# Real Wage Rigidities and the Cost of Disinflations*

**Guido Ascari**†

*University of Pavia*

**Christian Merkl**‡

*IfW, University of Kiel and IZA*

January 8, 2008

**Abstract**

This paper analyzes the cost of disinflations under real wage rigidities in a micro-founded New Keynesian model. The conventional view is that real wage rigidities can be a useful mechanism to generate a slump in output after a credible disinflationary policy, because they prevent the immediate adjustment of inflation. This view is flawed, since it depends on analyzing the model in a linearized framework. Once nonlinearities are taken into account, the results change both qual-

*We would like to thank Jean-Pascal Bénassy, Jordi Galí, Paul J. Kramer, Tiziano Ropele, Leo von Thadden, Roland Winkler, the participants of the IfW Symposium on "The Phillips Curve and the Natural Rate of Unemployment," the Bundesbank-IWH workshop on "Monetary and Financial Economics," the Annual Meeting of the German Economic Association, and the European Meeting of the Econometric Society, and two anonymous referees for very helpful comments.

†*Address*: Prof. Guido Ascari, Department of Economics and Quantitative Methods, University of Pavia, Via San Felice 5, 27100 PAVIA, Italy. *Tel: +39.0382.986211; e-mail:* gascari@eco.unipv.it

‡*Address*: Dr. Christian Merkl, Economist, Kiel Institute for the World Economy (IfW), Düsternbrooker Weg 120, 24105 KIEL, Germany. *Tel: +49.431.8814.260; e-mail:* christian.merkl@ifw-kiel.de

itatively and quantitatively. Disinflations actually lead to a permanently higher level of output, and real wage rigidities increase the output during the adjustment to the new steady state.

*JEL classification*: E31, E50.

*Keywords*: Disinflation, Sticky Prices, Real Wage Rigidities, Nonlinearities.

# 1 Introduction

Ball and Romer (1990) show that nominal rigidities need to be complemented by real rigidities in order to generate a suitable endogenous propagation mechanism of monetary policy. Indeed, it is well known that the widely used Calvo price staggering model is not able to generate inflation persistence, and it is believed that this shortcoming can be corrected by introducing real rigidities.

After the influential contribution of Hall (2005) a reduced form of a *real wage rigidity* assumption began to be incorporated in many dynamic stochastic general equilibrium (DSGE) New Keynesian models, starting with Krause and Lubik (2007) and applied in a number of papers (e.g., Blanchard and Galí, 2006, 2007, and Christoffel and Linzert, 2006).[1] Most of these papers show that real wage rigidities are important to improve the model performance, and to explain the sluggish behavior of inflation.

It is therefore natural to think that real wage rigidities may be useful to explain the output costs of a disinflationary policy because they induce a sluggish real wage adjustment, which is absent in the standard Calvo pricing model. The inertial dynamics of real wages then prevents the immediate adjustment of inflation to the new long run level, causing a recession. For a recent example of this argument see Section 4 of Blanchard and Galí (2007). Indeed, this is certainly the case in the log-linearized model.

This paper, however, shows that it is not the case if nonlinearities are taken into account. While real wage rigidities generate an output slump under a disinflation in

[1] A similar reduced form would also be implied by the famous "gift-exchange model" of Akerlof (1984), or by other labor market analyses (e.g., Holden, 1994, Oswald, 1979).

the log-linearized model, they lead to additional output during the adjustment to the new higher steady state in the nonlinear version of the model. The interaction between long-run effects and short-run dynamics leads, therefore, to completely different results in the linearized and in the nonlinear model. Real wage rigidities can, indeed, even cause an overshooting of output over its higher new steady state level. Hence, real wage rigidities do not imply any output costs after a disinflationary policy.

In contrast, the difference between the log-linear and the nonlinear model is only of quantitative nature when comparing the effects of real wage rigidities under a temporary shock. A temporary shock does not imply movement from one steady state to another. A disinflation, instead, implies a permanent change in the level of inflation. Thus, the effects of such a permanent shock cannot be analyzed using a version of the model that is log-linearized around one particular steady state.

Some papers in the literature (e.g., Ascari, 2004, Yun, 2005) show that nonlinearities may play an important role in DSGE New Keynesian models. Here, we show that this is the case regarding the effects of real wage rigidities, an increasingly common feature embedded into New Keynesian models. Researchers should be aware of the potentially big mistake of inferring the effects of permanent shocks through log-linearized models.

The rest of this paper is structured as follows. Section 2 presents the model and its calibration. Section 3 compares the steady state effects of a disinflation in the linearized and nonlinear model. Section 4 analyzes the effects of real wage rigidities in a disinflation experiment, and does the same for a temporary shock to the inflation target. Section 5 shortly concludes.

## 2 The Model and Calibration

The model is a standard NK model where:

(i) Firms produce a differentiated product using the following simple constant returns production function

$$Y_{i,t} = N_{i,t}, \tag{1}$$

where $Y$ is output and $N$ is the labor input.[2]

(ii) Firms' pricing is described by the usual Calvo mechanism, where $\theta$ is the fraction of firms not adjusting their price in any given period.

(iii) Households have the following instantaneous and separable utility function:

$$U\left(C_t, N_t\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - d_n \frac{N_t^{1+\varphi}}{1+\varphi}, \tag{2}$$

where $C$ is composite consumption (with elasticity of substitution between different types of goods equal to $\varepsilon$).

(iv) The following partial adjustment model for the real wage is assumed in order to introduce real wage rigidities à la Hall (2005):

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} \left(MRS_t\right)^{1-\gamma}, \tag{3}$$

where $MRS$ is the marginal rate of substitution between labor supply and consumption. For sufficiently big $\gamma$, this model implies a sluggish adjustment of the real wages. Note that this is the nonlinear counterpart of the partial adjustment model for the real wage

[2]Note that we choose the constant returns production function for expositional simplicity (the more general case is derived in the Appendix). Under diminishing returns, the differences between the linear and nonlinear model would even be more substantial. Results are available on request.

employed in many of the references above. For example, Blanchard and Galí (2007) assume $\hat{w}_t - \hat{p}_t = \gamma\left(\hat{w}_{t-1} - \hat{p}_{t-1}\right) + (1-\gamma)\widehat{mrs}_t$.[3]

(v) The monetary policy for the nominal interest rate, $i_t$, is assumed to be described by a standard Taylor rule:

$$\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \left(\frac{\pi_t}{\bar{\pi}}\right)^{\alpha_\pi}\left(\frac{Y_t}{\bar{Y}}\right)^{\alpha_y}, \tag{4}$$

where $\pi_t$ is the gross inflation rate (i.e., $P_t/P_{t-1}$), $\bar{\pi}$ is the central bank inflation target, and $\bar{Y}$ is the corresponding steady state level of output.[4]

The Appendix describes all model equations in detail and our benchmark calibration, which is standard. The qualitative results do not depend on the calibration values chosen.

