INFLATION BAND TARGETING AND

OPTIMAL INFLATION CONTRACTS

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ABSTRACT

In this paper we provide a theoretical treatment of how inflation target ranges cope with the time-inconsistency problem arising from incentives for the monetary policymaker to exploit the short-run tradeoff between employment and inflation to pursue short-run employment objectives, as in a Barro-Gordon (1983) model. Inflation band targets are able to achieve many of the benefits that arise under practically less attractive solutions such as the conservative central banker and optimal inflation contracts.   Our theoretical model also shows how an inflation targeting range should be set and how it should respond to changes in the nature of shocks to the economy.

1. Introduction

With the development of the literature on the time-inconsistency problem in Kydland and Prescott (1977), Calvo (1978) and Barro and Gordon (1983), there has been an increasing recognition both by policymakers and the academic literature that solving the time-inconsistency problem is crucial to the successful conduct of monetary policy. The time-inconsistency problem arises because there are incentives for the monetary policymaker to try to exploit the short-run tradeoff between employment and inflation to pursue short-run employment objectives using expansionary monetary policy, although the result is poor long-run outcomes – higher inflation, with no benefit on the output front.

Two approaches have been suggested in the literature to cope with the time-inconsistency problem: appointment of a conservative central banker (Rogoff 1985) or adoption of optimal contracts for monetary policymakers (Walsh 1995). Although both these approaches have attractive theoretical properties, they are difficult to implement in practice. It is likely to be quite difficult to find a central banker with the “right” preferences and it is hard to believe that politicians would naturally want to appoint central bankers with different preferences than theirs. Also, an opportunistic government would also be unlikely to appoint a conservative central banker, so that a regime based on having a conservative central banker is unlikely to be stable over time.

Optimal inflation contracts, usually thought of as pecuniary contracts between the government and the monetary authorities, are also unattractive because central bankers are not paid very highly, and this is particularly true in the United States, where the chairman of the Board of Governors is paid far less than many economics professors. Thus, it is highly unlikely that governments would be willing to write an inflation contract that would give a central banker sufficient incentives to produce optimal policy. Furthermore, public officials are almost never paid on the basis of their performance and we know of no central banker anywhere in the world that has performance-based pay. It also seems politically untenable to write an explicit inflation contract in which the central bank is rewarded for undershooting the optimal inflation rate.[1] Finally, in order for the inflation contract to be optimal it requires that we know how much the central banker values his office and that the monetary costs inflicted on the central baker would have to be translatable into utility units. This feature seems unappealing from a practical standpoint.

This paper argues that inflation band targeting, in which the central bank is assigned a target range and bears some cost if inflation goes outside the range, presents an alternative and more feasible approach to address the time-inconsistency problem. Indeed, inflation band targeting has already been adopted by many central banks. For example, the Reserve Bank of New Zealand is required to keep inflation within the range of 1-3% and in case of failure the Governor is subject to dismissal. The Bank of England has a target range of plus/minus one percent around a 2% inflation target, and if it fails the Governor is required to write a public letter to the government explaining why. We show that inflation band targeting regimes belong to a specific subclass of inflation contracts characterized by a particularly simple form, making them easy to adopt.[2] In fact, we are not arguing against the feasibility of inflation contracts in general, but rather the infeasibility of an *optimal* inflation contract as discussed above.

Inflation band targeting has several advantages relative to either appointment of a conservative central banker or optimal inflation contracts. First it eliminates the problem of finding the perfect central banker with the right preferences. Second, the framework is likely to be stable over time once the government has agreed to it. Third the target range provides added flexibility to the inflation targeting regime that is more palatable to politicians. Fourth, it is a simple framework that is easily implemented, and in contrast to optimal inflation contracts it does not require knowledge of how much the central banker values his office.

Although inflation band targeting has these attractive features, to our knowledge there is no research that provides a theoretical treatment of how inflation target bands can be designed to mitigate the time-inconsistency problem.[3] It is not clear that inflation band targeting has desirable properties. Indeed, some economists, including one of the authors of this paper, have been skeptical of inflation band targeting because it might produce too much focus on the edges of the range that can lead to the central bank to concentrate too much on keeping the inflation rate just within the range, rather than hitting the midpoint of the range (Bernanke et al 1999, and Mishkin 2001).

In this paper we analyze inflation target ranges in the context of the Barro-Gordon (1983) model because the simplicity of the framework provides an intuitive understanding of how target ranges can effectively deal with distortions arising from the time-inconsistency problem.[4] We demonstrate that inflation target bands or a range can achieve many of the benefits of these other strategies, providing a possible reason why this strategy has been used by so many central banks. Our theoretical model also enables us to outline how an inflation targeting range should be designed optimally and how it should change when there are changes in the nature of shocks to the economy.

To analyze the properties of inflation band targeting, we first examine a canonical benchmark model of optimal monetary policy and then illustrate the time-inconsistency problem which derives from discretionary monetary policy when policy preferences differ from the social welfare function. We then demonstrate what the optimal inflation contract would look like, and then develop the theory of inflation band targeting, showing how it replicates many of the features of the optimal inflation contract.

2. A Canonical Benchmark Model of Optimal Monetary Policy

The economy is described by the simple expectation-augmented Phillips curve,

$$u_t = u^n + b(\pi_t^e - \pi_t) + \varepsilon_t \qquad (1)$$

where, u^n is the natural rate, u_t is the realized unemployment rate in period *t*, ε_t is an i.i.d supply shock with mean 0 and fixed variance σ_ε^2, and b is a positive constant. The realized inflation and expected inflation in period *t* are denoted by π_t and π_t^e, respectively. It is worth commenting on the fact that equation (1) differs from recent specifications of the New Keynesian Phillips curve (e.g., Clarida et al 1999) in two important aspects. First, expectations are set over current inflation and not over future inflation as is the case under staggered price/wage setting. Second, the supply shock or cost-push shock, ε_t, does not exhibit persistence. These two assumptions greatly simplify the framework and have the advantage of providing a simple and attractive intuition on how inflation band targeting might mitigate the time-inconsistency problem.

The public forms its expectations rationally with respect to the complete information set up until and including period *t-1*, i.e.,

$$\pi_t^e = E_{t-1}[\pi_t] \qquad (2)$$

Each period the central bank sets its short-term inflation target, but does not have perfect control. That is,

$$\pi_t = \pi_t^{cb} + z_t$$

where z_t is a normally distributed control error with mean 0 and a fixed variance σ_z^2. Additionally, the central bank determines the inflation rate after observing the public's expectations and the supply shock. Since the supply shock is not included in the public's information set at *t-1*, the central bank can mitigate supply shock effects on real activity by creating surprise inflation.

Following a large literature we assume that the social welfare function at time *t* is quadratic and that welfare is negatively related to inflation and unemployment volatility. That is,

$$W_t = -\frac{1}{2}\left[(\pi_t - \pi^*)^2 + \omega(u_t - u^n)^2\right] \qquad (3)$$

where π^* is the socially optimal level of inflation and ω is the relative weight on unemployment volatility around its natural level.[5]

Now suppose that the monetary authorities could credibly commit to a state-contingent rule with respect to its short-run inflation target. To find the optimal rule we maximize the sum of current and future discounted welfare levels subject to the Phillips curve and rational expectations. Since there are no endogenous state variables, the maximization problem collapses to a one period problem and we can derive the optimal rule as:

$$\pi_t^{op} = \pi^* + \left(\frac{\omega b}{1+\omega b^2}\right)\varepsilon_t \qquad (4)$$

Thus, it is optimal for the central bank to set the inflation rate equal to the social optimal level of inflation and to respond to shocks that forces the current level of unemployment away from its long-run level. The shock management depends crucially on the preference parameter ω. The higher the relative weight is on unemployment stability the stronger is the response to a given supply shock. Combining (1) and (4), we can derive the equilibrium unemployment rate as:

$$u_t = u^n + \left(\frac{1}{1+\omega b^2}\right)\varepsilon_t - bz_t \qquad (5)$$

According to (4) and (5), average inflation and unemployment are equal to their target values. By substitution (4) and (5) into the welfare function we can obtain the unconditional expected welfare level in equilibrium as:

$$E[W] = -\frac{1}{2}\left[Var(\pi) + \omega Var(u)\right] \qquad (6)$$

where,

$$Var(\pi) = \left(\frac{\omega b}{1+\omega b^2}\right)^2 \sigma_\varepsilon^2 + \sigma_z^2 \qquad (7)$$

$$Var(u) = \left(\frac{1}{1+\omega b^2}\right)^2 \sigma_\varepsilon^2 + b^2\sigma_z^2 \qquad (8)$$

Since equations (4)-(8) represents optimal monetary policy, we will use this case as a benchmark in the analysis that follows. Any deviation from the optimal inflation rule described in equation (4) must hence lead to a decline in welfare.

