"Determinacy and Learnability of Monetary Policy Rules in Small Open Economies"*

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Abstract

This paper evaluates under which conditions different Taylor-type rules lead to determinacy and expectational stability (E-stability) of rational expectations equilibrium in a simple "New Keynesian" small open economy model, developed by Gali and Monacelli (RES, 2005). In particular, we extend Bullard and Mitra (JME, 2002) results of determinacy and E-stability in a closed economy to this small open economy framework. Our results highlight an important link between the Taylor principle and both determinacy and learnability of equilibrium in small open economies. More importantly, the degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways. A key finding is that, contrary to Bullard and Mitra, expectations-based rules that involve the CPI and/or the nominal exchange rate limit the region of E-stability and the Taylor Principle does not guarantee E-stability. We also show that some forms of managed exchange rate rules can help to alleviate problems of both indeterminacy and expectational instability, yet these rules might not be desirable since they can promote greater volatility in the economy.

Keywords: Learning; Indeterminacy; Monetary Policy Rules; Open Economy

JEL classification: E4; E5; F31: F41

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1 Introduction

The implementation of monetary policy in terms of interest rate feedback rules has been extensively studied in both closed and open economies contexts. However, the literature has devoted more attention to the closed-economy context and has left interesting open-economy questions aside. In open economies, there are two relevant issues in the design of a policy rule: a) whether the policy rule should react to consumer or domestic price inflation and b) whether the interest rule should react to changes in the exchange rate. Regarding the first issue, most small open economies define their goals in terms of consumer price inflation (CPI) implying that the dynamics of the targeted variable not only incorporates the movements of domestic inflation but also responds to changes in the exchange rate and world inflation. The second issue, as documented by Calvo and Reinhart (2002), is a widespread feature among developing countries.[1] Furthermore, this issue is also present in developed countries; for instance, Lubik and Schorfheide (2007) have found empirical evidence that the Bank of Canada and the Bank of England include the change in the nominal exchange rate in their policy rules. In this paper, we study those issues from the perspective of determinacy of the rational expectations and stability under adaptive learning.

One major question is whether a policy rule in open economies guarantees a (locally) determinate (i.e. unique and non-explosive) rational expectations equilibrium (REE). In a closed economy context, the usual condition for determinacy is the so-called "Taylor Principle".[2] A second major question is whether interest rate feedback rules perform satisfactorily if we relax the assumption of rational expectations by assuming that agents follow a learning process. A particular concern to this literature is the notion of *Expectational Stability* (or E-stability) developed by Evans and Honkapohja (1999, 2001): the conditions under which agents are able to learn the reduced form dynamics induced by the model given a monetary policy rule under the assumption of rational expectations.[3] Bullard and Mitra (2002, hereafter BM) have shown that if agents follow adaptive learning rules, then the stability of the Taylor-type rules might not be taken for granted. Yet, their results support the Taylor principle based on the learnability criteria. In particular, they find that if the monetary authority is able to commit to a Taylor-type interest rate rule, the REE is E-stable under learning dynamics as long as the Taylor principle is satisfied.

The aim of this paper is to evaluate the effects of trade openness coupled with a variety of Taylor-rules on determinacy and E-stability. We study both local determinacy and learnability properties[4] of the REE in the small open economy model proposed by Gali and Monacelli (2005, henceforth GM). In this sense, our work extends BM's (2002) closed-economy results to a small open economy framework. We perform the analysis using four simple monetary policy feedback rules: domestic inflation Taylor rule (DITR), CPI inflation Taylor rule (CPITR), and their extensions with responses to changes in the nominal exchange rate, DI-METR and CPI-METR, respectively. Following BM (2002), we evaluate the aforementioned rules under two specifications based on the way the central bank and private agents form expectations. In the first one, the monetary authority reacts to current values; this is called *contemporaneous data* specification. Our second specification assumes that policymakers react to forecasts; this is called *forecast-based rule* specification.

In general, our results highlight an important link between the Taylor Principle and both determinacy and learnability of REE. Yet, the degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways. The main findings of our analysis can be summarized as follows:

i) Under *contemporaneous data* specification, the determinacy condition under CPITR and DITR is given by the Taylor principle. Regarding E-stability, at least numerically, we find the same result. Openness affects the determinacy and learnability conditions quantitatively and its impact is ambiguous depending mainly on the degree of elasticity of substitution between foreign and domestic goods.

ii) Under the *forecast-based* CPITR and CPI-METR, the striking result is that there is an upper bound on the policy response to expected CPI inflation that ensures both determinacy and E-stability, and therefore the Taylor principle is not a sufficient condition for a determinate and learnable equilibrium. Such upper bound depends negatively on the degree of openness, thus as openness increases, the region of determinacy and E-stability shrinks. One important implication of these results is that the pure application of the Taylor principle in open economies could be misleading advice if policymakers target CPI inflation in a forward-looking fashion. Interestingly, we show that DITR or DI-METR do not have this flavor.[5]

iii) In the cases of the contemporaneous data CPIT-METR and DI-METR and forecast-based DI-METR, we find that the monetary policy authority in a small open economy can substitute inflation stabilization to some degree for exchange rate smoothing. That is, a managed exchange rate regime is more suitable than other monetary rules since it enlarges the areas of determinacy and learnability but it should be emphasized that such a policy promotes macroeconomic volatility.

The contribution of this paper is twofold. First, we obtain not only *analytical* conditions of determinacy, but also of learnability. Second, our analysis relies on a broad set of policy rules for small open economies, including those supported by the data, e.g. Taylor rules with managed exchange rates.[6] In that sense, our paper contributes to a growing literature that has been studying determinacy and stability under learning. Regarding to the issue of which inflation index must be included in the interest rate rule, Carlstrom et. al. (2006) show that the Taylor principle is a necessary and sufficient condition for determinacy in a two-sector closed economy model regardless which price index the central bank is targeting. Our analysis not only support such result in the context contemporaneous rules, but also indicates that forecast-based rules do not share that property. As for the role of openness, Zanna (2003) and De Fiori and Liu (2005) find that the conditions for determinacy depend crucially on the degree of openness to international trade.[7] Finally, a closer paper to ours is that of Bullard and Schaling (2006) who study determinacy and learnability in a two-country model. Some of their results regarding Taylor-type rules parallel ours.[8] For example, they show, as we do, that with *contemporaneous* domestic or CPI inflation targeting, openness alters determinacy and learning conditions at least numerically. Yet, the Taylor Principle is a necessary and sufficient condition to be met independently in both home and foreign economies. Still, we study instrument rules more extensively than Bullard and Schaling (2006), including different specifications (managed exchange rate and forecast-based) of Taylor-type rules.

The rest of the paper is organized as follows. Section 2 outlines the simple environment for the analysis of determinacy and learning. Here we specify the main equations of the GM (2005) model, emphasizing its differences with respect to the closed economy case. After that, we describe the different specifications of monetary policy rules. The analysis of determinacy and E-stability is addressed in Section 3. Within this section we also include additional exercises and discuss the link between E-stability and the implied macroeconomic volatility induced by managed exchange rate rules. Finally, Section 4 concludes.