# 3 Steady State Effects

The obvious starting point to analyze a disinflation experiment is to look at the steady state of the full nonlinear model, which is easy to compute, as there is no need for any approximation (see Appendix). As shown by Ascari (2004) and Yun (2005), the long-run trade-off between steady state inflation and output is highly nonlinear in the New Keynesian framework. This is because two effects are at work. First, trend inflation (i.e.

[3] Throughout the paper, capital letters refer to levels, whereas small hatted letters denote the log-deviations from the steady state. Note, morever, that the degree of real rigitidies, i.e., $\gamma$, does not affect the steady state of the model, since in the steady state $\frac{W}{P} = MRS$, whatever the value of $\gamma$.

[4] Note that each steady state inflation rate is associated with a different level of output, as will be shown below.

positive steady state inflation rates) affects the average mark-up (see King and Wolman, 1996). On the one hand, when intermediate firms are free to adjust their prices, they will set higher prices relative to their current marginal costs to offset the erosion of relative prices that trend inflation creates (see (47) in the Appendix). On the other hand, trend inflation mechanically erodes the relative prices that were set by firms in past periods (see (41) in the Appendix). The relative strength of these two countervailing effects determines how the average mark-up in the economy reacts to trend inflation.[5] At zero inflation, the average mark-up is decreasing in trend inflation. However, it starts to increase at very low levels of trend inflation. In other words, trend inflation affects the average mark-up in the economy nonlinearly. When the average mark-up is larger, the economy's monopolistic distortion increases and this has a negative effect on the steady state output.

Second, positive inflation increases price dispersion, which generates strong distortions in an economy with Calvo price staggering. In fact, aggregate employment is given by

$$
\begin{aligned}
N_t &= \left[\int_0^1 N_{i,t} di\right] = \left[\int_0^1 Y_{i,t} di\right] = \left[\int_0^1 \left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon} Y_t di\right] = \\
&= \underbrace{\int_0^1 \left[\left(\frac{P_{i,t}}{P_t}\right)^{-\varepsilon}\right] di}_{s_t} Y_t = s_t Y_t.
\end{aligned}
\tag{5}
$$

Since $s_t \geq 1$, price dispersion makes the economy less efficient (see Schmitt-Grohé and

[5] The terms in (47) and (41) are what King and Wolman (1996) call the "marginal mark-up" and the "price adjustment gap", respectively. Under constant returns to scale, the ratio between the two defines the average mark-up in the economy. See Appendix.

Uribe, 2007). In the steady state $s$ is increasing with inflation. Thus, the price dispersion generates a negative inflation-output trade-off.

Given the constant returns production function (1), output in steady state can be expressed as (see (51) in the Appendix)

$$Y = \left( \frac{1}{d_n \left(\frac{P}{MC}\right) s^{\varphi}} \right)^{\frac{1}{\varphi+\sigma}}. \tag{6}$$

The steady state output is a decreasing function of both the average mark-up of prices over marginal costs ($P/MC$) and the price dispersion in the economy (while $d_n, \varphi$ and $\sigma$ are structural deep parameters). Figure 1 plots the nonlinear steady state relationship between inflation and output, as implied by equation (6). Note that the right hand side of Figure 1 shows the trade-off for very low steady state inflation rates, while the left hand side does the same for somewhat higher rates.

– Figure 1 about here –

In contrast, the vast majority of papers on the New Keynesian literature employs a version of the model log-linearized around a zero inflation steady state, which yields the standard New Keynesian Phillips curve:[6]

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{y}_t. \tag{7}$$

Dropping the time indices implies a weak positive long-run trade-off between inflation and output: $\hat{y} = (1-\beta)/\kappa\hat{\pi}$. As it is evident from Figure 1, this conclusion is an artifact due to the model being linearized around zero inflation. In this case, the linearization abstracts from both the nonlinear relationship between average mark-up and

[6] $\kappa = (\sigma+\varphi)\frac{(1-\theta)(1-\beta\theta)}{\theta}$ under constant returns to labor. See Appendix.

trend inflation and the price dispersion effect, since there is no price dispersion up to a first-order Taylor approximation. Thus, only a time-discounting effect remains, generating a positive inflation-output relationship. Indeed, the tangent at zero inflation to the curve implied by the true nonlinear model in Figure 1 exhibits a positive slope equal to $\frac{1-\beta}{\kappa}$.[7]

# 4 Real Wage Rigidities and Model Dynamics

## 4.1 Permanent Shock

In this section, we look at a disinflation experiment, that is, an unanticipated and permanent reduction in the inflation target of the central bank, i.e., $\bar{\pi}$ in (4), say from 4% to 0%.

Figure 2[8] shows the path for output, inflation, real wages, and the nominal interest rate in response to such a policy change *in the log-linear model.*[9] In the absence of

[7]Note that this positive slope would disappear in the absence of discounting, i.e., $\beta = 1$. See Ascari (1998) and Graham and Snower (2007) for a detailed explanation of the time discounting effect.

[8]The disinflation policy is announced and credibly implemented in period 1 (both in the linear and nonlinear calibration). Thus, the output in period 0 refers to the value before the policy changed. In all the figures, the output path is normalized in terms of percentage deviations from the new steady state. The deviation in period zero, thus, reveals the steady state change due to the non-superneutrality of the model. We would obviously obtain the same graphical pattern if we show the development of the level of output or of the deviations from the old steady state instead. However, the scale of the ordinate would differ.

[9]The model is log-linearized around a zero inflation steady state. The dynamic adjustment is obtained by assuming that the initial values of the variables correspond to steady state values of

real wage rigidities, the model would immediately adjust to the new steady state, which implies a negligible output drop. There are no adjustment dynamics of real wages or inflation, and thus, no slump in output along the adjustment path. Introducing real wage rigidities in such a model, instead, generates sluggish real wages, which prevent an immediate adjustment of inflation, and, thus, cause a significant output slump along the adjustment path. These features become obviously more significant the higher the degree of real wage rigidity, i.e., the bigger $\gamma$. This is the type of result described, for example, in Section 4 in Blanchard and Galí (2007). Note, however, that real wage rigidities do not prevent a large initial drop in inflation towards the new long-run level.