3. Discretionary Monetary Policy under Distorted Policy Preferences

In Barro and Gordon (1983), the time-inconsistency problem arises because the central bank is aiming for an unemployment target below the natural rate. The rationale being that the central bank's target is consistent with the socially optimal level of unemployment which is lower than the natural level due to market imperfections. An alternative view, and one that we think is more realistic, is that the time-inconsistency problem arises through political influence on monetary policy. For instance, if the likelihood of reelection is higher when unemployment is low and stable, the government is likely to put pressure on the central bank to pursue excessively expansionary policy and over-stabilize economic activity.[6]

Consequently, we allow for the objective function of the central bank to differ from the social welfare function in the following manner:[7]

$$U_t^{cb} = -\frac{1}{2}\left[(\pi_t - \pi^*)^2 + (\omega + \delta)(u_t - u^*)^2\right] \qquad (9)$$

If $\delta > 0$, the monetary authorities gains some extra utility from lowering unemployment *volatility*, and if $u^* < u^n$ then they also get added utility from lowering the unemployment *level* below its natural level. Thus, the central bank aims for a lower and more stable unemployment rate than is socially optimal.

Clearly, it would be socially optimal if the monetary authorities could credibly commit to the inflation rule described by equation (4). However, as Kydland and Prescott (1977) argue, no such commitment exists when policy is discretionary in nature. Consequently, the monetary authorities, knowing that the public is aware that they are unable to resist the political influence from the government, do not believe that they can credibly manipulate inflation expectations. That is, they take inflation expectations as given when maximizing U_t^{cb}. The first order condition gives us the following reaction function of the central bank:

$$\pi_t^{cb} = \pi^* + (\omega + \delta)b(u^n - u^*) + (\omega + \delta)b(u_t - u^n) \qquad (10)$$

Equation (10) suggests that, under discretion, the monetary authorities have three concerns when determining the short-run inflation target, π^{cb}. The first is to set inflation as close as possible to the social optimal level of inflation, π^*, i.e., the long-run inflation target. The second concern is to appease the pressure from the government by raising the inflation rate above its long-run target in order to push the unemployment rate closer to u^*. The third concern of the monetary authorities is to stabilize unemployment around its natural rate.

Taking the expectations of (10) using equation (1) we get the following expressions for the equilibrium inflation and unemployment rate:

$$\pi_t = \pi^* + (\omega+\delta)b(u^n - u^*) + \left(\frac{(\omega+\delta)b}{1+(\omega+\delta)b^2}\right)\varepsilon_t + z_t \qquad (11a)$$

$$u_t = u^n + \left(\frac{1}{1+(\omega+\delta)b^2}\right)\varepsilon_t - bz_t \qquad (11b)$$

Equation (11a) and (11b) display the familiar results derived by Barro and Gordon (1983). That is, the expected long-run unemployment rate is simply equal to the natural rate while the expected long-run inflation is above the social optimal level i.e.,

$$\pi_{LR} = \pi^* + (\omega+\delta)b(u^n - u^*) \qquad (12)$$

This inflation bias arises because the monetary authorities will on average be pressured by the government to inflate the economy. The central bank will raise inflation until the marginal cost of pushing inflation above its socially optimal rate equals the perceived marginal benefits from giving in to political pressure. However, since the public behaves rationally, it will expect a higher level of inflation and hence there is no impact on real activity on average.

The long-run inflation and unemployment volatility under discretion can be derived as:

$$Var_D(\pi) = \left(\frac{(\omega + \delta)b}{1 + (\omega + \delta)b^2} \right)^2 \sigma_\varepsilon^2 + \sigma_z^2 \tag{13a}$$

$$Var_D(u) = \left(\frac{1}{1 + (\omega + \delta)b^2} \right)^2 \sigma_\varepsilon^2 + b^2 \sigma_z^2 \tag{13b}$$

When comparing equations (13a) and (13b) with equations (7) and (8), we see that if $\delta > 0$, inflation (unemployment) volatility is excessively high (low) under discretion.[8] The excess inflation volatility is positively related to how much extra the central bank cares about stabilizing real activity relative to what is socially optimal.[9] Given equations (12) and (13), the unconditional expected welfare level in equilibrium can be expressed as:

$$E[W] = -\frac{1}{2}\left[(\pi_{LR} - \pi^*)^2 + Var_D(\pi) + \omega Var_D(u) \right] \tag{14}$$

Thus, the two underlying determinants of the welfare distortions under discretion are:

(i) *The unemployment target,* u^* : The more misaligned the central bank's unemployment target is with the natural level, the higher is the long-run inflation bias and the lower is welfare.

(ii) *The stabilization bias,* δ : The central bank's excessive weight on unemployment stability has two negative effects on overall welfare. First, the more misaligned the government is with the monetary authorities regarding the stabilization objective the smaller is the unemployment volatility and hence the greater is inflation volatility. Second, the inflation bias is positively related to δ. This stems from the cost structure of the welfare function. The higher δ is, the higher is the perceived cost by the government of any given difference between the natural rate and the government's unemployment target and the more pressure it will lever on the monetary authorities to be overly expansionary[10]

4. Optimal Inflation Contracts

Walsh (1995) and Persson and Tabellini (1993) first introduced the concept of an inflation contract to solve the time-inconsistency problem. The idea is to write a contract between the government and the monetary authorities which penalizes the monetary authorities when they deviate from the optimal level of inflation. At first it might appear bizarre for the government to agree upon a contract that limits their ability to influence monetary policy. The point is, however, that the contract is written before expectations are determined and thus before the government is faced with the temptation to pressure the central bank to exploit the short-run trade-off between inflation and unemployment. Realizing its inability to refrain from this temptation once it arises, the government is willing to tie its hands by accepting the contract.

Most of the literature considers *linear* contracts that inflict a constant marginal penalty on the central bank when it raises inflation above its optimal level. As Svensson (1997) shows, these contracts are equivalent to the case of an appointment of a target-conservative central banker.[11] However, in our set-up we need a *quadratic* inflation contract in order to eliminate both the inflation bias and the excess volatility. To see this, define the quadratic inflation contract as an added penalty structure to the central banks objective function U_t^{cb} defined by equation (9) i.e.,

$$U_t^{cb} - \alpha\left(\pi - \pi^*\right) - \frac{\beta}{2}\left(\pi - \pi^*\right)^2 \qquad (15)$$

The first linear term of the contract inflicts a positive constant marginal cost on the monetary authorities when inflation is raised above the socially optimal level and thus can undo the inflation bias. The second quadratic term of the contract inflicts an increasing marginal cost and will hence discourage the monetary authorities from reacting too strongly to supply shocks and thus can undo the stabilization bias. The first order condition of the maximization problem can be expressed as:

$$\frac{\partial U_t^{cb}}{\partial \pi_t^{cb}} - \left[\alpha + \beta\left(\pi - \pi^*\right)\right] = 0 \qquad (16)$$

The second term of expression (16) is the marginal cost that the quadratic contract inflicts on the monetary authorities. Deriving the new reaction function from (16) we have:

$$\pi_t^{cb} = \pi^* + \frac{1}{1+\beta}\left[(\omega + \delta)b(u^n - u^*) - \alpha\right] + \frac{1}{1+\beta}(\omega + \delta)b\left(u_t - u^n\right) \qquad (17)$$

The parameters of the optimal inflation contract must in equilibrium replicate the optimal rule i.e., equation (4). This is true under the following parameterization:

$$\alpha = (\omega + \delta)b(u^n - u^*) \qquad \text{and,} \qquad \beta = \delta / \omega$$

Thus, the optimal *quadratic* inflation contract can completely eliminate the welfare distortions under discretion. Furthermore, it is straight forward to show that the quadratic contract is equivalent to the combination of a target-conservative and weight-conservative central banker (see Mishkin and Westelius 2006).

5. Modeling Inflation Band Targeting

We will now argue that inflation band targeting regimes can be seen as a subset of the class of inflation contracts. The government and the monetary authorities agree upon an inflation *range* within which inflation must be kept, and if the central bank fails to contain inflation within the specified range some form of punishment is inflicted on the monetary authorities. There are two main components of inflation band targeting; (1) the specified range and (2) the degree of accountability. Let us denote the upper and lower boundaries of the target range by $\overline{\pi}$ and $\underline{\pi}$, respectively, and the cost of overshooting (undershooting) the range by C.[12] Because the monetary authorities cannot perfectly control inflation there will always be a positive probability of failing to hit the target range. The new objective function facing the central bank at time *t* can thus be written as:

$$U_t^{cb} - C\{1 - F(\overline{\pi} - \pi)\} - C\{F(\underline{\pi} - \pi)\} \tag{18}$$

where U_t^{cb} is defined by (9), $F(\bullet)$ is the cumulative distribution function of the control error z_t, $\overline{\pi} = \pi^* + BW$, and $\underline{\pi} = \pi^* - BW$. For example, an inflation band target of 1-3% can be described by $\pi^* = 2$ and a bandwidth of $BW = 1$. The F.O.C can be derived as:

$$\frac{\partial U_t^{cb}}{\partial \pi_t^{cb}} - C\left[f(\pi^* + BW - \pi) - f(\pi^* - BW - \pi)\right] = 0 \qquad (19)$$

The first term of expression (19) corresponds to the F.O.C under discretion.[13] The second term is the marginal cost of missing the target range. Comparing (16) and (19) we can see how similar the set-up is to an inflation contract. The difference is simply the functional form of the marginal cost structure.