2 The Simple Environment

2.1 The model

We study the simple small open economy model developed by GM (2005). The model is built up by assuming a small open economy with staggered prices *a là* Calvo (1983) as one among a continuum of (infinitesimally small) economies making up the world economy. Before proceeding with the exposition of the model, we describe some useful notation used throughout the paper. Subscript H denotes any *domestic* or *home* variable, subscript F denotes *foreign* or *imported* variables (in domestic currency), superscript '$-$' denotes variables in their natural levels, and superscript * denotes *international* or *world* variables.[9]

The small open economy is log-linearized at a steady state and can collapse to the following two equations (equations 36 and 37 of GM),

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\alpha x_t \quad (1)$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\alpha}\left(r_t - E_t \pi_{H,t+1} - \overline{rr}_t\right) \quad (2)$$

where $\lambda \equiv \left[(1-\beta\theta)(1-\theta)/\theta\right], \kappa_\alpha \equiv \lambda(\sigma_\alpha + \varphi), \sigma_\alpha \equiv \frac{\sigma}{[1-\alpha+\alpha\varpi]}$, and $\omega \equiv \sigma\gamma + (1-\alpha)(\sigma\eta - 1)$.

The variables x_t, $\pi_{H,t}$ and r_t represent the *domestic output gap*, *domestic inflation*, and *domestic interest rate*, respectively. In the model, $\overline{rr}_t$ is the small open economy's natural level of real interest rate and E_t symbolizes the standard expectation operator. We implicitly base our analysis of learning and monetary policy on "Euler Equation" approach as it is suggested in Honkapohja, Mitra and Evans (2003). Therefore, throughout the paper we assume that our systems are valid under both rational expectations and learning. In this sense, the expectation operation is taken to describe aggregate behavior regardless of the precise nature of agents' expectation formation.[10]

Equation (1) is a *new Keynesian Phillips curve* (NKPC) and equation (2) is a dynamic IS-type. Both equations involve several deep parameters. The parameter β denotes the discount factor, σ is the coefficient of risk aversion, φ is the inverse of labor supply elasticity, η is the elasticity of substitution between domestic and foreign goods, γ is the elasticity of substitution between imported goods, α is the inverse of home bias in preferences and can be interpreted as a natural index of trade openness, and θ is the degree of price stickiness.

Notice that the coefficients κ_α and σ_α depend on parameters that are specific to the open economy, i.e., the degree of openness and the substitutability among goods of different origin. On one hand, the degree of openness, α, affects the dynamics of domestic inflation through its influence on the size of the slope of the Phillips curve, i.e., the size of response to any given variation in the output gap. In the open economy, a change in domestic output has an effect on marginal cost through its impact on employment (captured by φ) and the terms of trade (captured by σ_α). In particular, under the assumption that $\sigma\eta > 1$, an increase in openness dampens the impact of the adjustment on inflation after an output gap shock. On the other hand, the degree of openness influences the sensitivity of the output gap to interest rate changes. In particular, if $\sigma\eta > 1$, an increase in openness raises that sensitivity through the stronger effects of the induced terms if trade changes on demand. Considering the definitions of κ_α and σ_α given above, a special case arises. When the small open economy is totally autarkic (α is zero), σ_α reduces to σ. In this case, equations (1) and (2) collapse to the standard closed economy model of Woodford (2003*b*).[11]

Besides equations (1) and (2) we need to define auxiliary equations that will be useful for the analysis of determinacy and E-stability for a broad set of policy rules in open economies.

Under the assumption of complete international financial markets, GM (2005) obtain a version of the uncovered interest parity condition. Log-linearizing around a perfect foresight steady state,

$$E_t \Delta e_{t+1} = r_t - r_t^* \tag{3}$$

where e_t is the nominal exchange rate and r_t^* is the world interest rate. Equation (3) implies that an expected depreciation (appreciation) of the nominal exchange rate is necessary to counterbalance any positive (negative) difference between the domestic interest rate and the world interest rate.

GM define the log level of terms of trade s_t as $s_t = p_{F,t} - p_{H,t}$, where $p_{F,t}$ and $p_{H,t}$ are the log level of foreign prices and domestic prices, respectively. Then, we get an expression for the rate of change in the terms of trade:

$$\Delta s_t = \pi_{F,t} - \pi_{H,t} \tag{4}$$

where $\pi_{F,t} \equiv p_{F,t} - p_{F,t-1}$ and $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$. Combining the last equation with equation (14) of GM (2005), $\pi_t = \pi_{H,t} + \alpha \Delta s_t$, it is a matter of a few algebraic operations to obtain the following definition of CPI inflation,

$$\pi_t = (1 - \alpha)\, \pi_{H,t} + \alpha \pi_{F,t} \tag{5}$$

where $\pi_t \equiv p_t - p_{t-1}$ is CPI inflation. This makes CPI inflation a weighted average between domestic and foreign inflation in domestic currency, where the weighting factor is the degree of openness. Since the law of one prices holds for individual goods at all times, GM (2005) show that $p_{F,t} = e_t + p_t^*$ with p_t^* representing the log level of the world price index. The law of one price assumption implies that,

$$\pi_{F,t} = \Delta e_t + \pi_t^* \tag{6}$$

From (4) and (6) it follows that the rate of change in the terms of trade, the rate of change of nominal exchange rate, domestic inflation and world inflation are linked according to

$$\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t} \tag{7}$$

Let us define the terms of trade gap $\widehat{s}_t$ as the deviation of (log) domestic terms of trade s_t from its natural level $\overline{s}_t$, where the latter is in turn defined as the equilibrium level of terms of trade in the absence of nominal rigidities. Formally, $\widehat{s}_t \equiv s_t - \overline{s}_t$. Using this definition of terms of trade gap and equation (7) we have,

$$\widehat{s}_t = \widehat{s}_{t-1} + \Delta e_t + \pi_t^* - \pi_{H,t} + \Delta \overline{s}_t. \tag{8}$$

Manipulating equations (29) and (34) from GM (2005), we obtain an equivalence between the output gap and the terms of trade gap,

$$\widehat{s}_t = \sigma_\alpha x_t \tag{9}$$

Without loss of generality, it is assumed that world variables (r_t^*, π_t^*) are constant and equal to their steady state level. For the sake of simplicity, we further assume that the world steady state levels are centered at zero for both variables. Additionally, as in BM (2002), domestic variables at their natural levels $(\overline{rr}_t, \Delta \overline{s}_t)$ are driven by exogenous and mutually independent first-order autoregresive processes. We keep this assumption on the basis that $(r_t^*, \pi_t^*, \overline{rr}_t, \Delta \overline{s}_t)$ cannot be affected by the small open economy's policies or aggregate performance around its local equilibrium.