– Figures 2 and 3 about here –

The adjustment path changes dramatically, instead, when nonlinear simulations are employed.[10] As can be seen in Figure 1 the long-run equilibrium is radically different:

the log-linear model at a permanent inflation rate of 4% (where all variables are expressed in terms of log-deviations from the zero inflation steady state). The Taylor rule becomes: $\hat{\imath}_t = \alpha_\pi \hat{\pi}_t + \alpha_y \hat{y}_t + \widehat{\bar{x}}^{4\%}$, where $\widehat{\bar{x}}^{4\%} = \widehat{\bar{\imath}}^{4\%} - \alpha_\pi \widehat{\bar{\pi}}^{4\%} - \alpha_y \widehat{\bar{y}}^{4\%}$ summarizes the log-deviations of the targets from the zero inflation target. See Appendix for details.

[10]To simulate nonlinearly the impulse response functions, we used the software DYNARE, developed by Juillard at CEPREMAP. Impulse response functions are actually deterministic simulations: for a given path for the exogenous variable, one needs to find the response of the system, assuming that the initial values of the endogenous variables are at a steady state. As described in Boucekkine (1995) and Juillard (1996), DYNARE stacks up all the equations of the model for all the periods in the simulation (which we set equal to 100), and solves the resulting system en bloc by using the Newton-Raphson algorithm. Laffargue (1990) shows how this can be done by exploiting the special sparse structure of the Jacobian blocks.

a disinflation from 4% to 0 actually increases the permanent steady state output level rather than decreasing it, as in the log-linear model. Moreover, the steady state output change is now sizable. In Figure 3 we plot the output path in reaction to a disinflation. Output increases sluggishly to the new higher steady state level. Real wage rigidities have a striking implication: they may actually lead to an overshooting of the output above its new permanent natural level. Indeed, the higher the degree of real wage rigidities, the more likely the overshooting of the output. That is, the implications of the inclusion of real wage rigidities are exactly the opposite for the dynamics of the linear versus the nonlinear model.

The intuition for the completely different outcomes in the linear and nonlinear model is straightforward. It is due to the interplay between long-run effects and dynamic adjustment in the nonlinear model. In the nonlinear model, a disinflation, in fact, causes the real wage to increase in the long-run. A permanent decrease in the rate of inflation diminishes the price dispersion, $s$, in the long-run. Given (5), a permanent decrease in $s$ acts as a permanent increase in labor productivity. Thus, for a given labor supply curve, the labor demand schedule shifts outwards and causes a permanent rise in the real wage. In the dynamic adjustment with no real wage rigidities, the real wage actually overshoots its new higher long-run equilibrium level, because of the strong reaction of consumption (i.e., output) to a permanent shock and of the slow adjustment of the price dispersion term $s$, since

$$\frac{W_t}{P_t} = MRS_t = d_n(1+\varphi)s_t^{\varphi}Y_t^{\varphi+\sigma}. \tag{8}$$

Real wage rigidities, by construction, introduce a sluggish adjustment of the real wage in this adjustment process. It follows that the stronger the real wage rigidities are, the less the real wage will overshoot. The more muted adjustment path of the real wage increases output. Real wage rigidities basically transfer the overshooting from the real wage to output. Thus, real wage rigidities do not imply any output costs after a disinflationary policy.[11]

The intuition in the log-linear model, instead, goes exactly the other way around. A disinflation means a permanently lower level of output (due to the aforementioned discounting effect) and thereby reduces the real wage permanently. As the downward adjustment is more muted under real wage rigidities, they lead to an output slump. Although the latter may be more in line with empirical evidence, this result is a pure artefact of the log-linearization and does not correspond to the underlying micro-founded model.

[11] Finally, it is worth noting that one would obtain very similar results if a disinflation is envisaged as a sudden drop in inflation from 4% to 0%, as in Blanchard and Galí (2007). Indeed, in our simulations, the path of inflation is very close to that. Results, however, would be different if a disinflation is implemented through a drop in the money supply growth rate. In this case, real wage rigidities amplify the output slump induced by the monetary policy shift, as was shown in an earlier version of this paper, available on our Webpages (for more on this, see Ascari and Ropele, 2007). However, what is important for the argument in this work is that under both experiments nonlinearities play a crucial role. This role is simply more evident under the experiment in this version of the paper. We thank an anonymous referee for drawing our attention to this issue.

## 4.2 Temporary Shock

Figures 4 and 5 show the impulse responses of the log-linear and the nonlinear model after a *temporary* negative shock to the inflation target, $\bar{\pi}$.[12] A negative temporary shock to the inflation target causes a monetary tightening that induces a slump in output, and a temporary reduction in inflation and the real wage. The higher the real wage rigidities, the more sluggish the adjustment in the real wage and in inflation. The effects of the degree of real wage rigidities on output dynamics, however, are only marginal. The model's adjustment paths are in line with the role that real wage rigidities play in many of the papers mentioned in the introduction.

– Figures 4 and 5 about here –

Figures 4 and 5 show that for this policy experiment the log-linear and the non-linear model deliver very similar impulse response functions. They just differ slightly quantitatively. The message is that, as expected, the log-linear model is doing a pretty good job of approximating the model around a given steady state for sufficiently small shocks.[13] Temporary shocks indeed do not imply a movement from a steady state to another one, as permanent shocks do.

[12] The inflation target (initially at 0%) is temporarily reduced by 1 % during period 1 and follows an autoregressive process of first order (see Appendix for details on the calibration).

[13] The difference between the linear and the non-linear simulations becomes bigger with the size of the shock because the approximation implied by the log-linear model gets worse. However, this difference is still quantitatively negligible, as in Figure 5 and 6, even if we assume a 4% temporary shock to the inflation target (a sort of natural counterpart of the permanent shock in the previous Section). Hence, we prefer to show Figures 5 and 6, which use a more common size (1%) for the temporary shock.

# 5 Conclusions

This work shows that real wage rigidities in the microfounded New Keynesian model cannot explain the cost of disinflations. Actually, if the central bank permanently and credibly reduces the inflation target, the higher the degree of real wage rigidities, the more likely is an overshooting of the output. It is therefore flawed the idea that real wage rigidities can account for the output cost of disinflations, because they prevent an immediate adjustment of inflation to a permanent shock. Indeed, this idea is based on a log-linearized version of the model that is methodologically inaccurate, as the steady states change. Our analysis demonstrates that the interaction between long-run effects and short-run dynamics leads to completely different results in the log-linearized and in the nonlinear model. Indeed, in the nonlinear model: (i) inflation does not immediately adjust after a permanent shock, even in the absence of real wage rigidities, because of the sluggish adjustment of price dispersion; (ii) the long run effects on output, and so the role of real wage rigidities, are reversed.

