Temporary Boycotts as Self-Fulfilling Disruptions of Markets*

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Abstract

We consider a two-period durable goods monopoly model with demand uncertainty. When uncertainty is non-multiplicative, there can be equilibria in which, whenever the period 0 price exceeds a threshold, then with positive probability all consumers boycott in period 0. A consumer who in period 0 would purchase in the non-boycott equilibrium is willing to join a boycott because a boycott prevents the firm from learning demand. This dampens period 1 prices on average and makes the boycott self-fulfilling. Connections to the bank runs literature are discussed.

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1. Introduction

When each individual consumer has a negligible effect on the market, and derives utility only from his consumption bundle (of the good and money), it is usually assumed that collective action such as a consumer boycott cannot arise in equilibrium. If a consumer would purchase in the absence of a boycott, why would he forego the purchase to join a boycott? We show here that the expectation that all consumers will join a boycott can be self-fulfilling, without requiring a preference for punishing the firm, a preference for participating in a boycott or a fear of social pressure, bounded rationality, or other departures from the standard model of consumer behavior.

The setting is a two-period durable goods monopoly model with demand uncertainty. In the “non-boycott” equilibrium, the price in period 0, $p_0$, is such that consumers with sufficiently high valuations purchase, while consumers with lower valuations prefer to wait and hope for a lower price in period 1. The firm updates its beliefs about the state of demand based on period 0 sales, which affects the optimal price in period 1. A consumer with the cutoff valuation is indifferent between purchasing in period 0 and purchasing in period 1 at a price that will depend on period 0 sales. However, in the subgame following the firm’s choice of $p_0$, if consumers boycott the product then the firm must set $p_1$ having learned nothing about demand. If this $p_1$ is less than the expected price that would prevail without a boycott, then it is possible that all consumers would refuse to pay $p_0$ during a boycott, while consumers with valuations above a cutoff would pay $p_0$ in the absence of a boycott. We use the term boycott success probability to refer to the probability that a boycott equilibrium is selected in the subgame following $p_0$, when such an equilibrium exists. If the boycott success probability is small, then boycotts can take place on the equilibrium path of the full game. If the boycott success probability is large, then the firm will be induced to acquiesce and choose $p_0$ lower than what it would have chosen without the boycott threat.

To be sure, many real world boycott movements are led by activists who object to the policies of firms or nations. Baron (2001) and Baron and Diermeier (2007) model the interplay between a monopolist, who faces a cost of reducing its objectionable behavior, and an activist, who receives utility based on the firm’s behavior and its own efforts in organizing a boycott or otherwise punishing the firm. The activist, being a large player, avoids the free rider problem faced by an individual consumer. Innes (2006) models an environmental activist who interacts with duopolistic polluters, and shows that boycotts can arise on the equilibrium
path even with symmetric information. John and Klein (2003) study the purchase decisions of individual consumers who object to an “egregious act” on the part of the producer. Thus, consumers care directly about the firm’s policy, but there is a free rider problem due to the absence of a large activist. John and Klein (2003) find that a consumer will join a boycott only if he (i) can significantly affect the firm’s probability of abandoning the egregious act, (ii) incorrectly believes that his purchase decision directly influences the firm, (iii) is extremely altruistic, or (iv) derives utility from participating in the boycott, having a clean conscience from not purchasing, etc. All of these features are absent from the current model. Finally, Heijnen and van der Made (2012) consider a model in which it suddenly becomes common knowledge that the firm’s production is damaging to the environment, but the firm does not know the extent of consumers’ (common) disutility for the practice. A partially pooling equilibrium is derived in which some consumer types signal their high disutility by purchasing less of the firm’s product than they would under perfect information, in order to induce the firm to adopt a clean production process in the second period.

Not all real world boycotts involve egregious acts—sometimes consumers simply object to the high prices being charged. Friedman (1995) documents a series of consumer boycotts during the twentieth century, triggered by high prices and mostly supported by social pressure and local monitoring. Rea (1974) provides a theoretical analysis of boycotts, where again the only egregious act is the setting of high prices, but under the assumption that social pressure can enforce cooperation at the individual level.

When the market for e-books was first taking off in 2007 and 2008, some Amazon Kindle e-book owners attempted to boycott e-books priced over $10, and Amazon decided to incentivize its authors to price at or below $10. See the popular press and new media articles by Ganapati (2009), Rich (2010), Eckstein (2012), and Catan et al (2012). This is a complicated industry with competition from Apple and others, supply chain issues, and antitrust issues. This paper is not a model of the e-book industry. However, some features of this industry are captured by the present model. This is a market in which there is significant uncertainty about optimal prices, both for e-books relative to print books during 2007 and 2008 and for new titles in general. Consumers are forward looking and decide on the timing of their purchase. It is difficult to see how social pressure can be applied to enforce a boycott, so the question of individual incentives emerges. For a potential best seller, an equilibrium of the model could support a self-fulfilling boycott threat inducing the seller to accept an initial release price at $10, which
would avoid a boycott and allow demand to be learned. Then if the title turns out to have high demand, the price is increased, and if demand is moderate to low, the price is maintained or decreased.

The layout of the paper is as follows. Section 2 lays out the model, and Section 3 characterizes the “non-boycott” equilibrium. In Section 4, it is shown that, if demand uncertainty is multiplicative, boycott equilibria do not arise. Intuitively, when the firm knows the demand curve up to a multiplicative factor, it has nothing to learn about optimal prices and the model can be solved by backward induction. Section 4 also considers the case of non-multiplicative uncertainty and provides sufficient conditions under which the non-boycott equilibrium also has a boycott equilibrium in the subgame following \( p_0 \). We use this boycott threat to construct a continuum of equilibria in which the firm cuts its price to avoid a boycott. In Section 5, we augment the game to include a publicly observed sunspot variable, and it is shown that equilibrium exists in which boycotts occur on the equilibrium path with positive probability. Section 6 offers some concluding remarks, on the connection to the literature on experimentation and learning, and the close connection to the Diamond-Dybvig bank runs literature.