In fact, equation (19) suggests that the set of contracts involving inflation band targeting could be broadened further. For example, we could allow for an asymmetric accountability structure, greater penalties for larger deviations from the target, a midpoint that differs from the social optimal level, or simply defining an upper boundary but not a lower one, a so-called inflation cap. It is clear that such generalizations of the framework would allow us to find an optimal design that could move us even closer to optimum than the set-up that we consider here. However there are several reasons why we are not taking this direction. First, in many inflation band targeting regimes the main cost of missing the target range is the embarrassment effect and there is little reason for this cost to be asymmetric. Additionally, from a practical point of view, it is inherently difficult to set the right level of accountability in the first place, and it would be even harder to specify the correct difference between levels of accountability assigned to the lower and upper boundary. Second, setting a midpoint lower than the optimal inflation level may be politically infeasible. Indeed, the more opportunistic the government is, the lower the midpoint has to be in order to offset the increased inflation bias. For the same reasons we objected to optimal inflation contracts in which the central bank is rewarded for undershooting the optimal inflation rate, we believe that setting a midpoint below the social optimal inflation rate is politically untenable. We believe there is little incentive for a government to agree to such an arrangement. Third, as Eggertsson (2006) shows, discretionary policy could under certain circumstances give rise to a reversed inflation bias i.e., a deflation bias. In that case an inflation floor would be needed and just having an inflation cap would not make much sense. Furthermore, central banks particularly dislike deflation and the zero inflation level may thus provide an unavoidable lower boundary.[14]

Because we assume that the control error is normally distributed there is no closed form solution for the reaction function of the central bank. Consequently, in the proceeding analysis we rely upon simulations in order to understand the effects that the inflation target range has on the long-run inflation level and volatility. At first using numerical simulations may appear to be an undesirable feature. However, as will become clear, these simulations generate very straightforward intuition**.**[15]

5.1 The Marginal Cost Structure

The key to inflation band targeting is to understand how the marginal cost structure affects monetary policy under discretion. Figure 1 shows the marginal cost structure for inflation band targeting as a function of the inflation level, assuming a normally distributed control error. The solid line represents the marginal cost under inflation band targeting while the dashed line the marginal cost of the optimal inflation contract.[16] There are several characteristics of the marginal cost structure under inflation band targeting worth pointing out:

(i) *Symmetric around the midpoint*: The marginal cost is symmetric around the midpoint of the target range. Since the band is centered around the social optimal level of inflation, the cost of over and undershooting the target range is the same. Hence the marginal cost must be zero at the midpoint. If the inflation rate is above (below) the midpoint, the marginal cost is positive (negative) and encourages the monetary authorities to lower (raise) inflation. Consequently, inflation band targeting reduces the incentives of the monetary authorities to give in to political pressure and over-stabilize real activity. Additionally, since the marginal cost of increasing inflation above its socially optimal level is relatively higher than under discretion, the inflation bias must be lower. However, because the marginal cost is zero at the midpoint it is impossible to completely eliminate the inflation bias. Notice that this is in contrast to the inflation contract which allows for a positive marginal cost at the socially optimal level of inflation and can therefore completely eliminate the inflation bias.

(ii) *Increasing marginal cost inside the range:* The marginal cost of missing the target range increases as inflation moves away from the midpoint and towards the edges of the range. It peaks approximately at the boundaries.[17] Hence, similar to the optimal inflation contract, the slope of the marginal cost structure is positive within the target range. However, in contrast to the optimal inflation contract, the normality of the error term makes the marginal cost non-linear and therefore prevents the regime from completely eliminating the excess inflation level and volatility within the target range.

(iii) *Decreasing marginal cost outside the range*: As inflation goes beyond the bandwidth the absolute value of the marginal cost decreases.[18] The diminishing marginal cost outside the range is due to the probability distribution of the control error. The further away the optimal inflation rate is from the edges of the target range the less likely it is for the realized inflation to fall within the target range. That is, the marginal cost of exiting the target range is close to zero when inflation is well outside the target range and will therefore have no impact on monetary policy. For example, if the economy is hit by a large supply shock, the central bank might find it optimal to set the inflation rate outside the target range. At that point there is no incentive for the monetary authorities to restrain themselves from giving in to political pressure. As a result, the inflation level and volatility *outside* the target range remain close to their levels under discretion. As is clear from figure 1, the optimal inflation contract does not suffer from this drawback since the slope of the marginal cost structure does not depend on how far inflation is from its optimal level.

From the preceding discussion, it seems natural to design the optimal inflation band target in such a way that it approximates the marginal cost structure of the optimal inflation contract. The set-up presented in the previous section identifies two determinants of the shape of the marginal cost schedule.

(i) *Bandwidth (BW)*: Figure 2(a) shows the effect that a change in the bandwidth has on the marginal cost schedule. An increase in the bandwidth flattens the marginal cost around the midpoint, while a decrease has the opposite effect. An increase in the bandwidth would hence increase the incentives within the target range to give into political pressure.[19]

(ii) *Accountability (C)*: Figure 2(b) shows how a change in the degree of accountability affects the marginal cost structure. An increase in the cost of missing the target range increases the marginal cost proportionally at any given inflation level. Hence, increasing the degree of accountability should make the monetary authorities less inclined to give into political pressure.

5.2 Reaction Function of the Central Bank under Inflation Band Targeting

To show how inflation band targeting affects equilibrium inflation and volatility we proceed by simulating the reaction function of the central bank. Figure 3 shows the central bank's optimal inflation rate as a function of expected inflation. The solid and dashed lines represent the reaction function under inflation band targeting and pure discretion, respectively. The supply shock is assumed to be zero. Notice that the central bank has a relatively more muted response to changes in expected inflation *within* the target range than under discretion. This is because the target range imposes an additional marginal cost of raising inflation above the midpoint. The public understands this and expects a lower inflation level than under discretion. The lower equilibrium level of inflation is represented by the intersection between the solid line and the dotted 45-degree line.

Figure 4 shows the central bank's desired inflation rate as a function of the realized supply shock. The solid and dashed line represents the reaction function under inflation band targeting and pure discretion, respectively. In order to focus on the effect that the target range has on shock management we eliminate the inflation bias by setting the unemployment target equal to the natural rate. The response of the monetary authorities to a given supply shock is clearly lower *within* the target range relative to pure discretion. Indeed, the supply shock response within the range is approximately the same as under the optimal rule. However, if the supply shock is large such that the desired inflation rate is *outside* the range, then the response by the monetary authorities is similar to that under pure discretion. This means that inflation band targeting reduces inflation volatility within but less so *outside* the target range.

5.3 Optimal Bandwidth and Accountability

Figures 5(a)-(d) show the effect that the bandwidth has on welfare, equilibrium inflation and inflation/unemployment volatility, respectively. Figure 5(d) illustrates that a narrow bandwidth may result in high inflation volatility since the probability of missing the inflation target is great. On the other hand, a wide bandwidth does little to reduce volatility *within* the target range and hence can also generate high inflation volatility. Figure 5(b) shows the opposite result for unemployment volatility. Figure 5(b) and 5(d) therefore suggest that, given an initial narrow (wide) bandwidth, a widening of the bandwidth may indeed reduce (increase) inflation volatility and increase (decrease) unemployment volatility.

Figure 5(c) depicts the relationship between the bandwidth and equilibrium inflation. A narrow bandwidth suggests a higher marginal cost, given the inflation level, and thus should reduce the inflation bias. However, if the bandwidth is too narrow the probability of missing the target range may be so high that the inflation bias is greater than under a wider bandwidth. This is because inflation *outside* the target range remains close to its level under discretion. Hence an increase in the probability of missing the target range causes a rise in inflation expectations and forces the monetary authorities to increase inflation on average in order to alleviate the downward pressure on real activity.[20] Figure 5(c) also highlights that the inflation bias can never be fully eliminated with a symmetric inflation band target. As mentioned earlier, this is because the marginal cost due to the target range is zero at the social optimal level of inflation.[21]

Figure 5(a) shows the resulting welfare levels under various bandwidths. A too narrow bandwidth results in a high probability of missing the target range and will hence produce a high level and volatility of inflation. On the other hand, a too wide bandwidth, while decreasing the probability of missing the target range, does little to reduce the inflation level and volatility *within* the target range. Additionally, at the optimal level of the bandwidth, the welfare level is not very sensitive to changes in the bandwidth. This suggests that the cost of deviating from the optimal bandwidth may not be very large. Indeed, this may be one reason why inflation band targeting is an attractive regime to implement in practice.

Figures 6(a)-(d) show the effect that the degree of accountability has on welfare, equilibrium inflation and inflation/unemployment volatility, respectively. Not surprisingly, figures 6(c) and 6(d) show that increased accountability reduces both the inflation level and inflation volatility. Of course, a higher level of accountability also increases unemployment volatility. Consequently, as shown in figure 6(a), there exists a welfare maximizing level of accountability that trades off the reduction in the inflation bias and volatility against a higher level of unemployment volatility. In practice the level of accountability is probably the hardest parameter to specify optimally. However, if the overall welfare benefit of having monetary policy respond optimally to supply shocks is relatively small, then a too high level of accountability that reduces the inflation bias significantly, but over-stabilizes inflation, is less costly than a too low level of accountability which gives rise to a high inflation bias and under-stabilizes inflation.