However, under a temporary shocks at the zero inflation steady state log-linearizations only lead to a small quantitative difference compared to the underlying nonlinear model. The bottom line is that researchers, when using a model log-linearized around a particular steady state, should be very careful in extending results regarding temporary shocks to permanent shocks. We should be aware of the potentially big mistake of inferring the effects of permanent shocks through log-linearized models. The extent of this potential mistake obviously depends on how much nonlinearities matter, that is, how nonlinear the original model is around the point of the log-linearization. This message is particularly important for the standard New Keynesian model, because it is very nonlinear around the zero inflation steady state, where it is commonly log-linearized in the literature. Therefore, adding new features to the model, as real wage rigidities, may have very different effects when looking at the nonlinear versus the log-linearized version of the model.

# 6 References


**Akerlof, George A. (1984):** "Gift Exchange and Efficiency-Wage Theory: Four Reviews." *American Economic Review*, Vol. 74, No. 2, pp. 79-83.

**Ascari, Guido (1998):** "Superneutrality of Money in Staggered Wage-Setting Models." *Macroeconomics Dynamics*, Vol. 2, pp. 383-400.

**Ascari, Guido (2004):** "Staggered Prices and Trend Inflation: Some Nuisances." *Review of Economic Dynamics*, Vol. 7, No. 3, pp. 642-667.

**Ascari, Guido, and Ropele, Tiziano (2007):** "Disinflation in New Keynesian Models." mimeo, University of Pavia.

**Ball, Laurence, and Romer David (1990):** "Real Rigidities and the Non-Neutrality of Money." *Review of Economic Studies*, Vol. 57, No. 2, pp. 183-203.

**Bils, Mark, and Klenow, Peter (2004):** "Some Evidence on the Importance of Sticky Prices." *Journal of Political Economy*, Vol. 112, No. 5, pp. 947-985.

**Blanchard, Olivier, and Galí, Jordi (2007):** "Real Wage Rigidities and the New

Keynesian Model." *Journal of Money, Credit, and Banking*, Vol. 39, No. 1, Supplement, pp. 35-65.

**Blanchard, Olivier, and Galí, Jordi (2006):** "A New Keynesian Model with Unemployment." Massachusetts Institute of Technology, Department of Economics, Working Paper, No. 06-22, 18 July 2006.

**Boucekkine, Raouf (1995):** "An Alternative Methodology for Solving Nonlinear Forward-Looking Models." *Journal of Economic Dynamics and Control*, Vol. 19, No. 4, pp.711-734

**Christoffel, Kai P., and Linzert, Tobias (2006):** "The Role of Real Wage Rigidity and Labor Markt Frictions for Unemployment and Inflation Dynamics." Bundesbank Discussion Paper, Economic Research Centre: Series 1, No. 11/2006, April 2006.

**Galí, Jordi (2003):** "New Perspectives on Monetary Policy, Inflation, and the Business Cycle." In: Dewatriport, Mathias, Hansen, Lars P., and Turnovsky, Stephen J. (Eds), *Advances in Economics and Econometrics: Theory and Applications*, eighth World Congress, 2003, pp. 151-197.

**Graham, Liam, and Snower, Dennis (2007):** "Hyperbolic Discounting and the Phillips Curve." *Journal of Money, Credit, and Banking*, forthcoming.

**Hall, Robert (2005):** "Employment Fluctuations with Equilibrium Wage Stickiness." *American Economic Review*, Vol. 95, No. 1, pp. 50-65.

**Holden, Steinar (1994):** "Wage Bargaining and Nominal Rigidities." *European Economic Review*, Vol. 38, No. 5, pp. 1021-1039.

**Juillard, Michel (1996):** "Dynare: A Program for the Resolution and Simulation of Dynamic Models with Forward Variables." CEPREMAP Working Paper, No. 9602.

**King, Robert G., and Wolman, Alexander L. (1996):** "Inflation Targeting in a St. Louis Model of the 21st Century." *Review* (Federal Reserve of Saint Louis), Vol. 78, No. 3, pp. 83-107.

**Krause, Michael U., and Lubik, Thomas A. (2007): "**The (Ir)relevance of Real Wage Rigidity in the New Keynesian Model with Search Frictions", *Journal of Monetary Economics*, Vol. 54, No. 3, pp. 706-727.

**Laffargue, Jean-Pierre (1990):** "Résolution d'un Modèle Macroéconomique avec Anticipations Rationnelles." *Annales d'Économie et de Statistique*, Vol. 17, pp. 97-119.

**Oswald, Andrew J. (1979)**: "Wage Determination in an Economy with Many Trade Unions". *Oxford Economic Papers*, Vol. 31, pp. 369-385.

**Schmitt-Grohé, Stephanie, and Uribe, Martín (2007):** "Optimal Simple and Implementable Monetary and Fiscal Rules." *Journal of Monetary Economics*, Vol. 54, No. 6, pp. 1702-1725.

**Yun, Tack (2005):** "Optimal Monetary Policy with Relative Price Distortions**."** *American Economic Review*, Vol. 95, No. 1, pp. 89-109.

# Appendix

## I. Model Setup

### 1. Household

The instantaneous utility function is assumed to be separable

$$U\left(C_t\left(h\right), N_t\left(h\right)\right) = \frac{C_t^{1-\sigma}}{1-\sigma} - d_n \frac{N_t^{1+\varphi}\left(h\right)}{1+\varphi}, \tag{9}$$

and the period by period budget constraint is given by

$$P_t C_t + \left(1 + i_t\right)^{-1} B_t = W_t N_t - T_t + D_t + B_{t-1}, \tag{10}$$

where $i_t$ is the nominal interest rate, $B_t$ is one-period bond holdings, $W_t$ is the nominal wage rate, $N_t$ is the labor input, $T_t$ is lump sum taxes, and $D_t$ is the profit income. The representative consumer problem, maximizing the expected discounted (using the discount factor $\beta$) intertemporal utility subject to the budget constraints, yields the following first-order conditions

$$\text{Euler equation :} \quad \frac{1}{C_t^{\sigma}} = \beta E_t \left[\left(\frac{P_t}{P_{t+1}}\right)\left(1+i_t\right)\left(\frac{1}{C_{t+1}^{\sigma}}\right)\right], \tag{11}$$

$$\text{Labor supply equation} \quad \frac{W_t}{P_t} = -\frac{U_N}{U_C} = \frac{d_n N_t^{\varphi}}{1/C_t^{\sigma}} = d_n N_t^{\varphi} C_t^{\sigma}. \tag{12}$$

We introduce real wage rigidities, by changing this last equation to

$$\frac{W_t}{P_t} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma} MRS_t^{1-\gamma} = \left(\frac{W_{t-1}}{P_{t-1}}\right)^{\gamma}\left(-\frac{U_{N_t}}{U_{C_t}}\right)^{1-\gamma}, \tag{13}$$

where $\gamma$ measures the degree of real wage rigidities.