2. The Model

A monopoly seller with constant marginal production cost, normalized to zero without loss of generality, sells a durable good over two periods. Consumers demand either 0 or 1 unit of the good, and demand uncertainty is captured by the parameter \( \alpha \in [\alpha, \overline{\alpha}] \), so the measure of active consumers with valuation at least \( v \) in state \( \alpha \) is given by \( D(v, \alpha) \). We assume that \( D(v, \alpha) \) is twice continuously differentiable and strictly decreasing in \( v \), over the support of valuations, \([v, \overline{v}]\). The highest possible valuation, \( \overline{v} \), satisfies \( D(\overline{v}, \alpha) = 0 \) for all \( \alpha \). We also assume that \( -\frac{\partial D(v, \alpha)}{\partial p} \) is bounded from above and below, and that \( D(v, \alpha) \) is strictly increasing in \( \alpha \) for all \( v < v < \overline{v} \), and satisfies the revenue concavity condition,

\[
\frac{\partial^2 D(v, \alpha)}{\partial p^2} + 2 \frac{\partial D(v, \alpha)}{\partial p} < 0.
\]

The firm and all consumers share the same discount factor between period 0 and period 1, denoted by \( \delta < 1 \). Thus, if a consumer with valuation \( v \) purchases the good at price \( p \) in period 0, his utility is \( v - p \), and if he purchases in period 1, his utility is \( \delta(v - p) \).
The firm and consumers know the distribution of $\alpha$, characterized by the continuous density function $f(\alpha)$, but they do not observe the realization. Think of the following process generating the set of active consumers. First, nature draws the demand state according to $\Phi(\cdot)$. Then, out of the population of “potential” consumers $D(v, \alpha)$, nature randomly and independently selects each consumer to be active with probability $\frac{D(v, \alpha)}{D(\cdot, \alpha)}$. Finally, for the selected active consumers, nature randomly and independently selects a valuation $v$ from the distribution $(1 - \frac{D(v, \alpha)}{D(\cdot, \alpha)})$. To see that this procedure generates the appropriate demand function, the probability that an active consumer has a valuation less than or equal to $v$ equals $[1 - \text{measure of active with valuation } \geq v] = \frac{1 - \text{measure of active with valuation } \geq v}{D(\cdot, \alpha)}$

Because there is aggregate uncertainty, a consumer being active with valuation $v$ provides the consumer with information about $\Phi(\cdot)$. The probability of being active, given state $\cdot$, is $\frac{D(v, \alpha)}{D(\cdot, \alpha)}$. For small $\Delta v$, the probability of having valuation between $v$ and $v + \Delta v$, given state $\alpha$ and being active, is proportional to $\frac{\partial D(v, \alpha)}{\partial p} / D(v, \alpha)$. Intuitively, the slope of the demand curve at $v$ measures how many consumers have that valuation. Therefore, using Bayes’ rule, the density of $\alpha$, conditional on being active of type $\cdot$, is given by

$$f_{\cdot}(\alpha) \equiv \frac{\frac{\partial D(v, \alpha)}{\partial p} f(\alpha)}{\int_{\alpha} \frac{\partial D(v, \alpha)}{\partial p} f(\alpha) d\alpha}.$$  \hspace{1cm} (2.1)

Notice from (2.1) that a consumer of type $\cdot$ updates $f(\alpha)$ to assign higher probability weight to states in which there are a lot of active type $\cdot$ consumers. Our results rely on two regularity assumptions. Assumption 1 and Assumption 2 ensure that the price in period 1 is an increasing function of $\alpha$, and that higher valuation-types expect higher prices in period 1, due to the belief that higher $\alpha$ is more likely.

**Assumption 1:** The expression, $-\left[ \frac{\partial D(v, \alpha)}{\partial p} \right]$, is strictly increasing in $\alpha$ and the expression, $\left| \frac{\partial^2 D(v, \alpha)}{\partial p^2} \right|$, is weakly increasing in $\alpha$ for all $v$.

The timing of the game is as follows. At the beginning of period 0, the firm selects a price, $p_0$. Then, active consumers find out that they are active and

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1 See Deneckere and Peck (2012). In that paper, an active consumer has the additional information of his place in line, so the information effect is slightly different.
decide whether to purchase at the price \( p_0 \) or wait until period 1. The firm observes the quantity of sales in period 0. At the beginning of period 1, the firm chooses a price, \( p_1 \), and active consumers who did not purchase in period 0 decide whether to purchase at the price \( p_1 \) or to not consume. The solution concept is Perfect Bayesian Equilibrium (PBE). However, since all equilibria involve a cutoff valuation \( v^*(p_0) \) above which a consumer purchases in period 0 and below which a consumer does not purchase in period 0, the only relevant belief for the firm is the probability distribution over \( \alpha \) in period 1, contingent on observed purchases in period 0.

**Definition 1:** The subgame following \( p_0 \) has a boycott equilibrium if it has a PBE in which no consumer purchases in period 0 for any \( \alpha \) (that is, we have \( v^*(p_0) = \bar{\tau} \)).

We now characterize the value of \( p_0 \) above which a boycott equilibrium exists for the subgame. This result is not interesting by itself, because \( p_0 \) is treated as exogenous, but it will be useful later on. If consumers boycott in period 0, then consistency requires the firm to believe that \( \alpha \) is distributed according to \( f(\alpha) \). In period 1, sequential rationality on the part of a consumer with valuation \( v \) requires him to purchase at price \( p \) if and only if we have \( v \geq p \). Therefore, the sequentially rational period 1 price following a boycott, \( p^B_1 \), solves

\[
\max_{p_1} \int_\alpha p_1 D(p_1, \alpha) f(\alpha) d\alpha.
\]

Because of our concavity assumption on revenue, \( p^B_1 \) is defined to be the unique solution to the first order condition

\[
\int_\alpha [p_1 \frac{\partial D(p_1, \alpha)}{\partial p} + D(p_1, \alpha)] f(\alpha) d\alpha = 0. \tag{2.2}
\]

Also, define \( \bar{p} = \bar{\tau}(1 - \delta) + \delta p^B_1 \). One can interpret \( \bar{p} \) to be the period 0 price such that a type \( \bar{\tau} \) consumer is indifferent between purchasing and waiting, given that there will be a boycott by the other consumers and the period 1 price will be \( p^B_1 \). Given \( p^B_1 \) and \( \bar{p} \), we have the following characterization.