2.2 Simple Taylor Rules

We supplement equations (1) through (9) with a policy rule for the domestic interest rate r_t that represents the behavior of the monetary authority. We consider a handful of possible Taylor-type feedback rules with different sets of target variables. All the feedback rules have two alternative specifications: *contemporaneous data* and *forecast-based data*. In the first type, the interest rate reacts to information observed at time t, that is, current inflation (domestic or CPI), domestic output gap and/or nominal exchange rate changes. In the forecast-based specification, interest rates react to one period ahead expectations of the targeted variables.[12]

2.2.1 Contemporaneous Specification

We consider rules similar to the one proposed by the seminal work of Taylor (1993).

$$DITR \quad : \quad r_t = \phi_\pi \pi_{H,t} + \phi_x x_t \tag{10}$$

$$CPITR \quad : \quad r_t = \phi_\pi \pi_t + \phi_x x_t \tag{11}$$

$$DI-METR \quad : \quad r_t = \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_e \Delta e_t \tag{12}$$

$$CPI-METR \quad : \quad r_t = \phi_\pi \pi_t + \phi_x x_t + \phi_e \Delta e_t \tag{13}$$

2.2.2 Forecast-Based Specification

As in the case of contemporaneous specifications, we name each of these rules depending on which variables the central bank is reacting to. Throughout the paper we refer to these rules as forecast-based (FB) rules.

$$FB - DITR \quad : \quad r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1} \tag{14}$$

$$FB - CPITR \quad : \quad r_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1} \tag{15}$$

$$FB - DI - METR \quad : \quad r_t = \phi_\pi E_t \pi_{H,t+1} + \phi_x E_t x_{t+1} + \phi_e E_t \Delta e_{t+1} \tag{16}$$

$$FB - CPI - METR \quad : \quad r_t = \phi_\pi E_t \pi_{t+1} + \phi_x E_t x_{t+1} + \phi_e E_t \Delta e_{t+1} \tag{17}$$

It is worth to mention that (17) corresponds to the reaction function used by Lubik and Schorfheide (2007).[13]

2.3 Parameterization

In order to gain an insight into the effects of openness and the alternative policy rule specifications on determinacy and learnability conditions, we illustrate the results by using a calibrated case. Table 1 summarizes this Parameterization.

table 1 approx. here

Parameters γ, θ, and β are taken from GM (2005); η and σ from Chari et. al. (2002) and φ from Rotemberg and Woodford (1998). We let the degree of openness α take two possible values: 0 or 0.4, where the former characterizes our completely *closed economy,* whereas the latter characterizes our *open economy.*[14] Finally, we consider that all variables in their natural levels ($\overline{rr}_t, \Delta\overline{s}_t$) follow AR(1) processes with persistence less than one and zero cross correlation. As in BM (2002) we calibrate the policy reaction parameters for non-negative values.

3 Policy Rules, Determinacy and Learning

3.1 Contemporaneous Specifications

3.1.1 Domestic Inflation Taylor Rules (DITR)

First we study the case in which the central bank uses a contemporaneous Domestic Inflation Taylor Rule (DITR) of the form of (10). To obtain the determinacy and E-stability conditions, we combine equations (1), (2) and (10), so the model boils down to a two dynamic equation system involving domestic variables x_t and $\pi_{H,t}$,

$$\begin{aligned} y_t &= \Gamma + \Omega E_t y_{t+1} + \Theta w_t \\ w_t &= \rho w_{t-1} + \varepsilon_t \end{aligned} \tag{18}$$

where $y_t = [\pi_{H,t}, x_t]'$, $w_t = \overline{rr}_t$. To study the stability of REE under adaptive learning, we follow Evans and Honkapohja (2001, section 10.3) and assume that agents utilize a *perceived law of motion* (PLM) for y_t that corresponds to the *minimal state variable* (MSV) solution (see McCallum 1983) to the system (18). The PLM can be written as:

$$y_t = a + c\overline{rr}_t$$

Using this PLM, agents form expectations of y_{t+1} :

$$E_t y_{t+1} = a + c\rho\overline{rr}_t.$$

Plugging these expectations into (18) delivers a T-mapping from the PLM to the *actual law of motion* (ALM):

$$y_t = T_a(a) + T_c(c)\overline{rr}_t.$$

The rational expectations solution consists of values such that $\widetilde{a} = T_{\widetilde{a}}(\widetilde{a})$ and $\widetilde{c} = T_{\widetilde{c}}(\widetilde{c})$. The answer of the question of whether the system (18) is stable under learning is given by the principle of E-stability, which comes from analyzing the local asymptotic stability of the following matrix differential equation

$$\frac{\partial T(a,c)}{\partial \tau} = T(a,c) - (a,c)$$

evaluated at the REE solution $(\widetilde{a}, \widetilde{c})$. Specifically, the REE solution of the system (18) is E-stable or *learnable* if all real parts of the eigenvalues of

$$\begin{aligned} DT_a(\widetilde{a}) &= \Omega \\ DT_c(\widetilde{c}) &= \rho' \otimes \Omega \end{aligned}$$

are lower than 1. Determinacy is analyzed by asking under which conditions Ω has both of its eigenvalues inside the unit circle.[15]

Notice that equations (1) and (2) only involve domestic variables, thus the open economy effects come into the model through the coefficients that are altered relative to those of the closed economy case as discussed in section 2.1. In fact, an important case occurs when α is zero so that the economy is closed and the model is the same as in Woodford (2003*b*). Furthermore, DITR is in essence the same as the so-called contemporaneous data interest rule of BM (2002). Therefore, under this rule it should not be surprising that determinacy and learnability conditions of the small open economy coincide with conditions derived by BM (2002). Recalling Propositions 1 and 2 of BM (2002) we have that under DITR the necessary and sufficient condition for both determinacy and learnability is given by[16]

$$\kappa_\alpha (\phi_\pi - 1) + (1 - \beta) \phi_x > 0. \tag{19}$$

Condition (19) corresponds to the long-run version of the Taylor Principle: in the long run the nominal interest rate should be raised by more than the increase in inflation. As emphasized by BM (2002), such a policy succeeds in stabilizing the economy towards its rational expectations equilibrium. When there is no response to the output gap, the standard Taylor principle, i.e. $\phi_\pi > 1$, always implies its long-run and it becomes a necessary and sufficient condition for a determinate and learnable equilibrium.[17] Note that for values of $\phi_\pi < 1$, the policy authority can compensate for a relatively low value of ϕ_π by choosing a sufficiently large value of ϕ_x in such a way as to still satisfy condition (19).

figure 1 approx. here

In order to examine the effect of openness on the stability of the economy, Figure (1) depicts determinate and E-stable regions as functions of ϕ_π and ϕ_x under different degrees of openness. In all cases the rest of the parameters are set at their baseline values.[18] The numerical results reveal that the line between Determinate and E-stable and Indeterminate and E-unstable regions steps up as the degree of openness approaches to zero. Thus, whenever $\phi_\pi < 1$, relatively closed economies need greater responses to the output gap. Therefore, the more closed the economy, the tighter are the constraints faced by policymakers. The explanation behind this outcome relies on the effects of openness on κ_α. If $\sigma\eta > 1$, an increase of openness has a positive effect, increasing the area of determinacy and E-stability through the reduction of κ_α. This positive effect decays non-monotonically with the degree of openness: the area of determinacy and E-stability with a mild degree of openness (α is 0.4) is not greatly different from the corresponding area with a completely open economy (α is 1).

The intuition behind the enlargement of the determinate and learnable region stems from the terms of trade effect on inflation dynamics. Specifically, a positive (negative) output gap is offset by an increment (reduction) of the terms of trade, which causes an expenditure switching effect from domestic (foreign) towards foreign (domestic) goods. As a consequence, in relatively more open economies a central bank can be less concerned with the output gap because its fluctuations have a lower impact on domestic inflation. Note that when $\sigma\eta < 1$ the opposite result holds, whereby we would observe a reduction of both the determinate and learnable regions as the degree of openness increases.