### 2. Technology

Final good producers use the following technology

$$Y_t = \left[ \int_0^1 Y_{i,t}^{\frac{\varepsilon-1}{\varepsilon}} di \right]^{\frac{\varepsilon}{\varepsilon-1}}. \tag{14}$$

Their demand for intermediate inputs is therefore equal to $Y_{i,t+j} = \left( \frac{P_{i,t}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}$.

The production function of intermediate goods producers is

$$Y_{i,t} = N_{i,t}^{1-\alpha}. \tag{15}$$

The labor demand and the real marginal cost of firm $i$ is therefore

$$N_{i,t}^d = [Y_{i,t}]^{\frac{1}{1-\alpha}}, \tag{16}$$

and

$$MC_{i,t}^r = \frac{1}{1-\alpha} \frac{W_t}{P_t} Y_{i,t}^{\frac{\alpha}{1-\alpha}}. \tag{17}$$

Note that now marginal costs depend upon the quantity produced by the single firm, given the decreasing returns to scale. In other words, different firms charging different prices would produce different levels of output and hence have different marginal costs

$$MC_{i,t}^r = \frac{1}{1-\alpha} \frac{W_t}{P_t} \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} Y_t \right]^{\frac{\alpha}{1-\alpha}}. \tag{18}$$

## 3. Firms' Pricing

The price-resetting problem of an intermediate producer is hence

$$\max_{P_{i,t}} \quad E_t \sum_{j=0}^{\infty} \theta^j \Delta_{t,t+j} \left[ \frac{P_{i,t}}{P_{t+j}} Y_{i,t+j} - TC_{t+j}^r (Y_{i,t+j}) \right] \tag{19}$$

$$s.t. \quad Y_{i,t+j} = \left( \frac{P_{i,t}}{P_{t+j}} \right)^{-\varepsilon} Y_{t+j}, \tag{20}$$

where $P_{i,t}$ denotes the new optimal price of producer $i$, $TC^r_{t+j}(Y_{i,t+j})$ the real total cost function, and $\Delta_{t,t+j}$ the stochastic discount factor (from period $t$ to period $t+j$). The solution to this problem yields the familiar formula for the standard optimal resetted price in a Calvo setup

$$P_{i,t} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\frac{E_t\sum_{j=0}^{\infty}\theta^j\Delta_{t,t+j}\left[P^{\varepsilon}_{t+j}Y_{t+j}MC^r_{i,t+j}\right]}{E_t\sum_{j=0}^{\infty}\theta^j\Delta_{t,t+j}\left[P^{\varepsilon-1}_{t+j}Y_{t+j}\right]}. \tag{21}$$

Using (18), we can re-write equation (21) as

$$\left(\frac{P_{i,t}}{P_t}\right)^{1+\frac{\varepsilon\alpha}{1-\alpha}} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\psi_t}{\phi_t}\right), \tag{22}$$

$$\psi_t = E_t\sum_{j=0}^{\infty}(\theta\beta)^j u_c(t+j)\left[\Pi^{\frac{\varepsilon}{1-\alpha}}_{t,t+j}Y^{\frac{1}{1-\alpha}}_{t+j}\frac{1}{1-\alpha}\frac{W_{t+j}}{P_{t+j}}\right] \tag{23}$$

$$\phi_t = E_t\sum_{j=0}^{\infty}(\theta\beta)^j u_c(t+j)\left[\Pi^{\varepsilon-1}_{t,t+j}Y_{t+j}\right] \tag{24}$$

where $\Pi^z_{t,t+j} = \pi^z_{t+1}\pi^z_{t+2}...\pi^z_{t+j} = \prod_{i=1}^{j}\pi^z_{t+i}$for $j \geq 1$ and equal one for $j = 0$, $z$ is a power, and $\pi_t = P_t/P_{t-1}$. This allows us to write (23) and (24) recursively, to obtain

$$\psi_t = \frac{1}{1-\alpha}u_c(t)\,Y_t^{\frac{1}{1-\alpha}}\frac{W_t}{P_t} + \theta\beta E_t\left(\pi^{\frac{\varepsilon}{1-\alpha}}_{t+1}\psi_{t+1}\right) \tag{25}$$

$$\phi_t = u_c(t)\,Y_t + \theta\beta E_t\left[\pi^{\varepsilon-1}_{t+1}\phi_{t+1}\right]. \tag{26}$$

The aggregate price level evolves according to

$$P_t = \left[\int_0^1 P_{i,t}{}^{1-\varepsilon}di\right]^{\frac{1}{1-\varepsilon}} \Longrightarrow 1 = \left[\theta\pi_t^{\varepsilon-1} + (1-\theta)\left(\frac{P_{i,t}}{P_t}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}. \tag{27}$$

# 4. Aggregation and Price Dispersion

The aggregate resource constraint is simply given by

$$Y_t = C_t \tag{28}$$

and, as shown in the main text, the link between aggregate labor demand and aggregate output is provided by

$$N_t^d = [Y_t]^{\frac{1}{1-\alpha}} \underbrace{\int_0^1 \left[ \left( \frac{P_{i,t}}{P_t} \right)^{-\varepsilon} \right]^{\frac{1}{1-\alpha}} di}_{s_t} = s_t \, [Y_t]^{\frac{1}{1-\alpha}} \, . \tag{29}$$

Schmitt-Grohé and Uribe (2007) show that $s_t$ is bounded below at one, so that $s_t$ represents the resource costs due to relative price dispersion (under the Calvo mechanism) with long-run inflation. Indeed, the higher $s_t$, the more labor is needed to produce a given level of output. $s_t$ does not affect the real variables up to the first order whenever there is no trend inflation (i.e., $\bar{\pi}_t = 1$).

To close the model, we just need to solve for the dynamic of $s$, that is

$$s_t = (1-\theta) \left[ \frac{P_{i,t}}{P_t} \right]^{-\frac{\varepsilon}{1-\alpha}} + \theta \pi_t^{\frac{\varepsilon}{1-\alpha}} s_{t-1}. \tag{30}$$

# 5. Monetary Policy

The central bank follows a standard Taylor rule, with the weight $\alpha_\pi$ on deviations of inflation from the target level and the weight $\alpha_y$ on output deviations, i.e.,

$$\left( \frac{1+i_t}{1+\bar{\imath}} \right) = \left( \frac{\pi_t}{\bar{\pi}} \right)^{\alpha_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\alpha_y} . \tag{31}$$

## 6. System of Equations

The following systems of equations are simulated nonlinearly: Equations (11), (12), (22), (25), (26), (27), (28), (29), (30), and (31).