**Lemma 1:** The subgame following \( p_0 \) has a boycott equilibrium if and only if \( p_0 \geq \bar{p} \) holds.
Proof. Suppose $p_0 \geq \pi(1 - \delta) + \delta p_1^B$ holds. Having specified the period 1 actions and beliefs above, it remains to show that all consumers prefer not to purchase in period 0. This will be the case if the highest valuation consumer prefers not to purchase in period 0, which occurs if $\pi - p_0 \leq \delta(\pi - p_1^B)$. Thus, the subgame has a boycott equilibrium.

Now suppose $p_0 < \pi(1 - \delta) + \delta p_1^B$ holds. Then if no other consumers purchase in period 0, a consumer with valuation $\pi$ is better off purchasing, contradicting the possibility of a boycott equilibrium.

3. Non-Boycott Equilibrium

We now characterize the “non-boycott equilibrium,” based on a cutoff type $v^*(p_0)$ being indifferent between purchasing in period 0 and period 1, where $v^*(p_0)$ varies continuously from $\pi$ to $\pi^0$. Working backwards, given the price $p_1$, a type $v$ consumer will purchase in period 1 if and only if $v \geq p_1$. Therefore, given the cutoff type $v^*$ and the revealed state $\alpha$, the sequentially rational $p_1$ solves

$$\max p_1(D(p_1, \alpha) - D(v^*, \alpha)),$$

with necessary and sufficient first order condition,

$$D(p_1, \alpha) - D(v^*, \alpha) + p_1 \frac{\partial D(p_1, \alpha)}{\partial p} = 0.$$ (3.2)

Denote the unique solution to (3.2) as $p_1(v^*, \alpha)$. Our regularity assumptions guarantee that $p_1(v^*, \alpha)$ is strictly increasing in $v^*$, and we will also require that $p_1(v^*, \alpha)$ is weakly increasing in $\alpha$. Assumption 2, below, is satisfied, for example, if the price elasticity of residual demand, $D^{v^*}(p_1, \alpha) \equiv D(p_1, \alpha) - D(v^*, \alpha)$, is increasing in $p_1$ and decreasing in $\alpha$.

Assumption 2: $p_1(v^*, \alpha)$ is weakly increasing in $\alpha$ for all $v^*$.

For a consumer observing the price $p_0$ in period 0, the cutoff type $v^*(p_0)$ is determined by the indifference condition,

$$v^* - p_0 = \delta(v^* - \int_{\alpha}^{\pi} p_1(v^*, \alpha) f_{v^*}(\alpha) d\alpha),$$ or
which is nonnegative if and only if we have

\[
(1 - \delta)v^* - p_0 + \delta \frac{\int_\alpha^\pi p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}{\int_\alpha^\pi \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha} = 0. \tag{3.3}
\]

First we show that, if there is a cutoff solving (3.3), then higher valuations prefer to purchase in period 0 and lower valuations prefer to wait, verifying the cutoff property. The net advantage of purchasing for a type \( v \) consumer is

\[
(1 - \delta)v - p_0 + \delta \frac{\int_\alpha^\pi p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}{\int_\alpha^\pi \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}. \tag{3.4}
\]

We know from (3.3) that (3.4) is zero for \( v = v^* \), so it will suffice to demonstrate that the last term in (3.4) is weakly increasing in \( v \), holding \( v^* \) fixed, which is shown in the following lemma.

**Lemma 2:** The expression \( \frac{\int_\alpha^\pi p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}{\int_\alpha^\pi \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha} \) is weakly increasing in \( v \).

**Proof.** The derivative of the expression with respect to \( v \) is

\[
\frac{\int_\alpha^\pi p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}{\int_\alpha^\pi \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}.
\]

which is nonnegative if and only if we have

\[
\int_\alpha^\pi p_1(v^*, \alpha) \frac{\partial^2 D(v, \alpha)}{\partial p^2} f_v(\alpha) d\alpha \geq \int_\alpha^\pi p_1(v^*, \alpha) f_v(\alpha) d\alpha. \tag{3.5}
\]
This result follows from Assumption 1 and a weak-inequality version of Wang (1993, Lemma 2). \footnote{In Wang’s notation, \( x = f_v(\alpha) \), \( y = \left| \frac{\partial^2 D(v, \alpha)}{\partial \alpha^2} / \frac{\partial D(v, \alpha)}{\partial \alpha} \right| \), and \( z = p_1(v, \alpha) \).}

Second, we show that there is a unique interior solution to (3.3) whenever \( p_0 \) is below some upper bound. Differentiating the left side of (3.3) with respect to \( v^* \) yields

\[
(1-\delta) + \delta \int_\alpha \frac{\partial p_1(v^*, \alpha)}{\partial v^*} f(v^*)(\alpha) d\alpha + \delta \frac{\int_\alpha \left[ p_1(v^*, \alpha) - \frac{\int_\alpha p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial \alpha} f(\alpha) d\alpha}{\frac{\partial^2 D(v^*, \alpha)}{\partial \alpha^2} f(\alpha) d\alpha} \right] \frac{\partial^2 D(v^*, \alpha)}{\partial \alpha^2} f(\alpha) d\alpha}{\int_\alpha \frac{\partial D(v^*, \alpha)}{\partial \alpha} f(\alpha) d\alpha}
\]

Because the last term in (3.6) was shown to be nonnegative in Lemma 2, it follows that (3.6) is strictly positive. The left side of (3.3) is positive for \( v^*(p_0) \) and negative for sufficiently large \( p_0 \), so \( v^*(p_0) \) is uniquely determined by the indifference condition whenever \( p_0 \) is above the price for which \( v \) solves (3.3) and below the price for which \( \bar{v} \) solves (3.3). Define the price for which \( \bar{v} \) solves (3.3) as \( p^{\text{max}} \), given by

\[
p^{\text{max}} = (1-\delta)\bar{v} + \delta \int_\alpha p_1(\bar{v}, \alpha) f_{\bar{v}}(\alpha) d\alpha.
\]