3.1.2 Consumer Inflation Taylor Rule (CPITR)

In this section we assume that the central bank sets its interest rate according to a contemporaneous CPI Inflation Taylor Rule (CPITR), given by (11). In an open economy domestic inflation differs from CPI inflation due to the presence of the terms of trade as an additional endogenous variable. We depart from the earlier analysis by formulating the dynamics of the small open economy in terms of domestic inflation, nominal exchange rate and terms of trade gap. To do that, we combine equations (1), (3), (8), (11) and use definitions (5) and (9). Notice that the Taylor rule (11) can be re-expressed as:

$$r_t = \phi'_\pi \pi_{H,t} + \phi_x x_t + \phi'_e \Delta e_t \tag{20}$$

where $\phi'_\pi = (1-\alpha)\,\phi_\pi$ and $\phi'_e = \alpha\phi_\pi$. This rule embeds DITR since, instead of having the reaction to domestic inflation, $\pi_{H,t}$, equal to ϕ_π, under this rule the implied reaction to domestic inflation is smaller and equal to $(1-\alpha)\,\phi_\pi$. Yet, in addition there is an implicit reaction to contemporaneous changes to the exchange rate that will add some inertia to the rational expectations equilibrium. The model can be re-written as a system of three equations of the form

$$\begin{aligned}
\pi_{H,t} &= \beta E_t \pi_{H,t+1} + \kappa_\alpha \sigma_\alpha^{-1} \widehat{s}_t && (21)\\
\widehat{s}_t &= \widehat{s}_{t-1} + \Delta e_t - \pi_{H,t} + \Delta \overline{s}_t. && (22)\\
E_t \Delta e_{t+1} &= \phi'_\pi \pi_{H,t} + \phi_x \sigma_\alpha^{-1} \widehat{s}_t + \phi'_e \Delta e_t && (23)
\end{aligned}$$

Equation (21) is obtained by combining the aggregate supply equation, (1) , and definition (9). Equation (23) is derived by combining (20) with the UIP equation, (3), and (9) . The exogenous variable $\Delta\overline{s}_t$ follows,

$$\Delta\overline{s}_t = \rho\Delta\overline{s}_{t-1} + \epsilon_t$$

The system involving the endogenous variables $\pi_{H,t}, \Delta e_t$ and $\widehat{s}_t$ can be represented as

$$\begin{bmatrix} E_t q_{t+1} \\ z_{t+1} \end{bmatrix} = \Upsilon \begin{bmatrix} q_t \\ z_t \end{bmatrix} + \Lambda w_t \tag{24}$$

where $q_t = [\pi_{H,t}, \Delta e_t]', z_t = \widehat{s}_{t-1}$, $w_t = \Delta\overline{s}_t$. Variable q_t collects non-predetermined variables, whereas z_t collects states or predetermined variables. Vector w_t denotes the exogenous variables of the system. Matrix Υ is given by

$$\Upsilon = \begin{bmatrix} \frac{1}{\beta} + \frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} & -\frac{\kappa_\alpha}{\beta\sigma_\alpha} \\ -\frac{\phi_x}{\sigma_\alpha} + \phi'_\pi & \phi'_e + \frac{\phi_x}{\sigma_\alpha} & \frac{\phi_x}{\sigma_\alpha} \\ -1 & 1 & 1 \end{bmatrix} \tag{25}$$

Since there exists one predetermined variable (lag of terms of trade gap), the equilibrium is determinate if and only if the matrix Υ has exactly two eigenvalues outside the unit circle and one eigenvalue inside the unit circle. Woodford (2003*b*, Chapter 4) derives the necessary and sufficient conditions for a determinate equilibrium of a system like (24).[19] The following proposition resumes the result.

Proposition 1. *Under CPITR the necessary and sufficient condition for a rational expectations equilibrium to be determinate is that*

$$\kappa_\alpha\left(\phi'_\pi - \left(1 - \phi'_e\right)\right) + (1-\beta)\,\phi_x > 0 \tag{26}$$

Proof. See Appendix A. □

Apparently (26) is different from the long-run Taylor principle, but after replacing ϕ'_π and ϕ'_e, we can note that (26) becomes $\kappa_\alpha(\phi_\pi - 1) + (1-\beta)\,\phi_x > 0$. Therefore, as in the case of contemporaneous DITR, the long-run Taylor Principle completely characterizes determinacy. The reason behind this result relies on the fact that lower reaction to domestic inflation is canceled out by the implicit reaction to nominal exchange movements.[20] To analyze the stability under learning, we re-write the system (24) as

$$y_t = \Gamma + \Omega E_t y_{t+1} + \Phi y_{t-1} + \Theta w_t, \tag{27}$$

where $y_t = [\pi_{H,t}, \Delta e_t, \widehat{s}_t]'$, $w_t = \Delta \overline{s}_t$. Matrices are $\Gamma = 0$,

$$\Omega = \psi \begin{bmatrix} \beta(\phi_x + \sigma_\alpha \phi_e') & \kappa_\alpha & 0 \\ \beta(\phi_x - \sigma_\alpha \phi_\pi') & \kappa_\alpha + \sigma_\alpha & 0 \\ -\sigma_\alpha \beta(\phi_\pi' + \phi_e') & \sigma_\alpha & 0 \end{bmatrix} ; \Phi = \psi \begin{bmatrix} 0 & 0 & \kappa_\alpha \phi_e' \\ 0 & 0 & -(\phi_x + \kappa_\alpha \phi_\pi') \\ 0 & 0 & \sigma_\alpha \phi_e' \end{bmatrix}$$

$$\Theta = \psi \begin{bmatrix} \kappa_\alpha \phi_e' \\ -(\phi_x + \kappa_\alpha \phi_\pi') \\ \sigma_\alpha \phi_e' \end{bmatrix}$$

where $\psi = (\phi_x + \kappa_\alpha \phi_\pi' + (\kappa_\alpha + \sigma_\alpha)\phi_e')^{-1}$.

As stated in proposition 1 of McCallum (2007), determinacy is a sufficient (though not necessary) condition for E-stability (under t-dating expectations) for a broad class of models, including the one in this paper.[21] Hence, the Taylor principle given by condition (26) is sufficient for E-stability, i.e., all RE stable solution of (27) has the property of E-stability. Yet, we further need to check whether an indeterminate equilibrium is E-stable or not. We perform a numerical evaluation for the conditions of E-stability and find that none indeterminate equilibrium is learnable. Therefore, with contemporaneous data in the policy rule there is no difference between targeting domestic inflation or consumer price inflation.[22] However, as pointed out by GM, there is a difference between targeting CPI inflation and domestic inflation: the implied macroeconomic volatility of the endogenous variables will be larger under the CPITR.

3.1.3 Domestic Inflation Managed Exchange Rate Taylor Rule (DI-METR)

Similarly to the previous case, under contemporaneous Domestic Inflation Managed Exchange Rate Taylor Rule (DI-METR) the model can be re-written as a system of three equations of the form of (21) - (23). To study determinacy we re-express this set of equations as (24). Matrix Υ, given by (25), changes accordingly, i.e., $\phi_\pi' = \phi_\pi$ and $\phi_e' = \phi_e$.