In the presence of a real wage rigidity, equation (12) is replaced by equation (13).

## 7. The Log-Linear Model

The linear model is obtained by log-linearizing the original equations around the zero inflation steady state. The variables are expressed in terms of log-deviations from the zero inflation steady state (denoted by ˆ). The model equations are: the Phillips curve, the Euler equation and the labor supply/real wage rigidities equation (where $\hat{w}_t^r$ denotes the real wage).

$$\hat{\pi}_t = \beta \hat{\pi}_{t+1} + \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta\left[1+\alpha(\varepsilon-1)\right]}\hat{w}_t^r, \tag{32}$$

$$\hat{y}_t = E_t(\hat{y}_{t+1}) - \frac{1}{\sigma}(\hat{\imath}_t - E_t(\hat{\pi}_{t+1})), \tag{33}$$

$$\hat{w}_t^r = \gamma \hat{w}_{t-1}^r + (1-\gamma)(\sigma+\varphi)\hat{y}_t. \tag{34}$$

The steady state of such a model thus is

$$\hat{w}^r = (\sigma+\varphi)\hat{y}, \tag{35}$$

$$\hat{\pi} = \frac{\kappa}{1-\beta}\hat{y}, \tag{36}$$

$$\hat{\imath} = \hat{\pi}, \tag{37}$$

where $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)(1-\alpha)}{\theta[1+\alpha(\varepsilon-1)]}(\sigma+\varphi)$, and all the variables are in log-deviations from the zero inflation steady state level. Given a steady state level of inflation (e.g., 4% annual inflation would be $\hat{\pi}^{4\%} = \ln\left(1.04^{1/4}\right)$), we can hence easily calculate the corresponding steady state values for $\hat{y}, \hat{w}^r$ and $\hat{r}$, and use them as initial values for our disinflation experiment.

In the Taylor rule, instead, variables are expressed as deviations from the target. Therefore, given a generic non-zero inflation rate target ($\bar{\pi}$), the logarithm of the Taylor rule is

$$\ln\left(\frac{1+i_t}{1+\bar{\imath}}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + \ln\alpha_y\left(\frac{Y_t}{\bar{Y}}\right), \tag{38}$$

which is by definition an identity at the steady state (where targets are met). (38) can be transformed by defining the variables as deviations from the zero inflation rate steady state in the following simple way:

$$\ln\left(\frac{1+i_t}{1+\bar{\imath}^{0\%}}\frac{1+\bar{\imath}^{0\%}}{1+\bar{\imath}}\right) = \alpha_\pi \ln\left(\frac{\pi_t}{\bar{\pi}^{0\%}}\frac{\bar{\pi}^{0\%}}{\bar{\pi}}\right) + \ln\alpha_y\left(\frac{Y_t}{\bar{Y}^{0\%}}\frac{\bar{Y}^{0\%}}{\bar{Y}}\right) \Longrightarrow \tag{39}$$

$$\Longrightarrow \hat{\imath}_t = \alpha_\pi\hat{\pi}_t + \alpha_y\hat{y}_t + \widehat{\bar{x}}, \tag{40}$$

where $\widehat{\bar{x}} \equiv \widehat{\bar{\imath}} - \alpha_\pi\widehat{\bar{\pi}} - \alpha_y\widehat{\bar{y}}$ summarizes the log-deviations of the targets from the zero inflation target. Our disinflation experiment is an unanticipated and permanent reduction in the inflation target from 4% to zero. Hence this determines the initial value of $\widehat{\bar{x}}$ to be $\widehat{\bar{x}}^{4\%} = \widehat{\bar{\imath}}^{4\%} - \alpha_\pi\widehat{\bar{\pi}}^{4\%} - \alpha_y\widehat{\bar{y}}^{4\%}$, and then a permanent jump of $\widehat{\bar{x}}$ to zero.

# II. Steady State Calculations

The steady state values in the non-stochastic steady state can simply be obtained by dropping the time indices. The steady state inflation is equal to the central bank's inflation target: $\pi = \bar{\pi}$. By defining $x = \frac{P_i}{P}$, equation (27) determines the steady state relative price of firms which are able to reset the price

$$x = \left(\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}, \tag{41}$$

which in turn pins down the price dispersion in the economy from (30)

$$s = \frac{(1-\theta)\, x^{-\frac{\varepsilon}{1-\alpha}}}{1-\theta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}}. \tag{42}$$

Then (25), (26) and (22) become, respectively

$$\psi = \frac{\frac{1}{1-\alpha}\frac{W}{P}Y^{\frac{1}{1-\alpha}-\sigma}}{1-\theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}}, \tag{43}$$

$$\phi = \frac{Y^{1-\sigma}}{1-\theta\beta\bar{\pi}^{(\varepsilon-1)}}, \tag{44}$$

$$x^{1+\varepsilon\frac{\alpha}{1-\alpha}} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{\psi}{\phi}\right) = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{1-\theta\beta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}}\right)\frac{1}{1-\alpha}\frac{W}{P}Y^{\frac{\alpha}{1-\alpha}}. \tag{45}$$

Given (18) and $x = \frac{P_i}{P}$, this last equation can also be written as

$$\frac{x^{1+\varepsilon\frac{\alpha}{1-\alpha}}}{\frac{1}{1-\alpha}\frac{W}{P}Y^{\frac{\alpha}{1-\alpha}}} = \frac{\frac{P_i}{P}}{\frac{1}{1-\alpha}\frac{W}{P}\left[\left(\frac{P_i}{P}\right)^{-\varepsilon}Y\right]^{\frac{\alpha}{1-\alpha}}} = \frac{\frac{P_i}{P}}{MC_i^r} = \tag{46}$$

$$\frac{P_i}{MC_i} = \left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{1-\theta\beta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}}\right), \tag{47}$$

to define what King and Wolman (1996) call the marginal mark-up, that is the mark-up of the price resetting firm.

Equation (12) can be re-written as

$$\frac{W}{P} = d_n N^{\varphi} C^{\sigma} = d_n s^{\varphi} Y^{\sigma+\frac{\varphi}{1-\alpha}}. \tag{48}$$

Substituting it into (47) yields an expression for output as a function of $\bar{\pi}$ and $s$

$$Y = \left(x^{\frac{1-\alpha+\varepsilon\alpha}{1-\alpha}}\frac{(1-\alpha)(\varepsilon-1)\left(1-\theta\beta\bar{\pi}^{\frac{\varepsilon}{1-\alpha}}\right)}{\varepsilon\left(1-\theta\beta\bar{\pi}^{(\varepsilon-1)}\right)d_n}s^{-\varphi}\right)^{\frac{1-\alpha}{\varphi+\sigma+\alpha(1-\sigma)}}. \tag{49}$$

Substituting equations (41) and (42) into (49) delivers an equation for the long-run trade-off between inflation and output, that we use to plot Figure 1 in the main text.