Table 1shows inflation, inflation/unemployment volatility, and the welfare level under discretion, the optimal contract, and inflation band targeting for various values of δ and u^*. It also shows the corresponding optimal values of the bandwidth and accountability. The first thing to notice is that inflation band targeting does very well in reducing both the inflation bias and the excess volatility. For example, the lowest welfare level under discretion in table 1 is -11 ($\delta = 6$ and $u^* = 3$).The optimal inflation band target reduces the inflation level form 6.48% to 2.23%, inflation volatility from 1.3 to 0.64 standard deviations, and manages to increases the welfare level to -0.68. This can be compared with the optimal inflation contract which produces a welfare level of -0.61.

Table 1(b) shows that the inflation volatility under the optimal inflation targeting regime is lower than under the optimal inflation contract while the opposite is true for unemployment volatility. The reason is that the band target attempts to eliminate both the excess volatility and the inflation bias. However, at the point where the excess inflation volatility is eliminated, the marginal benefits from reducing the inflation bias is still greater than the marginal cost of over-stabilize inflation. Hence inflation (unemployment) volatility will generally be below (above) its optimal level.

Table 1(b) also gives us some indication of how to design an optimal inflation band target under various values of δ and u^*.The bandwidth only varies between 0.8 and 1.4

while the degree of accountability varies between 4.1 and 17.9. Furthermore, the optimal bandwidth does not seem to be affected much by the degree of the volatility bias, i.e., δ. Indeed, this distortion is most effectively reduced by a high degree of accountability since it reduces both the inflation bias and volatility. However, the simulations suggest that a country where the unemployment target is relatively low should have both a tighter band and a higher degree of accountability.[22]

Finally, in order to asses the likelihood of missing the target range, it may be convenient to express the optimal bandwidth in terms of standard deviation of inflation. The final matrix in table 1(b) shows that there are some variations in this measure across our set of simulation. Indeed, it varies between 1.25 and 1.90. Assuming that the inflation rate is at the midpoint and that the inflation distribution is approximately normal, these numbers would indicate that the optimal inflation band targeting regime implies a 6-21% probability that the target range will be breached.[23] Intuitively this makes sense since the marginal cost structure is fairly close to that of the optimal inflation contract as long as realized inflation is not too far outside the target range.[24]

5.4 Supply Shocks and Optimal Design

Because of the quadratic form of the welfare function any theoretical solution which completely eliminates the welfare distortions has to be characterized by a linear marginal cost structure. This means that the level of supply shock volatility becomes irrelevant to the optimal design of the inflation contract. One interesting feature of our inflation band targeting set-up is that its optimal design *is* sensitive to the level of supply shock volatility.

Table 2 shows how the optimal design of inflation band targeting changes as the volatility of the supply shock increases. The basic intuition for changing the width of the inflation bands and the degree of accountability is that higher supply shock volatility makes it more likely that inflation will end up outside the target range. As the first two rows of table 2 show, in this case it becomes optimal to increase both the bandwidth and the degree of accountability. The widening of the bandwidth makes it more likely that inflation will be contained within the boundaries of the target range. On the other hand, the inflation volatility within the target range increases. The increase in the degree accountability reduces this problem by making it increasingly costly to deviate from the socially optimal level of inflation i.e., the slope of the marginal cost structure within the target range increases. It is also worth pointing out that the ability of the inflation band targeting regime to reduce welfare distortions is lower at higher levels of supply shock volatility.

One potentially important implication of table 2 is that developing countries, which typically are characterized by greater supply shock volatility, should implement an inflation band targeting regime with a high degree of accountability and a wide target range. In addition, they will benefit less from an inflation band targeting regime relative to countries that are less prone to large supply shocks.

5.5 When does Inflation Band Targeting not help?

Inflation band targeting does not always improve the discretionary outcome. It may be the case that the equilibrium inflation rate lies outside the target range. In fact, figure 7 shows the case where the reaction function of the central bank intersects the 45-degree line at the same point as under discretion. This situation is likely to arise if the initial inflation bias is large, the target range is very narrow, and the degree of accountability very low. Under these circumstances the marginal cost of giving in to political pressure is not high enough to render the framework credible. Rational agents realizing this will still expect an inflation rate identical to that under discretion.

The optimal bandwidth of the target range is easy to calculate, so unless the designers of the inflation band target are incompetent, it is unlikely that they would choose a target range that is too narrow. However, there may well be limits on how high accountability can be set. We have seen an example of this in Europe with the implementation of the Growth and Stability Pact in which countries are supposed to be fined if they exceed the deficit limit of 3% of GDP. However, when countries have exceeded this deficit limit there has been a reluctance to fine them. It is true that there still has been a cost for these countries when they exceed the deficit limit, but it does illustrate that accountability may be limited because if a punishment is felt to be too draconian, it will not be imposed. This has an important implication. In order to avoid needing a level of accountability that is unenforceable, the inflation bias needs to be kept low and this requires that the amount of political influence over the central bank needs to be limited. This analysis thus provides another reason why the measures to promote central bank autonomy are important elements of a well-designed inflation targeting regime.

It is also possible that the there exists multiple equilibria. Figure 8 shows the same cases as in figure 7 but with a higher degree of accountability. The reaction function crosses the 45-degree line three times. The first equilibrium occurs within the target range but very close to the upper boundary. The second equilibrium occurs outside the target range but at a lower inflation level than the original inflation level under discretion. However, this equilibrium is not stable. If inflation expectations falls below the second equilibrium inflation will converge towards the equilibrium inside the target range. On the other hand, if for some reason expected inflation falls above the second equilibrium then inflation will converge to the initial inflation level under discretion. The situation of multiple equilibria is more likely to occur when the distortions are great. Thus, this analysis again emphasizes the importance of dealing with the distortions at their root base and to make the central bank more independent from the political process.

6. Conclusion

In this paper we have examined how target ranges work in the context of a Barro-Gordon (1983) type model. In contrast to conjectures in the literature that are skeptical about the benefits of inflation target ranges, we find that target ranges turn out to be an excellent way to cope with the time-inconsistency problem arising from incentives for the monetary policymaker to try to exploit the short-run tradeoff between unemployment and inflation. Target ranges provide incentives that get monetary policy to be very close to optimal policy in which the time-inconsistency problem is avoided altogether.

Our theoretical model also shows how an inflation targeting range should be set and how it should respond to changes in the nature of shocks to the economy. We find that inflation band targeting has a marginal cost structure that is very close to that of the optimal inflation contract as long as realized inflation is not too far outside the target range. This tells us that the target range has to be wide enough so that realized inflation ends up inside it most of the time, and this also tells us that the more uncertainty there is about the inflation process, the wider the target range has to be. Indeed, this is what we actually find in practice, where emerging market countries, which are more likely to have more uncertainty about inflation outcomes, tend to choose wider target ranges.

The theoretical framework here thus shows how inflation band targeting has desirable characteristics and why it makes sense for central banks to adopt inflation target ranges as part of their monetary policy framework.

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[1] Svensson (1997) also points out that if a low inflation level is associated with high unemployment, then an inflation contract may be politically difficult to implement since it rewards a central banker when inflation is low.

[2] Inflation band targeting regimes are also closely related to a dismissal rule as discussed by Walsh (2002).

[3] Amano, Black and Kasumovich (1997) examine whether the literature on exchange rate target zones can be applied to inflation band targeting. Erceg (2002) interprets the target range as a confidence interval of inflation derived from the preferences of the policymakers. The optimal bandwidth should then be chosen to reflect the desired point on the trade-off locus between inflation and unemployment volatility. Orphanides and Wieland (2000) recognise that inflation band targeting implies non-linear monetary policy and show that such policy is optimal when preferences are zone-quadratic or the structure of the economy exhibits zone linearity. Neither of these papers, however, consider inflation band targeting as a solution to the time-inconsistency problem. Athey, Atkeson, and Kehoe (2005) consider the Barro-Gordon set up when the central bank has private information about the state of the economy. Discretion allows the central bank to react to this information but it also gives rise to a desire to over-stimulate the economy. Thus the public has to decide the degree of

discretion it should give the central bank. The authors show that an inflation cap can implement the optimal solution.

[4]The time-inconsistency problem (arising from policymakers attempts to exploit the short-run tradeoff between unemployment and inflation) is however only one aspect of inflation targeting and does not exclude additional benefits such as better management of expectations through improved communication and policy transparency. We also do not address the stabilization bias that the time-inconsistency problem gives rise to in a New Keynesian framework (e.g., Clarida et al 1999) or how inflation targeting can serve as a commitment mechanism to optimal policy inertia as emphasized by Giannoni and Woodford (2003). Instead our analysis focuses on the case where the objectives of the central bank deviate from what is socially optimal.