Proposition 2. *Under DI-METR the necessary and sufficient condition for a rational expectations equilibrium to be determinate is that,*

$$\kappa_\alpha(\phi_\pi - (1 - \phi_e)) + (1 - \beta)\phi_x > 0 \tag{28}$$

Proof. The proof follows the same steps of Proposition 1. □

Notice that (28) can be re-written as $\kappa_\alpha(\phi_\pi + \phi_e - 1) + (1 - \beta)\phi_x > 0$. Therefore, *ceteris paribus*, the determinacy region increases with the degree of reaction of the interest rate to changes in the nominal exchange rate, which is clear from the term $(\phi_\pi + \phi_e - 1)$. This could be interpreted as a generalization of condition (19) which ensures determinacy and E-stability in a model without managed exchange rate. A novel implication that rises from (28) is that when interest rate reacts one-for-one to nominal exchange rate movements (i.e., ϕ_e is 1), monetary policy can induce determinacy even with a negligible response to inflation and/or the output gap. Analogous to DITR and CPITR, the degree of openness modifies determinacy conditional on whether $\sigma\eta > 1$ holds or not. As in the previous rule, in order to analyze E-stability we rely on McCallum (2007)'s result, i.e., condition (28) is sufficient for E-stability. The numerical evaluation of E-stability shows the correspondence between indeterminacy and instability under learning.[23]

figure 2 approx. here

Figure (2) shows our results under two different values of ϕ_e for a given degree of openness (α equals 0.4). The picture on the left plots the results when there is no response to nominal exchange rate (i.e., ϕ_e is zero): policymakers follow either a CPITR or DITR. In the picture on the right, we assume that monetary authority reacts to the nominal exchange rate besides domestic inflation and output gap. In the latter, we calibrate the value of ϕ_e to be 0.6. Interestingly a central bank reacting passively to inflation $(\phi_\pi < 1)$ and simultaneously targeting movements in the exchange rate in the policy rule $(\phi_e > 0)$ can induce a determinate and E-stable equilibrium even with null response to the output gap. For instance, when ϕ_x is zero, the lower bound of ϕ_π is around 0.4. Moreover, if ϕ_e is larger than one, any positive values for ϕ_π and ϕ_x would imply both determinate and E-stable equilibrium. Therefore, additional reaction to nominal exchange rate increases the determinate and learnable regions.[24]

3.1.4 Consumer Inflation Managed Exchange Rate (CPI-METR)

Under the contemporaneous Consumer Inflation Managed Exchange Rate Taylor rule (CPI-METR), we obtain a system of three equations like (21) to (23). To study determinacy we re-express this set of equations as (24). Matrix Υ, given by (25), changes accordingly, i.e., $\phi'_\pi = \phi_\pi (1 - \alpha)$ and $\phi'_e = \phi_e + \alpha\phi_\pi$.

Proposition 3. *Under CPI-METR the necessary and sufficient condition for a rational expectations equilibrium to be determinate is that*

$$\kappa_\alpha (\phi_\pi - (1 - \phi_e)) + (1 - \beta) \phi_x > 0 \tag{29}$$

Proof. The proof follows the same steps of Proposition 1. □

Condition (29) is exactly the same as we found under DI-METR. As previous analyzes, McCallum (2007)'s result applies and the numerical evaluation of E-stability shows the correspondence between indeterminacy and instability under learning.[25]

This class of rule elicits some interesting aspects of both determinacy and E-stability in small open economies. Compared with DI-METR, this type rule delivers the same results. Therefore, regardless of the inflation index targeted by the Central Bank, a certain degree of exchange rate management helps to avoid both indeterminacy and instability under learning. Furthermore, this implies that the direct reaction towards movements in the exchange rate is the factor that relaxes both determinacy and E-stability conditions. Instead, contemporaneous reaction to CPI inflation does not add anything in terms of determinacy and E-stability even if it implies an indirect reaction to nominal exchange rate changes. As noted above, such implicit reaction cancels out with the lower reaction to domestic inflation.[26] We also emphasize that managed exchange rate promotes both determinacy and learnability of equilibrium in open economies in the same way as the lagged interest rate in the policy rule (so-called policy inertia) does it in the closed economy counterpart, see Woodford (2003*a*) and Bullard and Mitra (2006). In fact, since the current nominal exchange rate varies one-for-one with the lagged of domestic interest rate, the inclusion of the former in the policy rule works as if there actually were inertia in the domestic interest rate.

3.1.5 Additional Exercises

$t-1$ **dating in expectations** The results presented above are based upon the assumption of t-dating in expectations, i.e., current endogenous variables are included in individual's information set. This assumption involves a problem with the simultaneous determination of expectations and current endogenous variables. If instead one considers $t-1$ dating in expectations, it is possible to avoid the problem of simultaneous determination but it is not clear whether determinacy is sufficient for E-stability; McCallum (2007) for a discussion. Since the model under CPITR, DI-METR or CPI-METR contains a lagged endogenous variable, the case of $t-1$ dating expectations implies different E-stability conditions. Following Evans and Honkapohja (2001, section 10.2.1), we obtain numerically the conditions of E-stability for CPITR, CPI-METR and DI-METR based on $t-1$ dating expectations formation. Our numerical experiments reveal that all of the previous findings hold, i.e., determinacy and E-stability regions are exactly the same compared to those in the previous sections.[27]

Reacting to the Real Exchange Rate (a general rule) In sections 3.1.3 and 3.1.4 we have shown that reacting to the change of nominal exchange rate relaxes the conditions for both determinacy and E-stability regardless which inflation (domestic or CPI) the Central

Bank is targeting. Yet, a key question is whether such result is robust to the inclusion of the real exchange rate (RER) rather than the nominal exchange rate. Taylor (2001) discuss a rule of the following general form,

$$r_t = \phi_\pi \pi_{H,t} + \phi_x x_t + \phi_{q,0} q_t + \phi_{q,1} q_{t-1} \tag{30}$$

where q_t is the level (in logs) of the real exchange rate. Taylor (2001) restricts his analysis to the case when $\phi_{q,0} > 0$, $\phi_{q,1} < 0$ and $\phi_{q,0} + \phi_{q,1} > 0$ then a lower RER would call on the central bank to reduce the interest rate.[28] To analyze determinacy and E-stability properties under (30), we use the definition of RER given in GM (page 713): $q_t = (1-\alpha)\, s_t$. We first study the case when $\phi_{q,0} = -\phi_{q,1}$, i.e., the interest rate reacts to the changes in the real exchange rate. Differencing the definition of RER and using (7), the rule (30) collapses to

$$r_t = \phi'_\pi \pi_{H,t} + \phi_x x_t + \phi'_e \Delta e_t$$

where $\phi'_\pi = \phi_\pi - \phi_{q,0}\,(1-\alpha)$ and $\phi'_e = \phi_{q,0}\,(1-\alpha)$. Notice that this rule is isomorphic to DIMETR and hence determinacy and E-stability coexist if $\kappa_\alpha\,(\phi'_\pi - (1-\phi'_e)) + (1-\beta)\,\phi_x > 0$. After plugging ϕ'_π and ϕ'_e, it is straightforward to show that reacting to RER changes does not alter the condition of determinacy and E-stability.[29] The reason is analogous to the case of CPITR: the lower reaction to inflation cancels outs with the higher reaction to the changes in the nominal exchange rate. We also study the general rule (30) in which $\phi_{q,0} > 0$, $\phi_{q,1} < 0$ and $\phi_{q,0} + \phi_{q,1} > 0$. Our numerical results, available upon request, show that the reactions to current and past levels of the RER delivers the same areas of both determinacy and E-stability as in the DITR or CPITR specifications.

