

# Econ 444 Elementary Econometrics

Fall 2007

## SUMMARY of CHAPTER 5

and

## HOMEWORK EXERCISE: 5

Due at the beginning of class on Wednesday, November 7

### 1 KEY TERMS AND CONCEPTS

Confidence intervals, Confidence level, The  $t$  test, Critical value, Type I error, Type II error, Level of significance, Two-tailed test, One-tailed test, Joint hypothesis testing,  $F$  test

### 2 FORMULAS AND RESULTS

$$t = \frac{\hat{\beta}_k - \beta_k}{SE(\hat{\beta}_k)} \quad (1)$$

is distributed as the  $t$ -distribution with  $(N - K)$  degrees of freedom for  $k=1,2,\dots,K$ .

To form a confidence interval, we first choose the confidence level, say 0.95, and find the  $t_C$  value such that

$$P[\hat{\beta}_k - t_C SE(\hat{\beta}_k) \leq \beta_k \leq \hat{\beta}_k + t_C SE(\hat{\beta}_k)] = 0.95. \quad (2)$$

For a two-tailed test, compute the  $t$  test statistic from Equation (1), get  $t_C$  (the *critical value*) from Table B-1, and reject  $H_0$  if  $|t| > t_C$  and accept  $H_0$  if  $|t| \leq t_C$ .

For a multiple regression with  $K$  independent variables,

$$F = \frac{R^2/K}{(1 - R^2)/(N - K - 1)} \quad (3)$$

follows the  $F$  distribution with  $K$  and  $N - K - 1$  degrees of freedom under the null hypothesis:

$$H_0 : \beta_1 = \dots = \beta_K = 0. \quad (4)$$

### 3 HOMEWORK EXERCISE: 5

1. Imagine that you have a time series data set for demand for coffee ( $Y$ ) and the price of coffee ( $X$ ), and that you run a regression:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \quad (5)$$

Suppose that the data are annual from 1981 to 2006, that Excel computer output for regression (1) is

	Coefficients	Standard Error
Intercept	4.217	2.357
X Variable	-0.92	0.36

with  $R$  square of 0.875.

(1.a) Report the regression results, using our class format.

(1.b) Interpret the point estimates of the regression coefficients in Regression (5).

(1.c) Establish a 95% confidence interval for  $\beta_1$ .

(1.d) Test the hypothesis  $H_0 : \beta_1 = 0$  at the 5% significance level in Regression (5), using a two-tailed test. Explain.

(1.e) Discuss which alternative hypothesis makes more sense;  $H_A : \beta_1 > 0$  or  $H_A : \beta_1 < 0$  for Regression (5). Then test the hypothesis  $H_0 : \beta_1 = 0$  at the 1% significance level. Explain.

2. Imagine that you have a time series data set for demand for coffee ( $Y$ ), the price of coffee ( $X_1$ ), and the price for tea ( $X_2$ ), and that you run two regressions:

$$Y_t = \beta_0 + \beta_1 X_t + \epsilon_t \quad (6)$$

$$Y_t = \beta_0 + \beta_1 X_{1t} + \beta_2 X_{2t} + \epsilon_t \quad (7)$$

Suppose that the data are annual from 1981 to 2006, and the results for Regression (6) were as in the previous question. The results for Regression (7) were

$$\hat{Y}_i = 4.317 - 0.95 X_{1i} + 0.25 X_{2i}$$

(0.34) (0.23)

$R^2 =$  0.86

(2.a) Interpret the point estimates of partial regression coefficients for Regression (7).

(2.b) Establish a 95% confidence interval for  $\hat{\beta}_2$ .

(2.c) Test the hypothesis  $H_0 : \beta_2 = 0$  at the 5% significance level in regression (2), using a two-tailed test. Explain.

(2.d) Discuss which alternative hypothesis makes more sense;  $H_A : \beta_2 > 0$  or  $H_A : \beta_2 < 0$  for Regression (7). Then test the hypothesis  $H_0 : \beta_2 = 0$  at the 1% significance level, using a one-tailed test. Explain.

(2.e) Test the hypothesis  $H_0 : \beta_1 = \beta_2 = 0$  in regression (7) at the 1% significance level. Explain.

(2.f) Compute the adjusted  $R^2$  for regression (6) and regression (7). Based on the adjusted  $R^2$ , which regression do you prefer?