[5] See Reis (2004) for a formal derivation of the Phillips-curve and the welfare function based on micro-foundations.

[6] McCallum (1995) and Mishkin (2000) points out that the time-inconsistency problem by itself does not imply that a central bank will pursue expansionary monetary policy which leads to inflation. Simply by recognizing the problem that forward-looking expectations in the wage- and price-setting process creates for a strategy of pursuing expansionary monetary policy, monetary policymakers can decide to "just not do it" and avoid the time-inconsistency problem altogether. Indeed, central bankers are fully aware of the time-inconsistency problem, but the time-inconsistency problem is still likely to remain nonetheless because of the politicians are able to put pressure on central banks to pursue overly expansionary monetary policy.

[7] See Mishkin and Westelius (2006) for a richer set-up where the central bank's objective function reflects political influence on the central bank. The set-up leads to similar conclusions of those found here. However, it explicitly illustrates that greater independence of the central bank from the political process helps mitigate the time-inconsistency problem. This suggests that insulating the central bank from political influence through communication strategies, price stability mandates and legislated central bank independence are another important dimension of institutional design to improve monetary policy performance.

[8] Svensson (1997) also finds inflation volatility too high under discretion. However, the source of the distortion stems from persistence in employment. If supply shocks have a persistent effect on employment, then there is an extra incentive for the monetary authorities to mitigate the effect of the supply shock on real activity. Clarida, Gali and Gertler (1999) examine discretionary monetary policy in a New Keynesian framework and find that persistence in the cost-push shock leads to excess inflation volatility. This, however, is due to the forward looking behavior of price setters in the economy which is absent in our and Svensson's (1997) models.

[9] Two other parameters that has previously been discussed in the literature in regards to the inflation bias result are ω and b. In our set-up, an increase in ω, the socially optimal weight on real stabilization, causes both the inflation bias and inflation volatility to increase. However, it also reduces the magnitude of δ relative to ω, thus *reducing* the excess inflation volatility under discretion. An increase in b causes an increase in the inflation bias due to a greater incentive to exploit the short-run trade off between inflation and unemployment. On

the other hand, it also allows for a more efficient shock management and thus decreases overall and excess inflation volatility.

[10] Svensson (1997) derives a similar prediction by introducing employment persistence. In his model, the persistence insures that surprise inflation has a positive effect on employment in the future and thus provides an additional incentive for the policymakers to both be overly expansionary *and* react stronger to supply shocks. In equilibrium this causes a larger inflation bias and more volatile inflation than in the absence of employment persistence. In contrast, the positive relationship between the level and volatility of inflation in our model stems entirely from the government's desire to push unemployment volatility below its socially optimal level.

[11] We refer to a target-conservative central banker as one that has an inflation target lower than the optimal rate. Svensson (1997) does not use the term "target-conservative central banker." Instead, he refers to this set-up as a constant inflation targeting regime.

[12] The framework suggests that the edges of the target range are "hard" in the sense that there is a zero tolerance of missing the range. Svensson (2001) on the other hand has argued for "soft" edges in order to avoid unnecessary instability in output, and interest rates. Although there might be advantages to "soft" edges, our results indicate that "hard" edges do quite well in getting close to the optimal contract and much better than we thought.

[13] Note that the supply shocks are assumed to be i.i.d. We make this assumption in order for our analysis of the inflation band target to be more tractable. One way in which previous literature has incorporated persistence in the unemployment rate is by including a lagged unemployment rate in the Phillips curve (e.g., Svensson 1997). This would not change our result substantially as long as inflation stays within the target range. However, if the supply

shock is large, the persistence in unemployment would cause inflation to be outside the range for several periods. In this case the central bank would pay the cost (C) every period until inflation converges within the target range. In reality, this repeated violation may cause the public to doubt the commitment by the central bank. Consequently, it may be optimal for the central bank to specify a state contingent inflation target range as long as inflation is outside the original target band. Indeed, this is exactly what occurred in 2003 in Brazil. Alternatively, if a country is adopting an inflation band targeting regime while inflation is well above its long-run target, such as in Canada and Chile, then it may be optimal to start with a appropriately specified midpoint target and then continuously adjust it until it converges to the desired long-run rate.

[14] For example, Switzerland only specifies an upper boundary of 2%, but has made it clear that they would not accept deflation.

[15] The parameter values chosen for the simulation exercises are based on U.S. annual inflation and unemployment data. The slope of the Philips curve, *b*, is equal to 0.64 as suggested by Reis (2003). The standard deviations of the control error and supply shock are set at 0.6 and 1, respectively. The natural rate is assumed to be 4% and the optimal level of inflation is set at 2%. The preference parameter in the social welfare is obviously chosen in a more ad hoc manner. We assume that unemployment and inflation stabilization are given equal importance (i.e., $\omega = 1$). The distortion parameters will vary as indicated for each simulation.

[16] The bandwidth is set at 1% and that accountability level at 3. These values were chosen to provide the clearest picture of the marginal cost structure under the inflation targeting regime.

[17] The maximum marginal cost occurs when

$$\pi = \pi^* + BW\left[1 + e^{\frac{2}{\sigma^2}BW\left(\pi^* - \pi\right)} \Big/ 1 - e^{\frac{2}{\sigma^2}BW\left(\pi^* - \pi\right)}\right].$$

Hence the maximum marginal cost occurs outside the boundaries of the target range. However, as BW/σ becomes large the maximum marginal cost approaches the boundary.

[18] As indicated in the previous footnote, the marginal cost is actually *increasing* within a small interval immediately outside the boundaries. However, when the bandwidth is reasonable large relative to the control error this interval becomes very small.

[19] Interestingly, if the bandwidth is smaller than one standard deviation of the control error i.e $BW < \sigma_Z$, then the slope of the marginal cost structure (close to the midpoint) is increasing when the bandwidth is widened. This is due to the increasing absolute value of the slope of the normal PDF that occurs within one std away from its mean.

[20] Additionally when the bandwidth is below one standard deviation of the control error, any further reduction in the bandwidth decreases the slope of the marginal cost curve and hence causes inflation to rise. Also notice that the equilibrium inflation rate is outside the target range for bandwidths that are extremely narrow (see section 5.5.)

[21] If the midpoint is allowed to be lower than the social optimal level or the level of accountability at the upper boundary is higher than that of the lower boundary, then it is possible to eliminate the bias completely.

[22] The optimal design of the inflation band target also depends on structural parameters such as b and ω. For instance, a higher value of ω increases the inflation bias and inflation volatility, but reduces the excess inflation volatility (see footnote 9). This would imply a higher degree of accountability to address the inflation bias and a wider bandwidth to

accommodate for the increased inflation volatility and reduced excess volatility bias. A higher value of b, on the other hand, increases the inflation bias but lowers both the overall and excess inflation volatility. From the optimal inflation contract, however, it is clear that the slope of the marginal cost structure remains the same. Thus a change in the slope of the Phillips curve should have a similar affect on the optimal design of the inflation band target as a change in the unemployment target.

[23] The distribution of inflation will not be normally distributed under inflation band targeting, but will have thicker tails because of the non-linear cost structure. However, the normal distribution is still likely to provide a good approximation.

[24] In Mishkin and Westelius (2006), it is shown that lower central bank independence requires a higher level of accountability but very little adjustments in the bandwidth.

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Table 1(a). Various Simulations under Discretion and Optimal Rule

Discretion

Inflation

δ

u*		2	3	4	5	6
	3	3.92	4.56	5.20	5.84	6.48
	3.25	3.44	3.92	4.40	3.92	5.36
	3.5	2.96	3.28	3.60	3.92	4.24
	3.75	2.48	2.64	2.80	2.96	3.12
	4	2.00	2.00	2.00	2.00	2.00

Inflation Volatility (std)

δ

u*		2	3	4	5	6
	3	1.05	1.14	1.21	1.26	1.30
	3.25	1.05	1.14	1.21	1.26	1.30
	3.5	1.05	1.14	1.21	1.26	1.30
	3.75	1.05	1.14	1.21	1.26	1.30
	4	1.05	1.14	1.21	1.26	1.30

Unemployment Volatility (std)

δ

u*		2	3	4	5	6
	3	0.59	0.54	0.51	0.48	0.46
	3.25	0.59	0.54	0.51	0.48	0.46
	3.5	0.59	0.54	0.51	0.48	0.46
	3.75	0.59	0.54	0.51	0.48	0.46
	4	0.59	0.54	0.51	0.48	0.46

Welfare

δ

u*		2	3	4	5	6
	3	-2.57	-4.07	-5.98	-8.29	-11.0
	3.25	-1.76	-2.64	-3.74	-2.76	-6.60
	3.5	-1.19	-1.62	-2.14	-2.76	-3.47
	3.75	-0.84	-1.26	-1.18	-1.37	-1.59
	4	-0.73	-0.80	-0.86	-0.91	-0.96

Optimal Rule/Contract

Inflation

δ

u*		2	3	4	5	6
	3	2.00	2.00	2.00	2.00	2.00
	3.25	2.00	2.00	2.00	2.00	2.00
	3.5	2.00	2.00	2.00	2.00	2.00
	3.75	2.00	2.00	2.00	2.00	2.00
	4	2.00	2.00	2.00	2.00	2.00