3.2 Forecast-Based Specifications

3.2.1 Forecast-Based Domestic Inflation Taylor Rule (FB-DITR)

Under a Forecast-Based Domestic Inflation Taylor Rule (FB-DITR), we can collapse the system of equations (1), (2) and (14) to two equations involving the endogenous variables x_t and $\pi_{H,t}$. These equations can be expressed as (18). Since the feedback policy rule (14) has the same form of the forward expectation rule studied in BM (2002), the same arguments discussed when DITR was analyzed (see section 3.1.1) apply here. Therefore, we use conditions for determinacy and E-stability given by Propositions 4 and 5 of BM (2002), respectively. Proposition 4 states that the necessary and sufficient conditions for a rational expectations equilibrium to be determinate under a forward expectation policy rule are[30]

$$\kappa_\alpha (\phi_\pi - 1) + (1+\beta)\phi_x < 2\sigma_\alpha (1+\beta) \quad (31)$$

$$\kappa_\alpha (\phi_\pi - 1) + (1-\beta)\phi_x > 0. \quad (32)$$

On the other hand, Proposition 5 indicates that a necessary and sufficient condition of the MSV solution to be *E-stable* is that,

$$\kappa_\alpha (\phi_\pi - 1) + (1-\beta)\phi_x > 0. \quad (33)$$

Figure (3) illustrates the intersections of the regions of determinacy and learnability of the MSV solution at the baseline Parameterization under both closed and open economies. Unlike contemporaneous rules, determinate equilibrium is always expectationally stable, but the opposite does not occur due to restriction (31).

In general, for both closed and open economies, a FB-DITR described by $\phi_\pi > 1$ and a relatively small response to output gap guarantees a determinate and learnable equilibrium, while an indeterminate but *E-stable* equilibrium exists for high values of ϕ_x and medium values of ϕ_π.[31] With our baseline Parameterization, an increase in the size of openness lowers the determinate and learnable area because restriction (31) tends to bind ϕ_x. This is due to the fact that an increase in openness reduces σ_α and thus reduces the upper bound of ϕ_x. Therefore, under FB-DITR, openness to trade jeopardizes the Central Bank's ability to stabilize the economy.

figure 3 approx. here

3.2.2 Forecast-Based CPI Inflation Taylor Rule (FB-CPITR)

Under Forecast-Based CPI Inflation Taylor Rule (FB-CPITR), the central bank follows a policy rule of the form of (15). Plugging (3) and (5) into the rule, the domestic interest rate can be rewritten as

$$r_t = \phi'_\pi E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1} \quad (34)$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$. By combining (1) and (2) with (34), we can reduce the system to two equations involving the endogenous variables x_t and $\pi_{H,t}$. The reduced system takes the form of (18), where Ω is defined by

$$\Omega = \begin{bmatrix} \frac{-\phi'_\pi \kappa_\alpha + \sigma_\alpha \beta + \kappa_\alpha}{\sigma_\alpha} & \frac{\kappa_\alpha}{\sigma_\alpha}\left(\sigma_\alpha - \phi'_x\right) \\ -\frac{\phi'_\pi - 1}{\sigma_\alpha} & \frac{\sigma_\alpha - \phi'_x}{\sigma_\alpha} \end{bmatrix}. \tag{35}$$

Since both x_t and $\pi_{H,t}$ are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. E-stability requires that all real parts of the eigenvalues of Ω and $\rho' \otimes \Omega$ are less than 1. The following two propositions summarize the conditions.

Proposition 4. *Under FB-CPITR, the necessary and sufficient conditions for a rational expectations equilibrium to be determinate are that*

$$\begin{aligned} \phi_\pi &< \frac{1}{\alpha} && (36) \\ \kappa_\alpha\left(\phi_\pi - 1\right) + (1+\beta)\phi_x &< 2\sigma_\alpha\left(1+\beta\right)\left(1 - \phi_\pi \alpha\right) && (37) \\ \kappa_\alpha\left(\phi_\pi - 1\right) + \left(1-\beta\right)\phi_x &> 0 && (38) \end{aligned}$$

Proof. See Appendix B. □

Proposition 5. *Suppose the time t information set is $(1, w_t)'$. Under FB-CPITR, the necessary and sufficient conditions for an MSV solution $(0, \widetilde{c})$ to be E-stable are that*

$$\begin{aligned} \phi_\pi &< \frac{1}{\alpha} && (39) \\ \kappa_\alpha\left(\phi_\pi - 1\right) + \left(1-\beta\right)\phi_x &> 0 && (40) \end{aligned}$$

Proof. See Appendix C. □

It is noticeable that FB-CPITR modifies both determinacy and learnability conditions with respect to FB-DITR. The main effect of openness is given by conditions (37) and (39), which clearly constraints the higher permissible values for ϕ_π. On the opposite, the lower bound for ϕ_π is still dictated by the long-run version of the Taylor Principle, conditions (38) and (40). For example, in the case of determinacy, if there is a null response to the expected output gap (i.e., ϕ_x is zero), the limits for ϕ_π are

$$1 < \phi_\pi < \frac{\kappa_\alpha + 2\sigma_\alpha(1+\beta)}{\kappa_\alpha + 2\sigma_\alpha \alpha(1+\beta)} \lesssim \frac{1}{\alpha}.$$

whereas in the case of E-stability the limits are,

$$1 < \phi_\pi < \frac{1}{\alpha}.$$

Thus, there exists a determinate and learnable equilibrium as long as the sensitivity of the interest rate to expected CPI inflation is approximately lower than the inverse of openness. Consequently, as the degree of openness increases, the scope of values for ϕ_π that guarantees determinacy and E-stability shrinks significantly. Remarkably, the Taylor Principle should be viewed as a necessary but not as a sufficient condition for learnability. This result contrasts with those of a closed economy, which suggest that the Taylor Principle guarantees E-stability; see BM (2002). The idea that the Taylor principle or "active" policy is a matter of changing nominal interest rates more than one-for-one with inflation is a celebrated result in the literature that has been almost always thought of as a pure inequality. The fact that open economy considerations create an upper bound on how aggressive policymakers can be with respect to inflation is striking and simple in this framework.

To clarify these results, Figure (4) depicts determinacy and learnability conditions at the baseline parameter values for closed and open economies. Because in a closed economy domestic and CPI inflation are the same concept,[32] the plot corresponding to the closed economy case coincides with the left panel of Figure (3). As discussed above, activism against future CPI inflation deviations from its target is remarkably bound not only for determinacy but also for E-stability. For example, in our benchmark calibration ϕ_π must lie between 1 and (around) 2.5 in order to achieve a determinate and learnable equilibrium in the open-economy case.[33] Unlike previous feedback rules, it is certain that the degree of openness together with the presence of expected CPI inflation in the policy rule *unambiguously* reduces both determinate and E-stable areas.

figure 4 approx. here

Our interpretation is that the reduction of determinate and E-stable areas comes from the interaction between activism in the policy rule and openness. Any increase (decrease) in the interest rate due to inflationary (deflationary) expectations triggered by an expected depreciation (appreciation) of nominal exchange rate reinforces the expectation of higher (lower) CPI inflation. In this context, the likelihood of a consequent movement in the interest rate relies on the preferences of the central bank, given by ϕ_π, and the degree of openness. Therefore, if either the degree of openness or the aggressiveness of the monetary policy with respect to expected CPI inflation is high, the economy is likely to be stuck in an indeterminate equilibria that private agents would not be able to learn.