Note that under constant returns to scale, i.e., $\alpha = 0$, the $MC$ is constant across firms and simply equal to $W/P$. In this case the average mark-up in the economy is simply given by the ratio between (47) and (41):

$$\frac{P}{MC} = \frac{P^i/MC}{P^i/P} = \frac{\left(\frac{\varepsilon}{\varepsilon-1}\right)\left(\frac{1-\theta\beta\bar{\pi}^{(\varepsilon-1)}}{1-\theta\beta\bar{\pi}^{\varepsilon}}\right)}{\left(\frac{1-\theta\bar{\pi}^{\varepsilon-1}}{1-\theta}\right)^{\frac{1}{1-\varepsilon}}} \tag{50}$$

and (49) reduces to

$$Y = \left(\frac{P^i}{P}\frac{MC}{P^i}\frac{s^{-\varphi}}{d_n}\right)^{\frac{1}{\varphi+\sigma}}. \tag{51}$$

# III. Calibration of the Model

$d_n$ is calibrated in such a way that people devote one third of their time to work (under zero steady state inflation). The elasticity of substitution between different product types, $\varepsilon$, is set to 10.[14] We use a standard quarterly discount rate of one percent, $\beta = 0.99$, a log-utility of consumption, $\sigma = 1$, and a quadratic disutility of labor, $\varphi = 1$ (see, e.g., Galí, 2003). The quarterly probability of not re-setting the prices, $\theta$, is set to 75 percent, as in most of the calibrations in the literature.[15]

[14] Note that the elasticity of substitution influences the size of the long-run trade-offs and thus the quantitative dynamic responses to permanent shocks. The more difficult it is to substitute between different goods types (i.e. the lower the elasticity of substitution), the smaller is the negative long-run trade-off at 4 percent permanent inflation (compared to the zero inflation steady state). However, the qualitative results of this paper are robust and not dependent on the specific chosen elasticity of substitution. We obtain the same qualitative results for $\varepsilon = 5$ or $\varepsilon = 3$. The detailed results are available on request.

[15] Note that setting the average price duration to two quarters (corresponding to the results in Bils and Klenow, 2004) does not affect our qualitative outcomes.

As usual in the literature, we set the coefficients in the Taylor rule as follows: $\alpha_\pi = 1.5$ and $\alpha_y = 0.5$. In order to make our results comparable to the existing literature, under a temporary shock to the inflation target, we assume an autoregressive process of first order, namely

$$\ln \bar{\pi}_t = \rho \ln \bar{\pi}_{t-1} + \varepsilon_t \tag{52}$$

for the linear model, and

$$\bar{\pi}_t = \bar{\pi}_{t-1}^{\rho} \exp \varepsilon_{t,} \tag{53}$$

for the nonlinear model, where $\varepsilon_t$ is an i.i.d. shock. We set the autoregressive parameter to $\rho = 0.5$.

# Figures

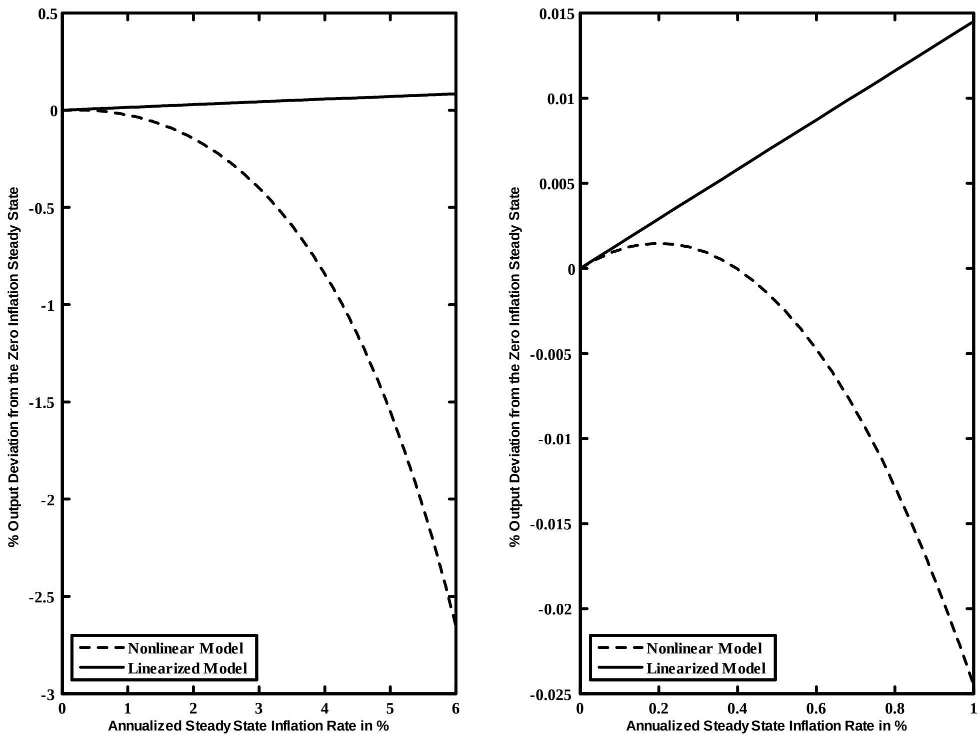


Figure 1: Relation between steady state inflation and output in the linearized and nonlinear model (the right panel shows the trade-off for low steady state inflation rates, while the left panel does the same for higher rates).

