# Econ 444 Elementary Econometrics (Fall 2007)

## ANSWERS TO HOMEWORK EXERCISE: 3

(a) $VAR(\epsilon_i) = E(\epsilon^2)$. $\epsilon^2$ is $3^2$ with probability 0.5 and $(-3)^2$ with probability 0.5. Hence $VAR(\epsilon_i) = 0.5 \times 9 + 0.5 \times 9 = 9$.

(b) From (a), $\sigma^2 = 9$. The sample mean of $X$ is $\bar{X} = (32+34+42)/3 = 36$. Hence $\sum_{i=1}^{3}(X-\bar{X})^2 = (-4)^2+(-2)^2+6^2 = 16+4+36 = 56$. Using Equation (5), $VAR(\hat{\beta}_1) = 9/56 = 0.161$.

(c) Using Equation (8), the variance is

$$\frac{2 \times 9}{(42-34)^2} = 0.281. \tag{1}$$

(d) Using Equation (8), the variance is

$$\frac{2 \times 9}{(34-32)^2} = 4.5. \tag{2}$$

(e) Using Equation (8), the variance is

$$\frac{2 \times 9}{(42-32)^2} = 0.18. \tag{3}$$

(f) Comparing the results for (c)-(e), the two-point estimator with i=1 and j=2 in (d) has the maximum variance. The two-point estimator with i=1 and j=3 in (e) has the minimum variance.

(g) Among the two-point estimators, the variance should increase as the distance between $X_j$ and $X_i$ decreases. This is because the slope that goes through the two observations can be very different from the true slope coefficient when $X_j$ and $X_i$ are close. Because $X_1$ and $X_2$ are closer than $X_2$ and $X_3$ in this example, the maximum variance results when we choose $i = 1$ and $j = 2$. Because the distance between $X_j$ and $X_i$ is the largest when we choose $X_1$ and $X_3$, the minimum variance results when we choose $i = 1$ and $j = 3$.

(h) The OLS estimator in (b) has the minimum variance.

(i) Yes. The Gauss-Markov Theorem states that the OLS estimator is the BLUE. Because the two-point estimators are linear unbiased estimators, the variance of the OLS estimator must be smaller than or equal to the variance of the any two-point estimator.