To recap, consumer behavior in the subgame following \( p_0 \) is as follows. If \( p_0 < p^{\text{max}} \), then \( v^*(p_0) \) solves (3.3), types \( v \geq v^*(p_0) \) purchase in period 0, and types \( v < v^*(p_0) \) wait. \footnote{If \( p_0 \) is so low that all types strictly prefer to purchase, then they do so, \( v^*(p_0) = \alpha \).} If \( p_0 \geq p^{\text{max}} \), then no one purchases in period 0. In period 1, following \( p_0 \) and \( q_0 > 0 \), the firm assigns probability one to the \( \alpha \) solving \( D(v^*(p_0), \alpha) = q_0 \), and sets the price \( p_1(v^*(p_0), \alpha) \). \footnote{If \( D(v^*(p_0), \alpha) > q_0 \) holds, then the firm assigns probability one to state \( \alpha \) and considers the cutoff to solve \( D(v^*, \alpha) = q_0 \). If \( D(v^*(p_0), \bar{\alpha}) < q_0 \) holds, then the firm assigns probability one to state \( \bar{\alpha} \) and considers the cutoff to solve \( D(v^*, \bar{\alpha}) = q_0 \).} In period 1, following \( p_0 \) and \( q_0 = 0 \), the firm believes that \( \alpha \) is distributed according to \( f(\alpha) \), and sets the price \( p_0^F \). In period 1, for any history given \( p_1 \), a consumer of type \( v \) purchases if and only if \( v \geq p_1 \) holds.

To complete the characterization of the non-boycott equilibrium, we determine the sequentially rational \( p_0 \). Clearly, the firm will choose \( p_0 < p^{\text{max}} \) in order to learn \( \alpha \) and will want to induce an interior cutoff. It is convenient to think of the
firm as choosing \( v^* \), where \( p_0(v^*) \) is the inverse of \( v^*(p_0) \), given by
\[
p_0(v^*) = (1 - \delta)v^* + \frac{\int_\alpha \pi p_1(v^*, \alpha) \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}{\int_\alpha \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha}.
\] (3.8)

We have already shown that \( p_0(v^*) \) is strictly increasing in \( v^* \). The firm’s total profit can be written as
\[
\pi(v^*) = \int_\alpha \left[ p_0(v^*) D(v^*, \alpha) + \delta [p_1(v^*, \alpha)(D(p_1(v^*, \alpha), \alpha) - D(v^*, \alpha))] \right] f(\alpha) d\alpha.
\] (3.9)

Differentiating, we have
\[
\pi'(v^*) = \int_\alpha \left[ p_0(v^*) \frac{\partial D(v^*, \alpha)}{\partial p} + D(v^*, \alpha)p_0'(v^*) \right] f(\alpha) d\alpha +
\delta \int_\alpha p_1(v^*, \alpha) \left( \frac{\partial D(p_1(v^*, \alpha), \alpha)}{\partial p} \frac{\partial p_1(v^*, \alpha)}{\partial \alpha} - \frac{\partial D(v^*, \alpha)}{\partial \alpha} \right) f(\alpha) d\alpha +
\delta \int_\alpha (D(p_1(v^*, \alpha), \alpha) - D(v^*, \alpha)) \frac{\partial p_1(v^*, \alpha)}{\partial \alpha} f(\alpha) d\alpha.
\]

Using the necessary first order conditions for \( p_1(v^*, \alpha) \), this expression simplifies to
\[
\pi'(v^*) = p_0'(v^*) \int_\alpha D(v^*, \alpha)f(\alpha) d\alpha + \int_\alpha \left[ p_0(v^*) - \delta p_1(v^*, \alpha) \right] \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha.
\]

Substituting for \( p_0(v^*) \) from (3.8), and after much manipulation, \( \pi'(v^*) \) can be written as
\[
\pi'(v^*) = p_0'(v^*) \int_\alpha D(v^*, \alpha)f(\alpha) d\alpha + (1 - \delta)v^* \int_\alpha \frac{\partial D(v^*, \alpha)}{\partial p} f(\alpha) d\alpha.
\] (3.10)

Establishing the concavity of \( \pi(v^*) \) would guarantee that there is a unique non-boycott equilibrium, but sufficient conditions would be extremely complicated and involve assumptions about third derivatives of demand. However, the following proposition demonstrates that a non-boycott equilibrium exists, by demonstrating that \( \pi(v^*) \) is maximized for some \( v^* < \overline{\alpha} \). Typically the equilibrium involves an interior cutoff type, but it is possible for the firm to sell to all consumers in period 0, as would be the case, for example, if \( \overline{\alpha} \) is close to \( \overline{\alpha} \) and \( \delta \) is small.
Proposition 1: There exists a non-boycott equilibrium characterized by a period 0 price, $p_0^{NB}$, satisfying $p_0^{NB} < p^{\text{max}}$.

Proof. The subgame following each $p_0$ has been uniquely characterized above, and sequential rationality and consistency have been shown to be satisfied. From (3.9), it is clear that $\pi(v^*)$ is a continuous function on $[\underline{\pi}, \bar{\pi}]$, and therefore achieves its maximum for some $v^* \in [\underline{\pi}, \bar{\pi}]$. Evaluated at $\pi$, we have

$$
\pi'(\pi) = (1 - \delta)\pi \int_{\underline{\pi}}^{\bar{\pi}} \frac{\partial D(\pi, \alpha)}{\partial p} f(\alpha) d\alpha < 0,
$$

so any profit maximizing $v^*$ on $[\underline{\pi}, \bar{\pi}]$ satisfies $v^* < \pi$, as required in order to have period 0 sales reveal $\alpha$. Given a profit maximizing $v^*$, choose $p_0^{NB} = p_0(v^*)$ from (3.8). Since $v^* < \pi$ holds, it follows that $p_0^{NB} < p^{\text{max}}$ holds. ■

4. Boycott Equilibrium

In this section, we show that boycotts and boycott threats do not occur under multiplicative uncertainty. It is then shown that, if Assumption 2 holds strictly, then for $\delta$ sufficiently close to one, the subgame following $p_0^{NB}$ has a boycott equilibrium.

