward-sloping yield curves are observed, short-term interest rates tend to be low. When downward-sloping yield curves are observed, short-term interest rates tend to be high.

3. Yield curves are usually upward-sloping.

2.2 Expectation Theory

The key assumption: Bonds with different maturities are *perfect substitutes*.

Consider the following two investment strategies:

1. Purchase a one-year bond, and when it matures in one year, purchase another one-year bond.

2. Purchase a two-year bond and hold it until maturity.

i_t: today's (time t) interest rate on the 1-year bond

i_t^e:interest rate on the 1-year bond expected for the next year (time $t+1$)

i_{2t}: today's (time t) interest rate on the 2-year bond

The expected cash flow from \$1 today over two years from the first investment strategy is:

$$(1 + i_t)(1 + i^e_{t+1}) \tag{1}$$

The expected cash flow from $1 today over two years from the second investment strategy is:

$$(1 + i_{2t})^2 \tag{2}$$

If bonds are perfect substitutes, then these cash flows should be the same:

$$(1 + i_{2t})^2 = (1 + i_t)(1 + i^e_{t+1}) \tag{3}$$

Taking logs of both sides,

$$2\log(1+i_{2t}) = \log(1+i_t) + \log(1+i^e_{t+1}) \tag{4}$$

Using the log approximation in the Lecture Note for Chapter 4,

$$2i_{2t} \cong i_{1t} + i^e_{t+1}) \tag{5}$$

Dividing both sides by 2, we have an approximate relationship:

$$i_{2t} = \frac{i_{1t} + i^e_{t+1}}{2} \tag{6}$$

The two-year interest rate is the average of the current and expected future one-year interest rates.

By a similar argument, for an n-year bond, we have

$$i_{nt} = \frac{i_t + i^e_{t+1} + ... + i^e_{t+n-1}}{n}. \qquad (7)$$

The n-year interest rate is the average of the current and expected future one-year interest rates.

Problem 1: Answer the following problems, assuming that the expectations theory holds.

- If the one-year interest rate is 7% today and is expected to stay at 7% in the future, what are the two=, three-, four-, five-year interest rates today? Draw the yield curve for one- to five-year maturities.

- If the one-year interest rate is 7% today and is expected to rise to 8%, 9%, 10%, and 11% over the next five years, what are the two=, three-, four-, five-year interest rates today? Draw the yield curve for one- to five-year maturities.

- If the one-year interest rate is 7% today and is expected to fall to 6%, 5%, 4%, and 3% over the next five years, what are the two=, three-, four-, five-year interest rates today? Draw the yield curve for one- to five-year maturities.

Problem 2: Do you think that the expectations theory explains stylized facts 1, 2, and 3? Explain.

2.3 Segmented Markets Theory

This theory sees markets for different-maturity bonds as completely separate and segmented. The interest rate for each bond is determined by the supply and demand for that bond. The key assumption: bonds of different maturities are not substitutes at all.

Problem 3: Do you think that the segmented markets theory explains stylized facts 1, 2, and 3? Explain.

2.4 Liquidity Premium and Preferred Habitat Theories

2.4.1 Liquidity Premium Theory

The key assumption: Bonds of different maturities are substitutes.

$$i_{nt} = \frac{i_t + i^e_{t+1} + ... + i^e_{t+n-1}}{n} + l_{nt}. \tag{8}$$

where l_{nt} is the liquidity (term) premium for the n-period bond at time t.

2.4.2 Preferred Habitat Theory

The key assumption: Investors have a preference for bonds of one maturity over another.

2.4.3 The Relationship with the Expectations Theory

See Figure 5.

Problem 4: Answer the following problems, assuming that the liquidity premium theory holds, and that the liquidity premium for one- to five-year bonds are 0%, 0.25%,0.5%,0.75%, and 1.0$.

- If the one-year interest rate is 7% today and is expected to stay at 7% in the future, what are the two=, three-, four-, five-year interest rates today? Draw the yield curve for one- to five-year maturities.

- If the one-year interest rate is 7% today and is expected to fall to 6%, 5%, 4%, and 3% over the next five years, what are the two=, three-, four-, five-year interest rates today? Draw the yield curve for one- to five-year maturities.

Problem 5: Do you think that the liquidity premium and preferred habitat theories explain stylized facts 1, 2, and 3? Explain.

See Figure 6.