Econ 444 Elementary Econometrics (Fall 2007)

SUMMARY of CHAPTER 4
and
HOMEWORK EXERCISE: 3

Due at the beginning of class on Wednesday, October 17

1 KEY TERMS AND CONCEPTS

variance, correlation, sample variance, sample correlation coefficient, Classical Assumptions, perfect multicollinearity, normal distributions, sampling distributions of OLS estimators, Linear unbiased estimators, Variances of OLS estimators, Gauss-Markov theorem, BLUE

2 FORMULAS AND RESULTS

The Classical Assumptions

I. The regression model is linear.

$$Y_i = \beta_0 + \beta_1 X_{1i} + ... + \beta_K X_{Ki} + \epsilon_i, \qquad i = 1, 2, ..., N \tag{1}$$

II. The error term has a zero population mean.

$$E[\epsilon_i] = 0, \qquad i = 1, 2, ..., N. \tag{2}$$

III. All explanatory variables are given and non-random.

IV. The error terms are uncorrelated with each other.

$$Cor(\epsilon_i, \epsilon_j) = 0, \qquad for i \neq j. \tag{3}$$

V. The error term has a constant variance (no heteroskedasticity)

$$Var(\epsilon_i) = \sigma^2, \qquad i = 1, 2, ..., N \tag{4}$$

VI. There is no perfect linear relationship between the explanatory variables (no perfect multicollinearity).

VII. The error term is normally distributed.

The variance of the OLS estimator for the slope coefficient when K=1

$$VAR(\hat{\beta}_1) = \frac{\sigma^2}{\sum_{i=1}^{N}(X_i - \bar{X})^2)} \tag{5}$$

Gauss-Markov Theorem: Under the Classical Assumptions I-VI, the Ordinary Least Squares (OLS) estimator of $\hat{\beta}_k$ is the Best Linear Unbiased Estimators (BLUE).

3 HOMEWORK EXERCISES: 3

Imagine that the expected cost of Susie's ice cream stand is given by

$$E(Y|X) = 30 + 0.5X \tag{6}$$

where Y is the cost (in dollars) and X is the number of ice cream bars sold.

At each level of X, actual cost depends on whether or not it is hot or cold. The probability that it is hot 1/2, and the probability that it is cold is 1/2. If it is hot, the cost rises from the expected value by 3 dollars; and if it is cold, the cost falls by 3 dollars. We have data for three levels of the number of ice cream bars sold.

Consider a two-point estimator that passes through two points through two points you choose, (X_i, Y_i) and (X_j, Y_j). We call the slope of this line, a two-points estimator, and denote it by $\hat{\beta}_1^*$. Note that

$$\hat{\beta}_1^* = \frac{Y_j - Y_i}{X_j - X_i}, \tag{7}$$

when $X_j > X_i$. Under the Classical Assumptions, the two-point estimator is a linear unbiased estimator with the variance

$$VAR(\hat{\beta}^*{}_1) = \frac{2\sigma^2}{(X_j - X_i)^2}. \tag{8}$$

The expected cost is given in the following table.

Table 4.A Expected Cost for Susie's Ice cream stand

Observation	The number of bars	The expected cost
i	X_i	$E(Y_i\|X_i)$
(1)	(2)	(3)
1	32	46
2	34	47
3	42	51

Answer the following questions.

(a) Compute the variance of the error term, $VAR(\epsilon_i)$.

(b) Compute the variance of the OLS estimator for the slope coefficient, $VAR(\hat{\beta}_1)$.

(c) Compute the variance of the two-point estimator when you choose i=2, and j=3.

(d) Compute the variance of the two-point estimator when you choose i=1, and j=2.

(e) Compute the variance of the two-point estimator when you choose i=1, and j=3.

(f) Compare your results for (c), (d), and (e). Which two-point estimator has the maximum variance among these? Which two-point estimator has the minimum variance among these?

(g) Give an intuitive explanation for your answer to (f).

(h) Compare your results for (b), (c), (d), and (e). Which estimator has the minimum variance among these?

(i) Is your answer to (h) consistent with the Gauss-Markov Theorem?