Definition 2: Demand uncertainty is multiplicative if there exists a function $\tilde{D}$ such that we have $D(p, \alpha) = \alpha \tilde{D}(p)$.

Proposition 2 below shows that when demand uncertainty is multiplicative, for any equilibrium period 0 price $p_0$, there are positive sales in period 0, revealing the state, and there is no boycott equilibrium to the subgame. Intuitively, under multiplicative uncertainty the firm knows the per capita demand curve $D(p)$ but not the size of the market. Since pricing decisions do not depend on $\alpha$, there is no benefit from boycotting in period 0. A boycott would increase the residual demand in period 1, to the disadvantage of consumers.

Proposition 2: If demand uncertainty is multiplicative, then in any PBE, sales are positive in period 0, and the subgame following the equilibrium $p_0$ does not have a boycott equilibrium.
Proof. In any PBE, following any $\pi_0$, there is a cutoff valuation $v^*(\pi_0)$ above which a consumer purchases in period 0 and below which a consumer waits. The reason is that $f_v(\alpha)$ is independent of $v$ under multiplicative uncertainty, so if type $v$ weakly prefers to purchase in period 0, a higher valuation type will strictly prefer to purchase in period 0. Looking ahead to period 1, the firm observes $\pi_0$ and $q_0$, and therefore infers $v^*(\pi_0)$. Sequential rationality requires that a consumer with valuation $v^*$ purchases at price $\pi_1$ if and only if we have $v^* \geq \pi_1$. Denoting the firm’s beliefs about the state by $\mu(\alpha)$, the sequentially rational $\pi_1$ solves

$$
\max_{\pi_1} \int_\alpha \pi_1[\alpha \tilde{D}(p) - \alpha \tilde{D}(v^*(\pi_0))] \mu(\alpha) d\alpha
$$

$$
= \pi_1[\tilde{D}(\pi_1) - \tilde{D}(v^*(\pi_0))] \int_\alpha \alpha \mu(\alpha) d\alpha,
$$

characterized by the first order condition given by the unique solution to the first order condition

$$
p_1 \frac{d\tilde{D}(\pi_1)}{dp} + \tilde{D}(\pi_1) - \tilde{D}(v^*(\pi_0)) = 0.
$$

Notice that $\pi_1$ depends only on $v^*$ and not the firm’s beliefs about $\alpha$. We denote the price by $\pi_1(v^*(\pi_0))$, which is strictly increasing in $v^*(\pi_0)$.

An interior cutoff satisfies the indifference condition,

$$
(1 - \delta)v^*(\pi_0) - \pi_0 + \delta \pi_1(v^*(\pi_0)) = 0.
$$

It is without loss of generality to consider $\pi_0$ such that (4.2) holds. If the cutoff is $\underline{v}$, the firm loses profits if that type strictly prefers to purchase; if the cutoff is $\overline{v}$, nothing is gained if $\pi_0$ is such that type $\overline{\pi}$ strictly prefers to purchase. Thus, finding the firm’s sequentially rational $\pi_0$ is equivalent to finding the $v^*$ to maximize profits,

$$
\pi(v^*) = \int_\alpha [p_0(v^*) \alpha \tilde{D}(v^*) + \delta p_1(v^*)[\alpha \tilde{D}(\pi_1(v^*)) - \alpha \tilde{D}(v^*)]] f(\alpha)d\alpha,
$$

or

$$
\pi(v^*) = [p_0(v^*) \tilde{D}(v^*) + \delta p_1(v^*)[\tilde{D}(\pi_1(v^*)) - \tilde{D}(v^*)]] \int_\alpha \alpha f(\alpha)d\alpha,
$$

where $p_0(v^*)$ is given by

$$
p_0(v^*) = (1 - \delta)v^* + \delta \pi_1(v^*). \tag{4.4}
$$
Substituting (4.4) into (4.3), differentiating with respect to $v^*$, and using (4.1) to simplify, we have

$$\pi'(v^*) = (1 - \delta)[v^* \tilde{D}'(v^*) + \tilde{D}(v^*)] + \delta \tilde{D}(v^*)p'_1(v^*).$$  \hfill (4.5)

Evaluated at $v^* = \overline{v}$, we have $\pi'(\overline{v}) = (1 - \delta)\overline{v}\tilde{D}'(\overline{v}) < 0$. Thus, the firm will induce a cutoff strictly below $\overline{v}$. From (4.4), we have $p_0 < (1 - \delta)\overline{v} + \delta p_1(\overline{v}) = \overline{v}$. Therefore, the firm strictly prefers to induce an interior cutoff, sales in period 0 are strictly positive, and given the equilibrium $p_0$, there does not exist a boycott equilibrium.

When demand uncertainty is not multiplicative, then the optimal price in period 1 depends on the firm’s beliefs about $\alpha$. We show here that, whenever Assumption 2 holds strictly and $\gamma$ is sufficiently close to one, the non-boycott equilibrium satisfies $p_0^{NB} > \overline{v}$, so the subgame following $p_0^{NB}$ also has a boycott equilibrium. We then use this boycott threat to construct equilibria with $p_0 < p_0^{NB}$. In the next section, we introduce the augmented game with a sunspot variable that triggers a boycott with a probability between zero and one, which allows for equilibrium with actual (and not just threatened) boycotts on the equilibrium path.

**Assumption 2':** $p_1(v^*, \alpha)$ is strictly increasing in $\alpha$ for all $v^*$.

For the case of multiplicative uncertainty, $p_1(v^*, \alpha)$ is independent of $\alpha$, so Assumption 2 holds but Assumption 2' does not hold.

**Proposition 3:** If, in addition to our maintained assumptions, Assumption 2' holds and $\gamma$ is sufficiently close to one, the subgame following $p_0^{NB}$ has a boycott equilibrium.