3.2.3 Forecast-Based Domestic Inflation Managed Exchange Rate Taylor Rule (FB-DI-METR)

The central bank follows a policy rule of the form of (16). Plugging (3) into the rule, the domestic interest rate can be rewritten as

$$r_t = \phi'_\pi E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1}$$

where $\phi'_\pi = \left(\frac{\phi_\pi}{1-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_e}\right)$. Notice that ϕ_e modifies ϕ'_π and ϕ'_x. The reduced system takes the form of (18). Matrix Ω is given by (35) but with the previously defined ϕ'_π and ϕ'_x. Since both x_t and $\pi_{H,t}$ are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. E-stability requires that all real parts of the eigenvalues of Ω and $\rho' \otimes \Omega$ are less than 1. The following two propositions summarize the conditions.

Proposition 6. *Under FB-DI-METR, the necessary and sufficient conditions for a rational expectations equilibrium to be determinate are that*

$$\begin{aligned} \phi_e &< 1 && (41) \\ \kappa_\alpha (\phi_\pi - (1-\phi_e)) + (1+\beta)\phi_x &< 2\sigma_\alpha (1+\beta)(1-\phi_e) && (42) \\ \kappa_\alpha (\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x &> 0 && (43) \end{aligned}$$

Proof. The proof follows the same steps of Proposition 4. □

Proposition 7. *Suppose the time t information set is $(1, w_t)'$. Under FB-DI-METR, the necessary and sufficient conditions for an MSV solution $(0, \widetilde{c})$ to be E-stable are that*

$$\begin{aligned} \phi_e &< 1 && (44) \\ \kappa_\alpha (\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x &> 0 && (45) \end{aligned}$$

Proof. The proof follows the same steps of Proposition 5. □

First, note that the degree of managed exchange rate ϕ_e affects both determinacy and learnability conditions. On one side, ϕ_e restricts the determinacy region through condition (42). On the other side, a positive ϕ_e relaxes both determinacy and E-stability conditions through the *generalized* Taylor Principle, conditions (43) and (45) respectively. However, although ϕ_e helps, reacting excessively to expected exchange rate movements causes indeterminacy and expectational instability.

figure 5 approx. here

Figure (5) illustrates the intersections of the regions of determinacy and learnability of the MSV solution at the baseline Parameterization assuming the open economy case. The graph on the left shows the case of FB-DITR or no managed exchange rate whereas the graph on the right shows the case of FB-DI-METR. We can note that a managed exchange rate is detrimental in terms of determinacy because shrinks the upper limit to ϕ_x. However, as in contemporaneous rules with managed exchange rate, FB-DI-METR guarantees stability even if a central bank reacts passively to domestic inflation $(\phi_\pi < 1)$.

3.2.4 Forecast-based CPI Inflation Managed Exchange Rate Taylor Rule (FB-CPI-METR)

In this section we suppose that the monetary authority follows a Forecast-Based CPI Inflation Managed Exchange Rate (FB-CPI-METR). First, with the same procedure used for FB-CPITR, the interest rate feedback rule (17) can be rewritten as

$$r_t = \phi'_\pi E_t \pi_{H,t+1} + \phi'_x E_t x_{t+1}$$

where $\phi'_\pi = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha-\phi_e}\right)$ and $\phi'_x = \left(\frac{\phi_x}{1-\phi_\pi\alpha-\phi_e}\right)$. Notice that ϕ_e modifies ϕ'_π and ϕ'_x with respect to FB-CPITR case. The reduced system takes the form of (18). Matrix Ω is given by (35) but with the previously defined ϕ'_π and ϕ'_x. Since both x_t and $\pi_{H,t}$ are free variables, determinacy requires both the eigenvalues of Ω to be inside the unit circle. E-stability requires that all real parts of the eigenvalues of Ω and $\rho' \otimes \Omega$ are less than 1. The following two propositions summarize the conditions.

Proposition 8. *Under FB-CPI-METR, the necessary and sufficient conditions for a rational expectations equilibrium to be determinate are that*

$$\phi_\pi < \frac{(1-\phi_e)}{\alpha} \tag{46}$$

$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1+\beta)\phi_x < 2\sigma_\alpha(1+\beta)(1-\phi_e-\phi_\pi\alpha) \tag{47}$$

$$\kappa_\alpha(\phi_\pi - (1-\phi_e)) + (1-\beta)\phi_x > 0 \tag{48}$$

Proof. The proof follows the same steps of Proposition 4. □

Proposition 9. *Suppose the time t information set is $(1, w_t)'$. Under FB-CPI-METR interest rate rules, the necessary and sufficient conditions for an MSV solution $(0, \widetilde{c})$ to be E-stable are* *that*

$$\phi_\pi < \frac{(1-\phi_e)}{\alpha} \quad (49)$$

$$\kappa_\alpha \left(\phi_\pi - (1-\phi_e)\right) + (1-\beta)\,\phi_x > 0 \quad (50)$$

Proof. The proof follows the same steps of Proposition 5. □

figure 6 approx. here

Figure (6) plots the numerical results under two possible values for ϕ_e. The graph on the left depicts the case of FB-CPITR, i.e., when there is a null response to the expected nominal exchange rate (ϕ_e is zero), whereas the graph on the right depicts the case of FB-CPI-METR (ϕ_e is 0.6). Comparing the conditions under FB-CPI-METR with the conditions obtained under FB-CPITR, we can note that the degree of managed exchange rate, measured by ϕ_e, has affected both determinacy and learnability conditions. There are two major effects through which ϕ_e impact on the stability of the system. The first effect comes form the *generalized* Taylor Principle as discussed before. The second effect couples with the degree of openness: any positive reaction to expected nominal exchange rate movements reduces the area of determinacy and learnability through (47) and (49), respectively. For example, when ϕ_e is 0.6, those conditions imply that the upper limit for ϕ_π is around 1.

Consequently, highly open economies joint with a central bank reacting too strongly to either future CPI inflation or expected nominal exchange rate movements are more prone to indeterminacy and instability under learning. Yet, if the degree of openness and activism towards CPI and exchange rate are moderate, the monetary authority is able to push the economy towards the determinate and E-stable region, even with no response to the output gap. More important, a passive reaction to expected CPI inflation could success in generating a determinate and E-stable path.

Analyzing this type of rule helps us to disentangle some key features observed under forecast-based rules. As we stressed earlier (see section 3.2.2), reacting to expected CPI inflation imposes an upper bound to ϕ_π approximately equal to the inverse of openness. With the analysis of FB-DI-METR and FB-CPI-METR, we confirm that reacting to the expected changes in nominal exchange rate does not threaten determinacy and E-stability as long as the central bank is not targeting future CPI inflation. Therefore, different from the case of contemporaneous rules, the definition of inflation that is targeted in the policy rule is key for open economies when the central bank is forward-looking.

3.2.5 Learnability and Volatility

As shown in previous sections, some forms of managed exchange rate rules make the conditions of determinacy and learnability less stringent in small open economies. For example, in the particular case of the FB-DI-METR, (16), to the extent that ϕ_e lies between zero and one, the region of both *E-stability* and determinacy gets larger. In fact, the larger ϕ_e the less likely the economy will fall in an indeterminate or an expectationally unstable region. The above result suggests that a FB-DI-METR might be desirable based on the criterion of both determinacy and learnability compared to a FB-DITR, (14).