Inflation Volatility (std)

δ

u*		2	3	4	5	6
	3	0.75	0.75	0.75	0.75	0.75
	3.25	0.75	0.75	0.75	0.75	0.75
	3.5	0.75	0.75	0.75	0.75	0.75
	3.75	0.75	0.75	0.75	0.75	0.75
	4	0.75	0.75	0.75	0.75	0.75

Unemployment Volatility (std)

δ

u*		2	3	4	5	6
	3	0.81	0.81	0.81	0.81	0.81
	3.25	0.81	0.81	0.81	0.81	0.81
	3.5	0.81	0.81	0.81	0.81	0.81
	3.75	0.81	0.81	0.81	0.81	0.81
	4	0.81	0.81	0.81	0.81	0.81

Welfare

δ

u*		2	3	4	5	6
	3	-0.61	-0.61	-0.61	-0.61	-0.61
	3.25	-0.61	-0.61	-0.61	-0.61	-0.61
	3.5	-0.61	-0.61	-0.61	-0.61	-0.61
	3.75	-0.61	-0.61	-0.61	-0.61	-0.61
	4	-0.61	-0.61	-0.61	-0.61	-0.61

Note: The following parameterization was used: $b = 0.64$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, and $\sigma_\varepsilon = 1$

Table 1(b). Optimal Inflation Band Targeting

Inflation

δ

u*		2	3	4	5	6
	3	2.22	2.23	2.24	2.24	2.23
	3.25	2.23	2.22	2.22	2.24	2.23
	3.5	2.20	2.21	2.21	2.21	2.21
	3.75	2.13	2.13	2.14	2.14	2.14
	4	2.00	2.00	2.00	2.00	2.00

Accountability

δ

u*		2	3	4	5	6
	3	7.0	9.2	11.7	13.9	17.0
	3.25	6.0	8.7	12.8	14.8	17.6
	3.5	5.8	8.0	10.5	15.0	17.9
	3.75	5.0	8.0	8.6	10.6	12.8
	4	4.1	6.0	7.2	9.3	11.2

Inflation Volatility (std)

δ

u*		2	3	4	5	6
	3	0.64	0.64	0.64	0.65	0.64
	3.25	0.66	0.66	0.65	0.66	0.66
	3.5	0.68	0.69	0.68	0.68	0.68
	3.75	0.71	0.72	0.73	0.73	0.73
	4	0.76	0.74	0.75	0.75	0.75

Bandwidth

δ

u*		2	3	4	5	6
	3	0.8	0.8	0.8	0.8	0.8
	3.25	1.0	1.0	1.1	1.1	1.1
	3.5	1.2	1.2	1.2	1.3	1.3
	3.75	1.3	1.3	1.3	1.3	1.3
	4	1.4	1.3	1.3	1.3	1.3

Unemployment Volatility (std)

δ

u*		2	3	4	5	6
	3	0.95	0.94	0.94	0.94	0.94
	3.25	0.91	0.92	0.92	0.92	0.92
	3.5	0.88	0.88	0.88	0.88	0.88
	3.75	0.85	0.84	0.83	0.83	0.83
	4	0.80	0.82	0.81	0.81	0.81

Bandwidth/STD inflation

δ

u*		2	3	4	5	6
	3	1.25	1.25	1.24	1.24	1.25
	3.25	1.52	1.53	1.69	1.68	1.68
	3.5	1.76	1.75	1.75	1.90	1.90
	3.75	1.82	1.81	1.78	1.78	1.79
	4	1.84	1.76	1.73	1.73	1.74

Welfare

δ

u*		2	3	4	5	6
	3	-0.68	-0.68	-0.68	-0.68	-0.68
	3.25	-0.66	-0.66	-0.66	-0.66	-0.66
	3.5	-0.64	-0.64	-0.64	-0.64	-0.64
	3.75	-0.62	-0.62	-0.62	-0.62	-0.62
	4	-0.61	-0.61	-0.61	-0.61	-0.61

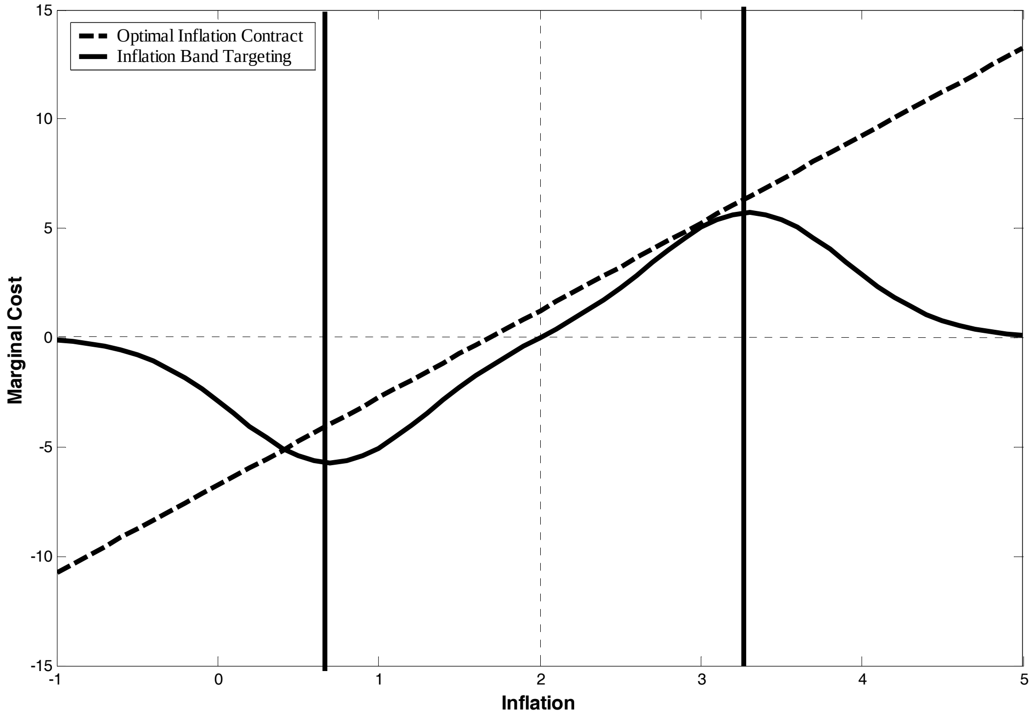
Note: The following parameterization was used: $b = 0.64$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, and $\sigma_\varepsilon = 1$

Table 2. Supply Shock Volatility and Optimal Design of Inflation Band Targeting

	Supply shock volatility (std)				
	0.5	**0.75**	**1**	**1.25**	**1.5**
Optimal accountability (C)	8.3	8.5	10.5	10.6	11.5
Optimal bandwidth (BW)	0.6	0.9	1.2	1.3	1.3
Inflation					
Full discretion	3.60	3.60	3.60	3.60	3.60
Inflation targeting	2.14	2.17	2.21	2.23	2.22
Optimal rule	2.00	2.00	2.00	2.00	2.00
Inflation volatility (std)					
Full discretion	0.80	0.99	1.21	1.44	1.69
Inflation targeting	0.61	0.64	0.68	0.75	0.79
Optimal rule	0.64	0.69	0.75	0.83	0.91
Unemployment volatility (std)					
Full discretion	0.42	0.46	0.51	0.56	0.62
Inflation targeting	0.57	0.72	0.88	1.04	1.24
Optimal rule	0.52	0.66	0.81	0.97	1.13
Welfare level					
Full discretion	-1.68	-1.87	-2.14	-2.48	-2.89
Inflation targeting	-0.36	-0.48	-0.64	-0.85	-1.10
Optimal rule	-0.34	-0.45	-0.61	-0.81	-1.05
Reduction in welfare distortion (%)	-98.6	-98.1	-97.8	-97.7	-97.5

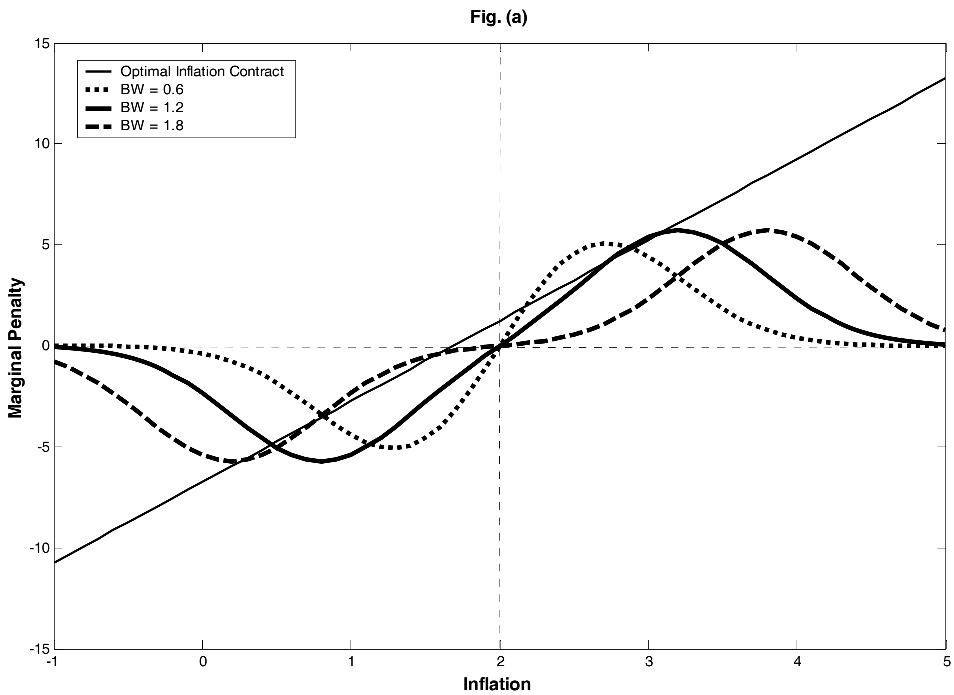
Note: The following parameterization was used: $b = 0.64$, $u^n = 4$, $u^* = 3.5$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\delta = 4$