**Proof.** A necessary condition for a non-boycott equilibrium is that $v^*(p_0^{NB})$ solves $\pi'(v^*) = 0$ in (3.10). It is easy to see that $p'_0(v^*)$ is the expression in (3.6), which strictly exceeds $(1 - \delta)$. Then for all $\varepsilon > 0$, there exists $\overline{\delta} < 1$ such that $\delta \geq \overline{\delta}$ implies $\bar{\overline{v}} - v^*(p_0^{NB}) < \varepsilon$. Otherwise, if for some $\varepsilon$, we have $\overline{\overline{v}} - v^*(p_0^{NB}) > \varepsilon$ for all $\delta$ sufficiently close to one, then $\int_{\alpha} D(v^*, \alpha)f(\alpha)d\alpha$ is bounded above zero, so the first term in (3.10) is bounded above zero. However, the second term in (3.10) is arbitrarily close to zero, so we would have $\pi'(v^*(p_0^{NB})) > 0$, contradicting that $v^*(p_0^{NB})$ solves $\pi'(v^*) = 0$. Thus, as $\delta$ converges to one, $v^*(p_0^{NB})$ converges to $\overline{v}$,
which implies that \( p_0^{NB} \) converges to \( p^{\text{max}} \). That is, for all \( \varepsilon > 0 \), there exists \( \tilde{\delta} < 1 \) such that \( \delta \geq \tilde{\delta} \) implies \( p^{\text{max}} - p_0^{NB} < \varepsilon \).

By definition of \( p^{\text{max}} \) and \( \overline{p} \), we have

\[
p^{\text{max}} - \overline{p} = \delta \int_{\alpha}^{\overline{p}} p_1(\overline{\alpha}, \alpha) \frac{\partial D(\overline{\alpha}, \alpha)}{\partial \alpha} f(\alpha) d\alpha - \delta \int_{\alpha}^{\overline{p}} p_1(\overline{\alpha}, \alpha) f(\alpha) d\alpha. \tag{4.6}
\]

Since \(-\frac{\partial D(\alpha, \overline{\alpha})}{\partial \alpha} \) is strictly increasing in \( \alpha \) by Assumption 1, and \( p_1(\overline{\alpha}, \alpha) \) is strictly increasing in \( \alpha \) by Assumption 2', it follows from Wang (1993, Lemma 2) that the right side of (4.6) is strictly positive. Therefore, we have \( p^{\text{max}} > \overline{p} \), so there exists \( \varepsilon \) such that \( p^{\text{max}} > \overline{p} + \varepsilon \). Since there exists \( \tilde{\delta} \) such that \( p^{\text{max}} - p_0^{NB} < \varepsilon \) also holds for \( \delta \geq \tilde{\delta} \), it follows that \( p_0^{NB} > \overline{p} \) holds, so the subgame following \( p_0^{NB} \) has a boycott equilibrium for \( \delta \geq \tilde{\delta} \).

We now use the result in Proposition 3 to show that the game has multiple equilibria. Besides the non-boycott equilibrium, there are a continuum of equilibria in which the firm chooses \( p_0^{\ast} \in (\overline{p}, p_0^{NB} - \varepsilon) \), in order to avoid a threatened boycott.

**Proposition 4:** Suppose that, in addition to our maintained assumptions, Assumption 2' holds. Then for all \( \varepsilon > 0 \), there exists \( \overline{\delta} < 1 \) such that \( \delta \geq \overline{\delta} \) implies there is a continuum of PBE, indexed by \( p_0^{\ast} \in (\overline{p}, p_0^{NB} - \varepsilon) \), in which a boycott threat induces the firm to choose \( p_0 = p_0^{\ast} \).

**Proof.** From Proposition 3, we know that \( p_0^{NB} > \overline{p} \) holds, so there is a continuum of prices within the range, \( (\overline{p}, p_0^{NB} - \varepsilon) \) for sufficiently small \( \varepsilon \). Fix \( p_0^{\ast} \in (\overline{p}, p_0^{NB} - \varepsilon) \).

Here is the strategy profile. The firm chooses \( p_0 = p_0^{\ast} \). For any history with \( p_0 \leq p_0^{\ast} \), the continuation strategies (and beliefs) are exactly as in the non-boycott equilibrium. It follows that sequential rationality and consistency hold for any such history.

For the subgame following \( p_0 > p_0^{\ast} \), all consumers boycott (no one buys), and \( q_0 = 0 \). In period 1, for all histories with \( p_0 > p_0^{\ast} \) and \( q_0 = 0 \), the firm believes that \( \alpha \) is distributed according to \( f(\alpha) \) and sets \( p_1 = p_1^{\ast} \). In period 1, for all histories with \( p_0 > p_0^{\ast} \) and \( q_0 > 0 \), the firm assigns probability one to the belief that \( \alpha = \overline{\alpha} \) holds, considers the cutoff to solve \( D(v^{\ast}, \overline{\alpha}) = q_0 \), and sets \( p_1 = p_1(v^{\ast}, \overline{\alpha}) \).

Then a type \( v \) consumer purchases in period 1 if and only if \( v \geq p_1 \). Sequential rationality and consistency are satisfied for the subgame following \( p_0 > p_0^{\ast} \). In
particular, since we have \( p_0 > p_0^* > \overline{p} \), it is sequentially rational for all consumers to boycott. Also, given \( p_0 > p_0^* \) and \( q_0 > 0 \), any beliefs are consistent, but the price in period 1 is sequentially rational given those beliefs.