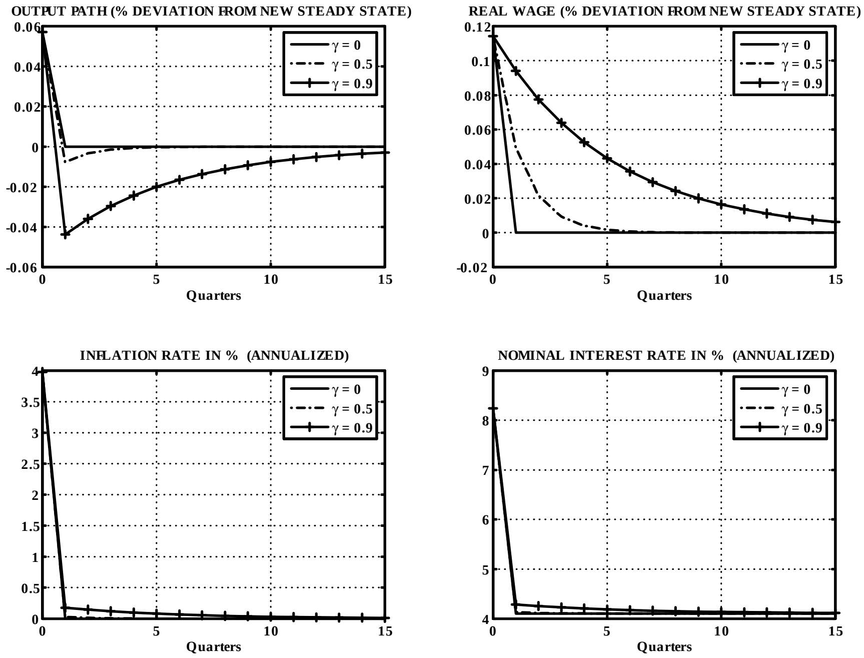


Figure 2: Permanent shock (disinflation from 4% to 0) in the linear model.

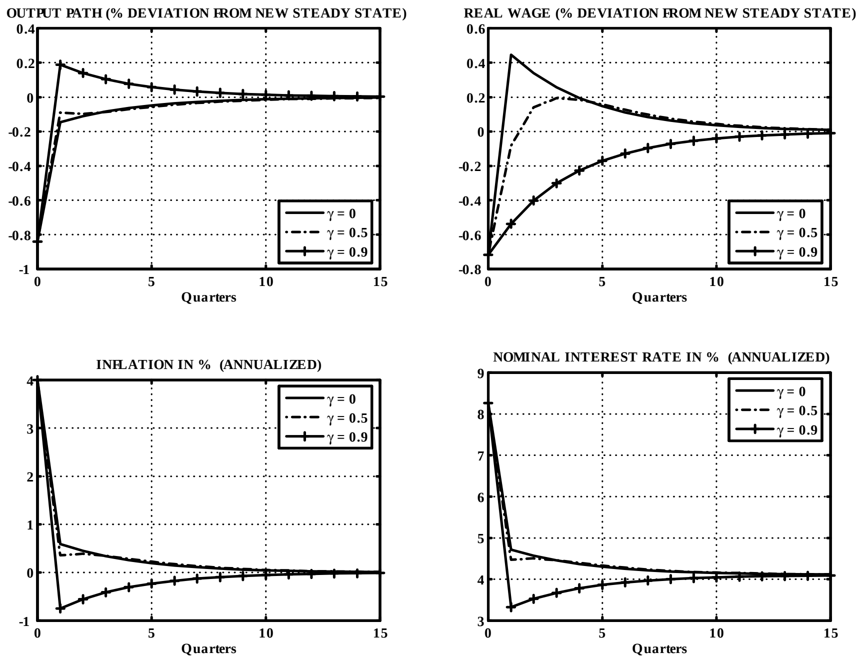


Figure 3: Permanent shock (disinflation from 4% to 0) in the nonlinear model.

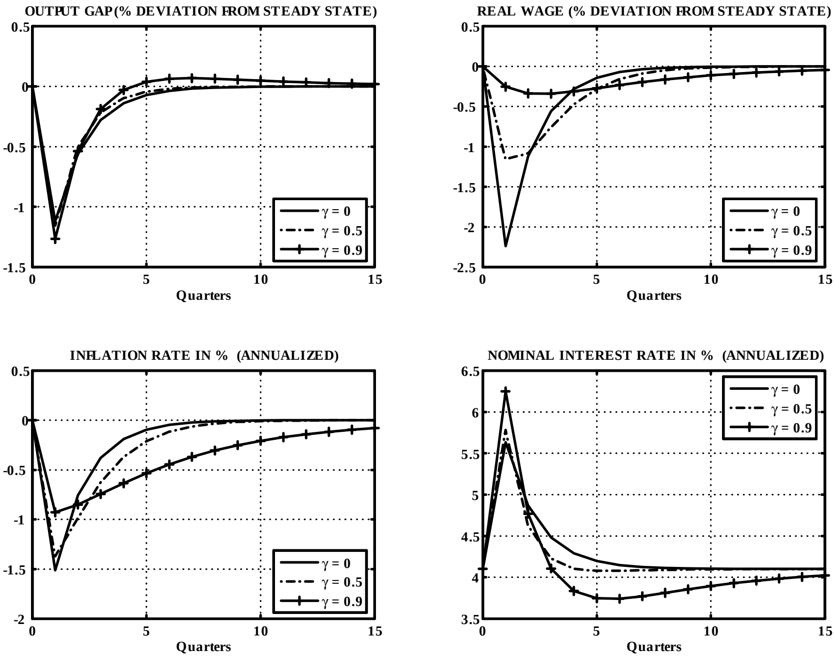


Figure 4: Temporary shock in the linear model.

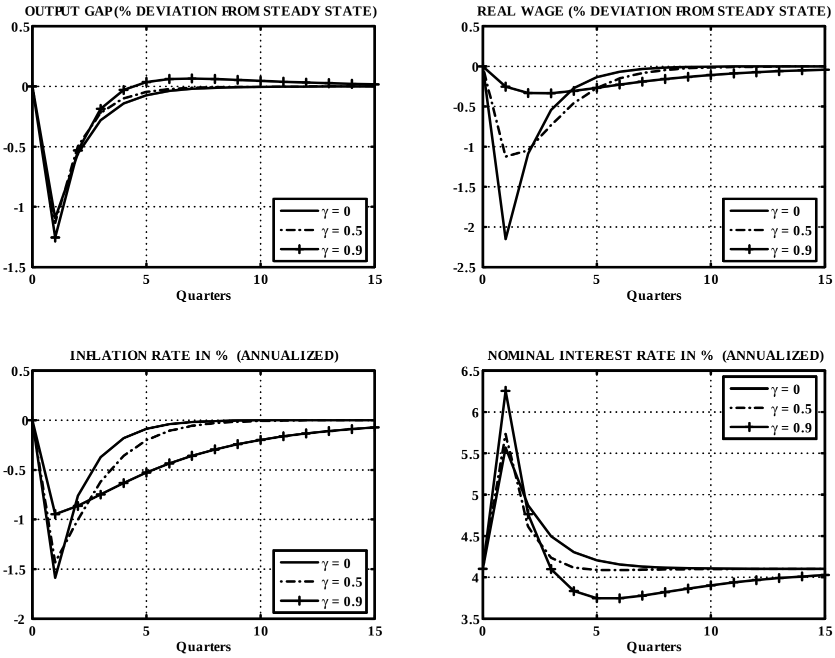


Figure 5: Temporary shock in the nonlinear model.