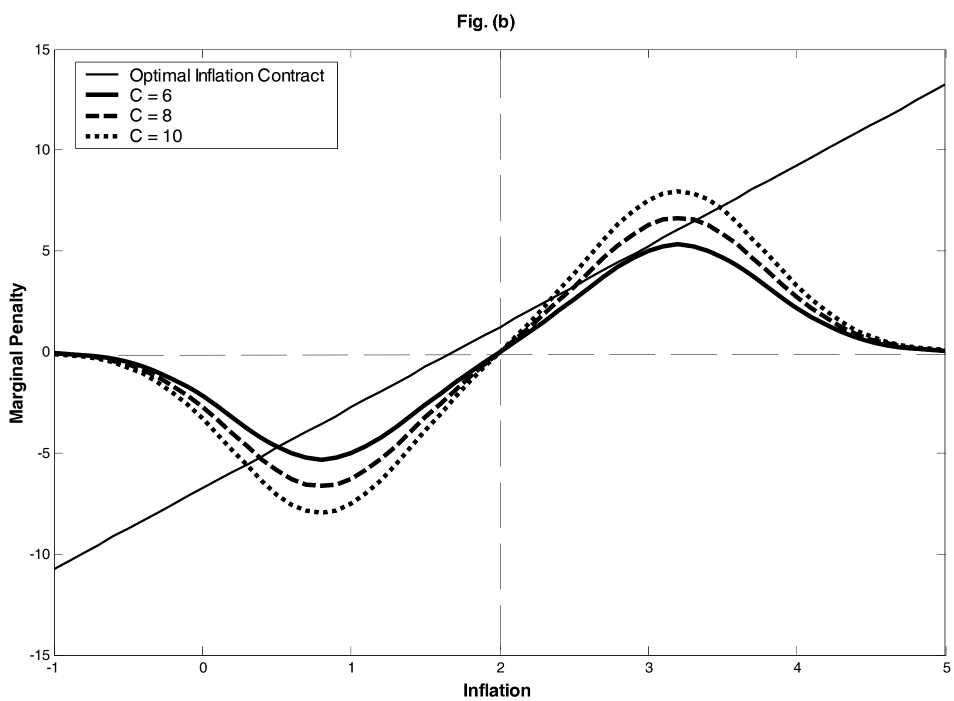
Figure 1. Marginal Cost Structure



Note: The following parameterization was used: $b = 0.64$, $u^* = 3.75$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$. $BW = 1.3$, $C = 8.6$

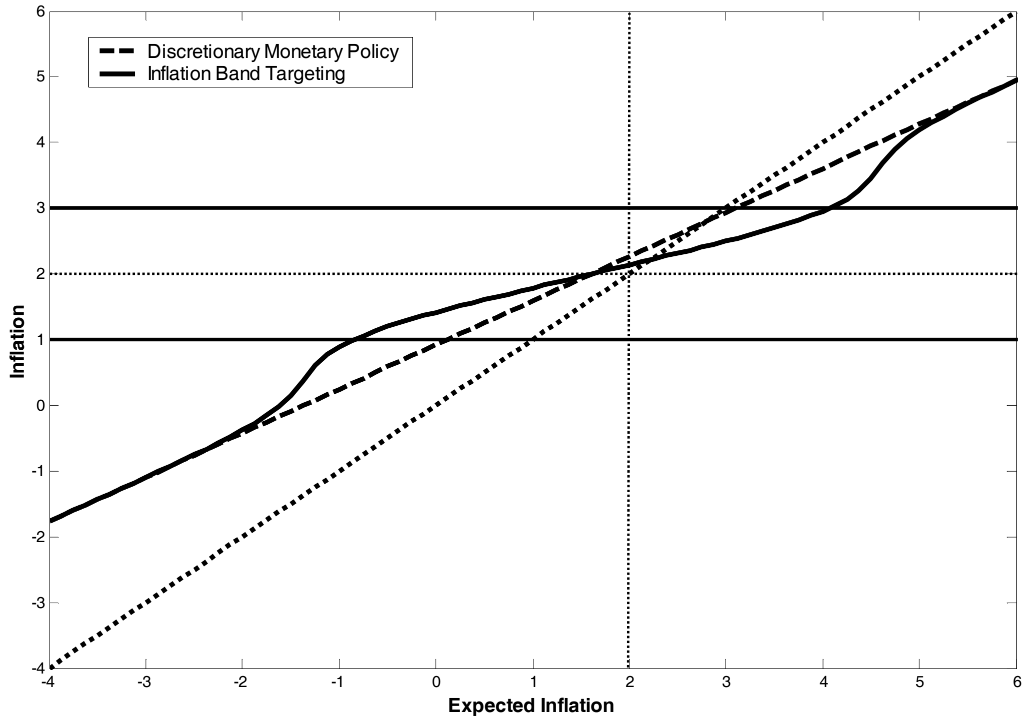
Figure 2. Marginal Cost Structure, Bandwidth and Accountability





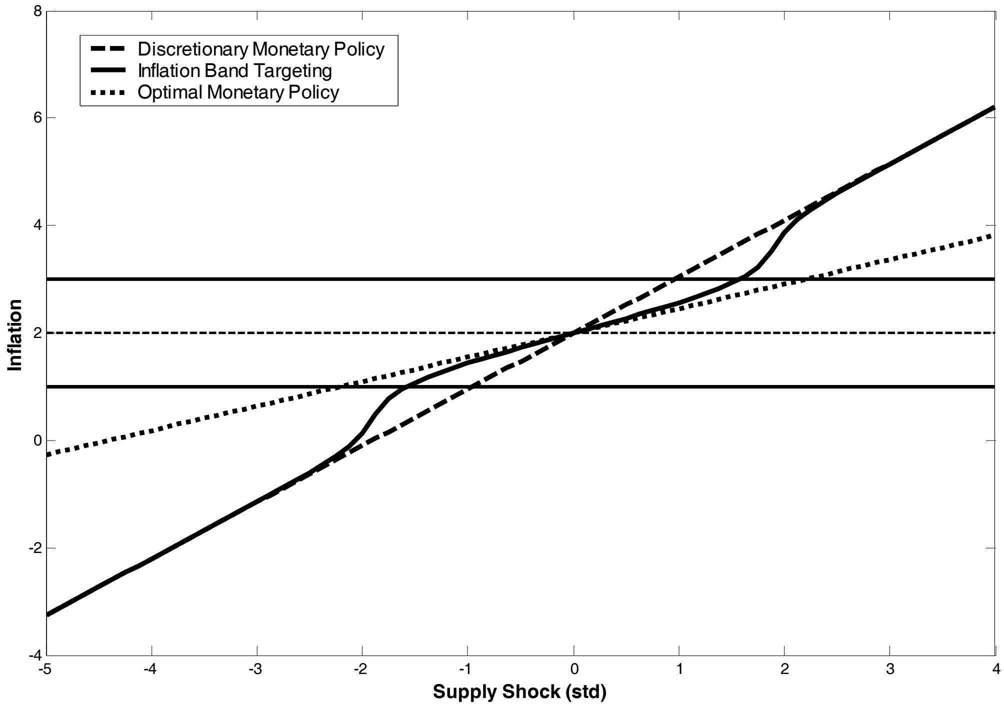
Note: The following parameterization was used: $b = 0.64$, $u^* = 3.75$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, For figure 2(a) the degree of accountability is fixed at 8.6. For figure 2(b) the bandwidth is fixed at 1.2.