We now show that \( p_0 = p_0^* \) is sequentially rational. Based on equation (3.9) for the non-boycott equilibrium, the profits are given by

\[
\pi(v^*(p_0^*)) = \int_0^\overline{\pi} \left[ p_0^* D(v^*(p_0^*), \alpha) + \delta[p_1(v^*(p_0^*), \alpha)(D(p_1(v^*(p_0^*), \alpha) - D(v^*(p_0^*), \alpha))] f(\alpha) d\alpha.
\]

Given \( \varepsilon \), choose \( \overline{\delta} \) such that \( \pi'(v^*) > 0 \) holds for all \( v^* < v^*(p_0^{NB} - \varepsilon) \). We know there exists such a \( \overline{\delta} \) from the proof of Proposition 3. Since we have \( p_0^* < p_0^{NB} - \varepsilon \), it follows that \( \pi(v^*(p_0^*)) > \pi(v^*(\overline{p})) \) holds. We have

\[
\pi(v^*(\overline{p})) = \int_0^\overline{\pi} \left[ \overline{p} D(v^*(\overline{p}), \alpha) + \delta[p_1(v^*(\overline{p}), \alpha)(D(p_1(v^*(\overline{p}), \alpha) - D(v^*(\overline{p}), \alpha))] f(\alpha) d\alpha.
\]

\[
> \int_0^\overline{\pi} \left[ \overline{p} D(v^*(\overline{p}), \alpha) + \delta[p_1^B(D(p_1^B, \alpha) - D(v^*(\overline{p}), \alpha))] f(\alpha) d\alpha.
\]

Inequality (4.7) follows from the fact that \( p_1(v^*(\overline{p}), \alpha) \) is sequentially rational in period 1, yielding higher profits than \( p_1^B \). If, instead of setting \( p_0 = p_0^* \), the firm set \( p_0 > p_0^* \), this would trigger a boycott, and profits would be

\[
\delta \int_0^\overline{\pi} p_1^B(D(p_1^B, \alpha), \alpha) f(\alpha) d\alpha.
\]

Thus, the deviation would result in lower profits. If the firm set \( p_0 < p_0^* \), profits would be \( \pi(v^*(p_0)) \), where \( v^*(p_0) < v^*(p_0^*) \) holds. Because \( \pi'(v^*) > 0 \) holds in this range, the deviation would result in lower profits. \( \blacksquare \)

**Remark:** A maintained assumption is \( \delta < 1 \), but it is interesting to consider the limiting case of \( \delta = 1 \). Since any positive quantity of sales reveals \( \alpha \) in the non-boycott equilibrium, and \( v^*(p_0^{NB}) \) converges to \( \overline{p} \) as \( \delta \) converges to one, there is a discontinuity in the limiting case, \( \delta = 1 \). In the limit, \( v^*(p_0^{NB}) = \overline{p} \) holds, there are no sales, and nothing about the state is revealed. On the other hand, if nothing about the state is revealed, the firm would be better off choosing a lower price and learning demand. Thus, a non-boycott equilibrium does not exist in the limit.
if Assumption 2′ holds. Interestingly, the equilibrium constructed in Proposition 4 continues to hold in the limit, since \( v^*(p_0^*) < \bar{v} \) holds. For the limiting case of \( \delta = 1 \), and under Assumption 2′, boycott threats are required for the existence of equilibrium.

5. Boycotts on the Equilibrium Path: The Augmented Game

In the previous section, we showed how the threat of a boycott can credibly force the firm to offer lower prices. In this section, we show that boycotts can occur on the equilibrium path. To model the uncertainty over whether a boycott effort will be successful, we introduce to the game a stage in which agents observe a public random variable that has no intrinsic effect on payoffs but may serve to coordinate actions. In the bank runs and macro literatures, this is called a sunspot variable. Here is the timing of the augmented game. At the beginning of period 0, the firm selects a price, \( p_0 \). Then, all agents (including the firm) observe the realization of a sunspot variable, \( \sigma \), which without loss of generality is uniformly distributed on the unit interval. Next, active consumers find out that they are active and decide whether to purchase at the price \( p_0 \) or wait until period 1. The firm observes the quantity of sales in period 0. At the beginning of period 1, the firm chooses a price, \( p_1 \), and active consumers who did not purchase in period 0 decide whether to purchase at the price \( p_1 \) or not consume.

We now construct an equilibrium of the augmented game in which boycotts occur on the equilibrium path, with positive probability.

**Proposition 5:** Suppose that, in addition to our maintained assumptions, Assumption 2′ holds. Then there exists \( \tilde{\delta} < 1 \) such that \( \delta \geq \tilde{\delta} \) implies there is a continuum of PBE of the augmented game, in which boycotts occur with positive probability along the equilibrium path.

**Proof.** We will construct a class of equilibria, parameterized by a boycott success probability \( s \). That is, a boycott occurs if the subgame following \( p_0 \) and \( \sigma \) has a boycott equilibrium and \( \sigma \leq s \). Here is the strategy profile. The firm chooses \( p_0 = p_0^{NB} \). For any history with \( \sigma > s \) or any history with \( \sigma \leq s \) and \( p_0 \leq \overline{p} \), the continuation strategies (and beliefs) are exactly as in the non-boycott equilibrium. It follows that sequential rationality and consistency hold for any such history.
For the subgame following \( \sigma \leq s \) and \( p_0 > \overline{p} \), all consumers boycott (no one buys), and \( q_0 = 0 \). In period 1, following \( \sigma \leq s \), \( p_0 > \overline{p} \), and \( q_0 = 0 \), the firm believes that \( \alpha \) is distributed according to \( f(\alpha) \) and sets \( p_1 = p_1^\beta \). In period 1, following \( \sigma \leq s \), \( p_0 > \overline{p} \), and \( q_0 > 0 \), the firm assigns probability one to the belief that \( \alpha = \overline{\alpha} \) holds, considers the cutoff to solve \( D(v^*, \overline{\alpha}) = q_0 \), and sets \( p_1 = p_1(v^*, \overline{\alpha}) \). For any history, a type \( v \) consumer purchases in period 1 if and only if \( v \geq p_1 \). Sequential rationality and consistency are satisfied for the subgame following \( \sigma \leq s \) and \( p_0 > \overline{p} \). In particular, following \( \sigma \leq s \), \( p_0 > \overline{p} \), and \( \theta_0 > 0 \), the firm assigns probability one to the belief that \( \alpha = \overline{\alpha} \) holds, considers the cutoff to solve \( D(v^*, \overline{\alpha}) = q_0 \), and sets \( p_1 = p_1(v^*, \overline{\alpha}) \). For any history, a type \( v \) consumer purchases in period 1 if and only if \( v \geq p_1 \). Sequential rationality and consistency are satisfied for the subgame following \( \sigma \leq s \) and \( p_0 > \overline{p} \). In particular, following \( \sigma \leq s \), \( p_0 > \overline{p} \), and \( \theta_0 > 0 \), any beliefs are consistent, but the price in period 1 is sequentially rational given those beliefs.