Figure 3. Equilibrium Inflation under Inflation Band Targeting



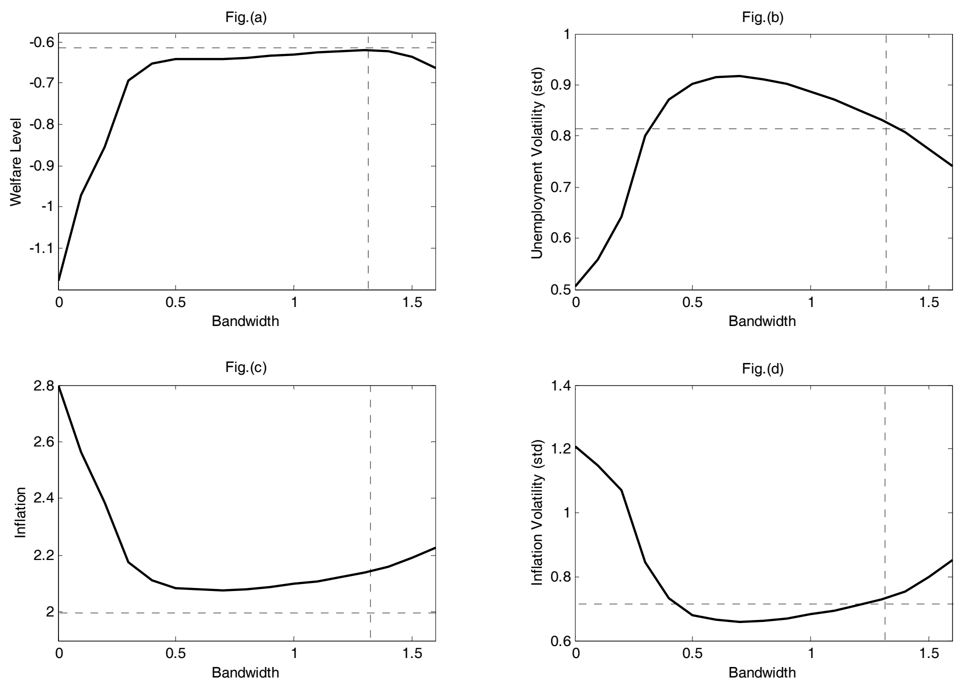
Note: The following parameterization was used: $b = 0.64$, $u^* = 3.75$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$. $C = 3$ $BW = 1$

Figure 4. Shock Management under Inflation Band Targeting



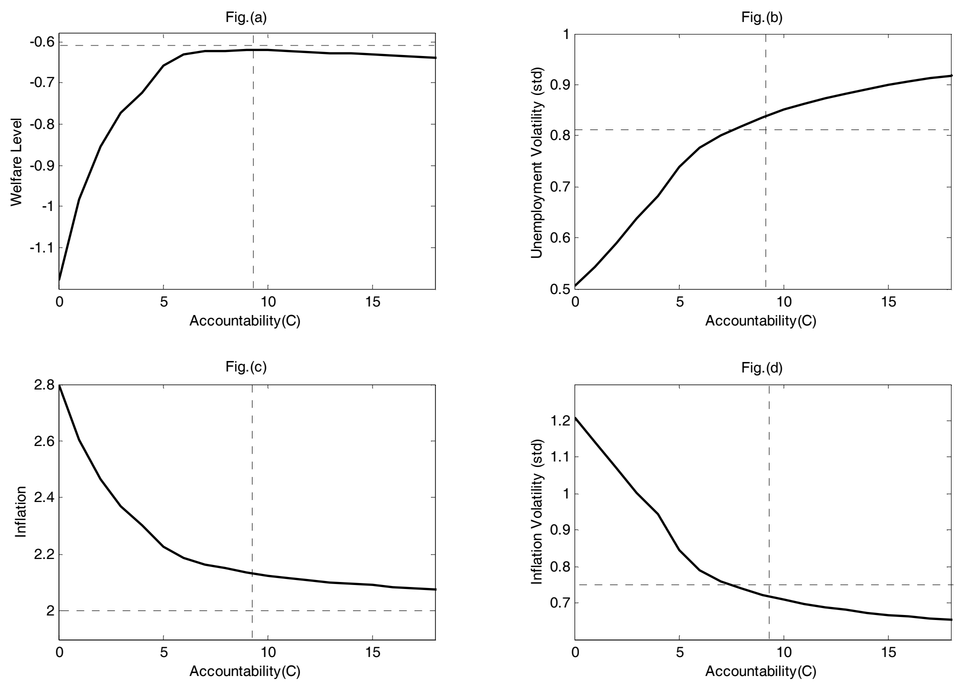
Note: The following parameterization was used: $b = 0.64$, $u^* = 4$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$. $BW = 1$, $C = 3$

Figure 5. Optimal Bandwidth given the Degree of Accountability



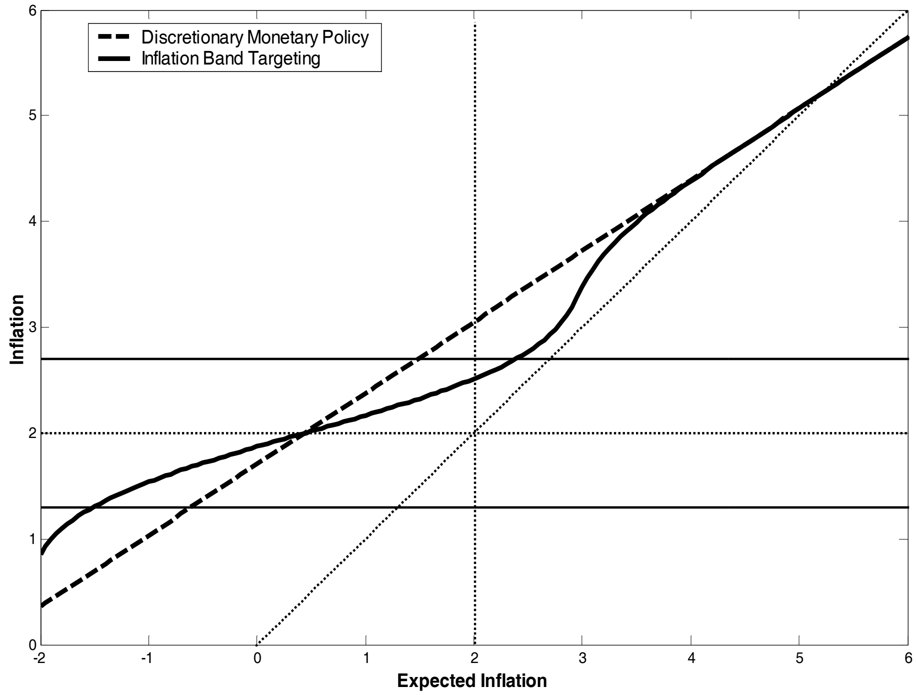
Note: The following parameterization was used: $b = 0.64$, $u^* = 3.75$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, and $C = 8.6$

Figure 6. Optimal Degree of Accountability given the Bandwidth



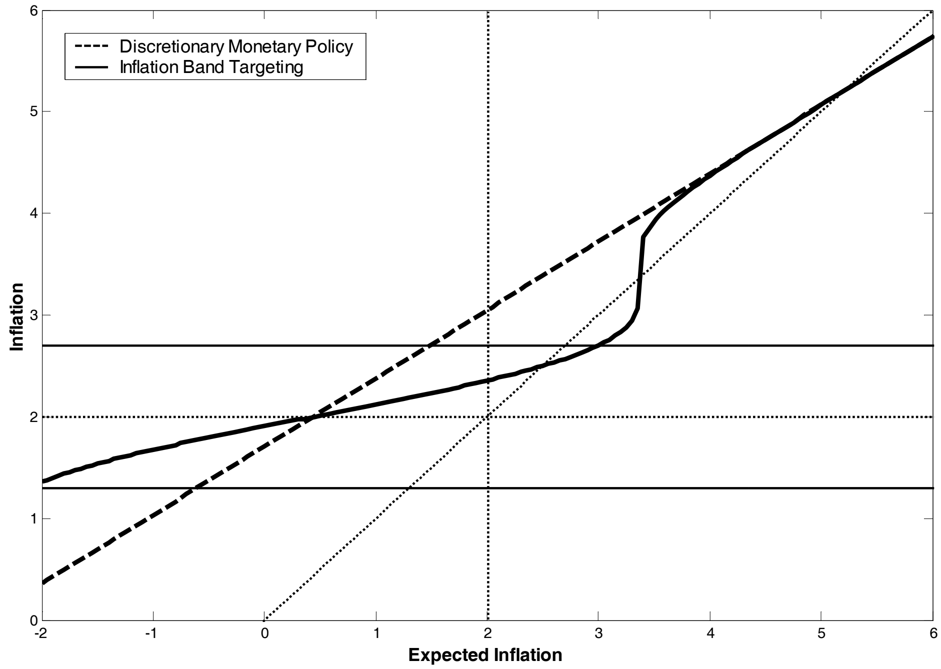
Note: The following parameterization was used: $b = 0.64$, $u^* = 3.75$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$, and $BW = 1.3$

Figure 7. Inflation Band Targeting without Equilibrium within the Range



Note: The following parameterization was used: $b = 0.64$, $u^* = 3$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$. $BW = 0.7$, $C = 3$

Figure 8. Multiple Equilibria under Inflation Band Targeting



Note: The following parameterization was used: $b = 0.64$, $u^* = 3$, $u^n = 4$, $\omega = 1$, $\pi^* = 2$, $\sigma_z = 0.6$, $\sigma_\varepsilon = 1$, $\delta = 4$. $BW = 0.7$, $C = 5$