We now show that \( p_0 = p_0^{NB} \) is sequentially rational. We choose \( \overline{\delta} \), based on Proposition 3, to satisfy \( p_0^{NB} > \overline{p} \). Denote profits in the boycott equilibrium, given in (4.8), as \( \pi_B \). Then profits in the augmented game, by choosing \( p_0 = p_0^{NB} \), are

\[
(1 - s)\pi(v^*(p_0^{NB})) + s\pi_B. \tag{5.1}
\]

If the firm were to deviate to some other price, \( p_0 > \overline{p} \), profits are

\[
(1 - s)\pi(v^*(p_0)) + s\pi_B. \tag{5.2}
\]

Because \( p_0^{NB} \) is sequentially rational in the non-boycott equilibrium, we have \( \pi(v^*(p_0^{NB})) \geq \pi(v^*(p_0)) \), so the expression in (5.1) is greater than or equal to the expression in (5.2), and the deviation is not beneficial.

If the firm were to deviate to some other price, \( p_0 \leq \overline{p} \), profits are \( \pi(v^*(p_0)) \). However, we must have \( \pi(v^*(p_0^{NB})) \geq \pi(v^*(p_0)) \) for all \( p_0 \leq \overline{p} \), so there exists \( s > 0 \) such that \( (1 - s)\pi(v^*(p_0^{NB})) + s\pi_B \geq \pi(v^*(p_0)) \) holds. That is, for sufficiently small \( s \), the deviation is not beneficial. This establishes that \( p_0 = p_0^{NB} \) is sequentially rational. A boycott occurs with positive probability, \( s \), along the equilibrium path.

The assumption that the firm observes \( \sigma \) is not needed for our results, because whenever sales are zero the firm can infer that a boycott is taking place. It seems more natural to specify a model in which the firm observes \( \sigma \); for most markets, consumers could not organize a boycott without the firm finding out about it.

The equilibrium has the property that, with fixed probability \( s \), the consumers boycott whenever a boycott equilibrium exists. Different equilibria are also possible. One could construct an equilibrium in which, with fixed probability \( s \), consumers boycott only a subset of the period 0 prices admitting a boycott equilibrium. Alternatively, one could construct a more complicated equilibrium in which the probability of a boycott following \( p_0 \) depends on the value of \( p_0 \).
There are many demand functions with non-multiplicative uncertainty satisfying our assumptions, including Assumption 2'. One such example is \( D(v, \alpha) = 1 - v^\alpha \). While numerical computations for this demand system are easy to do, there do not exist closed-form solutions for \( v^*(p_0) \).

6. Concluding Remarks

One aspect of the boycotts modeled in this paper seems very unrealistic. That is, even the most successful of boycotts will not be 100% effective. Are there equilibria of the model in which only a fraction of consumers participate in a boycott? If a known fraction of consumers do not participate, then the firm would be able to infer demand, undermining the incentive for others to join a boycott. However, if signals are not perfectly correlated, I conjecture that there can be equilibria with partial boycotts on the equilibrium path, supported by uncertainty about the extent of the boycott. Consumers receiving a signal to join a boycott would update their beliefs in favor of large boycotts and fewer sales in period 0, while consumers not receiving a signal to join a boycott would update their beliefs in favor of small boycotts and more sales in period 0. Thus, for a given valuation, a consumer receiving a signal to join a boycott would expect a lower price in period 1 than a consumer not receiving a signal to join a boycott, thereby incentivizing the former consumer to join and the latter consumer not to join.

Rothschild (1974), Easley and Kiefer (1988), and other papers on learning and experimentation also model a monopolist learning about demand through its pricing decisions. However, this literature differs from the current paper in several crucial respects. These papers assume that the demand function in each period contains unknown permanent parameters and a period-specific shock. The monopolist learns about the demand parameters over time by observing the quantity sold, may choose prices that do not maximize within-period profits in order to better learn demand, and learning may be incomplete. In this literature, demand is tied to a single period, so that the price charged and quantities sold in one period has no effect on the demand curve in future periods. In the present paper, demand is not tied to a single period, and consumers explicitly optimize over when to purchase. The residual demand in period 1 is affected by sales in period 0. In equilibria of the sort presented in Proposition 4, the firm chooses a price in period 0 that is low enough to prevent a boycott from taking place, thereby allowing the firm to learn demand. However, it would be wrong to say that the firm is experimenting or investing in information. In Rothschild (1974) and Easley and
Kiefer (1988), there is uncertainty about permanent parameters and about the realization of temporary shocks. In the present paper, there are no temporary shocks because consumers active in any period are active in all periods.5

We conclude with a discussion on the connection to the Diamond-Dybvig (DD) bank runs model. In the DD model, the bank offers a deposit contract and agents deposit their endowment in period 0. In period 1, some agents learn that they are “impatient” and must consume during that period, while others are “patient” and can consume during either period 1 or period 2. In the non-run equilibrium, patient consumers prefer to withdraw their deposits in period 2, and this is the best equilibrium for consumers. However, there is also a run equilibrium, since if everyone withdraws in period 1, the bank will liquidate all of its resources in period 1 and there will be nothing left for a patient deviator who waits until period 2.6

The boycott equilibrium considered in this paper is a kind of “anti-run.” That is, the boycott equilibrium corresponds to the non-run equilibrium, because consumers coordinate on the most efficient equilibrium (for them) and wait to transact. The non-boycott equilibrium characterized here corresponds to the run equilibrium; we can think of consumers as panicking and failing to coordinate on the boycott equilibrium. The boycott success probability, in which sunspots serves to coordinate whether or not a boycott equilibrium occurs in the subgame, plays the same role as the propensity to run in Peck and Shell (2003), in which sunspots serves to coordinate whether or not a run equilibrium occurs in the post-deposit subgame.

References


5An interesting extension would be to allow some consumers to become active in period 0 and others to become active in period 1. However, with similar regularity conditions on demand as assumed in this paper, sales in period 0 would still reveal the batch of demand that becomes active in period 0. Complicating the model to have multi-dimensional uncertainty in each “batch” of demand would take us closer to the learning and experimentation literature.