Yet, there is another dimension to consider in order to conclude whether managed exchange rate rules are more desirable. In particular, it is important to quantify the volatility that these types of rules induce to the endogenous macro variables, such as the output gap and inflation. We illustrate this issue by obtaining numerically the unconditional volatility under adaptive learning of both domestic inflation and output gap under both Taylor rules, i.e. FB-DITR and FB-DI-METR.

We calculate second moments under least squares learning following the recent contribution of Carceles-Poveda and Giannitsarou (2007). We implement their toolbox to the present model by using the reduced form implied by equations (1) and (2) along each rule.[34] Table 2 below shows the unconditional volatilities for different shocks under both least squares learning and rational expectations.[35] We perform the analysis by assuming the same AR(1) process for both shocks ($\overline{rr}_t$ and r_t^*).

table 2 approx. here

Depending on the source of shocks a managed exchange rate rule (FB-DI-METR) might be more or less desirable in terms of volatility under both learning and REE. Under a natural interest rate shock, the managed exchange rate rule ($\phi_e > 0$) induces smaller volatility under both learning and rational expectations relative to DITR ($\phi_e = 0$), then the FB-DI-METR will be more desirable since it also induces E-stability. On the other hand, when the economy is hit by a foreign nominal interest rate shock, the FB-DITR induces smaller volatility compared to that generated by a FB-DI-METR rule, hence the latter rule is less desirable. When we activate both shocks the desirability of any rule becomes ambiguous. Therefore, we argue that, in addition to the *E-stability* criterion that a Taylor rule has to meet, it is important to evaluate which are the implications in terms of volatility of a Taylor-type rule in order to conclude whether this rule is desirable or not.[36]

4 Conclusions

Using GM (2005)'s small open economy model, we have studied the determinacy and learnability conditions of rational expectations equilibrium under a handful of possible Taylor-type instrument rules. Our analytical results highlight an important link between the Taylor Principle and both determinacy and learnability of REE in small open economies. The degree of openness coupled with the nature of the policy rule adopted by the monetary authorities might change this link in important ways.

With *contemporaneous rules*, we show that openness affects determinacy and E-stability conditions quantitatively. The final impact of openness, in terms of enlargement of the determinate and E-stable region, is ambiguous and depends on the degree of the elasticity of substitution between tradable goods. More importantly, conditions for unique and learnable REE do not depend on whether the central bank responds to domestic or CPI inflation, i.e., the long-run Taylor Principle is a necessary and sufficient condition under both policies. Yet, we have shown that a managed exchange rate regime relaxes the constraint on the degree of response to inflation and alleviates problems of indeterminacy and expectational instability.

We have stressed that in the case of *forecast-based* monetary rules, openness imposes an additional constraint, making it more difficult to induce a determinate and learnable solution. Indeed, the *Taylor Principle does not guarantees E-stability,* as it is the case in a closed economy (BM 2002). When the central bank follows a CPI inflation targeting (with or without a managed exchange rate), the determinacy and learnability regions shrink significantly. Domestic inflation targeting does not suffer from this problem, instead suggesting that more aggressive reaction towards inflation is all to the good as in the closed economy case. Therefore, in order to avoid indeterminacy and expectational instability problems forward-looking central banks in open economies should adopt some kind of “inward-looking” policy by focusing on domestic inflation.

Finally, one important question is: If a rule is desirable in terms of both macroeconomic volatility and both determinacy and E-stability, how fast do private agents learn this rule? Analyzing the speed of learning under the broad set of rules studied in this paper will add another dimension through which the desirability of a rule should be evaluated, and we think this would be highly useful undertaking.

5 Appendices: Proofs

5.1 Appendix A: Proof of Proposition 1

Here we closely follow Woodford's proof of determinacy of a Taylor rule with some form of partial adjustment of the short term interest rate (Woodford 2003*b*, Chapter 4). Let the characteristic equation of the matrix Υ (defined in (25)) be written in the form

$$P(\mu) = \mu^3 + A_2\mu^2 + A_1\mu + A_0 = 0$$

where

$$A_2 = -\beta^{-1}\left(1 + \kappa_\alpha\sigma_\alpha^{-1}\right) - 1 - \sigma_\alpha^{-1}\phi_x - \phi_e' < 0 \tag{A1}$$

$$A_1 = \beta^{-1} + \beta^{-1}\sigma_\alpha^{-1}\left(\kappa_\alpha\phi_\pi' + \phi_x\right) + \left(1 + \beta^{-1} + \beta^{-1}\sigma_\alpha^{-1}\kappa_\alpha\right)\phi_e' > 0 \tag{A2}$$

$$A_0 = -\phi_e'\beta^{-1} < 0 \tag{A3}$$

and where $\phi_\pi' = (1-\alpha)\phi_\pi$ and $\phi_e' = \alpha\phi_\pi$. Woodford (2003*b*) shows that the above equation has one root inside the unit circle and two roots outside if and only if one of three cases holds. It is straightforward to rule out the first case based on coefficients A_i , then we focus on the other two cases.

(Case II):

$$1 + A_2 + A_1 + A_0 > 0; \quad -1 + A_2 - A_1 + A_0 < 0;$$
$$A_0^2 - A_0A_2 + A_1 - 1 > 0$$

(Case III):

$$1 + A_2 + A_1 + A_0 > 0; \quad -1 + A_2 - A_1 + A_0 < 0;$$
$$A_0^2 - A_0A_2 + A_1 - 1 < 0; \quad |A_2| > 3$$

Notice that both cases share the first condition $(1 + A_2 + A_1 + A_0 > 0)$, which can be reduced to

$$\kappa_\alpha\left(\phi_\pi' - \left(1 - \phi_e'\right)\right) + (1-\beta)\phi_x > 0$$

By replacing ϕ_π' and ϕ_e' we obtain

$$\kappa_\alpha(\phi_\pi - 1) + (1-\beta)\phi_x > 0 \tag{A4}$$

which is a necessary condition for determinacy. By considering the signs of coefficients A_i , $-1+A_2-A_1+A_0 < 0$ holds. The additional condition required for Case II $\left(A_0^2 - A_0A_2 + A_1 - 1 > 0\right)$

can be written after some manipulation as

$$\left(\phi_\pi - \beta^{-1}\phi_e'\right) + \frac{(1-\phi_e')}{\kappa_\alpha}\phi_x + \left(\beta^{-1}-1\right)\left[\kappa_\alpha^{-1}\sigma_\alpha\left(1-\phi_e'\right)\left(\beta-\phi_e'\right)\right] > 0 \tag{A5}$$

and the remaining condition needed for Case III ($|A_2| > 3$) can be written as

$$\beta^{-1}\left(1+\kappa_\alpha\sigma_\alpha^{-1}\right) + \phi_e' + \sigma_\alpha^{-1}\phi_x > 2. \tag{A6}$$

Equilibrium is determinate if and only if the coefficients of the policy rule (11) satisfy (A4) and either (A5) or (A6) . We will show that under the sign assumption, (A4) is both necessary and sufficient for determinacy. We prove this by showing that any value that satisfies (A4) but not (A6) *must* necessarily satisfy (A5). First let's write (A6) as,

$$\beta^{-1}\kappa_\alpha\sigma_\alpha^{-1} + \left(\phi_e' - \beta\right) + \left(\beta^{-1}+\beta\right) + \sigma_\alpha^{-1}\phi_x > 2. \tag{A7}$$

Note that under the sign assumption, the above equation can fail to hold only if $\phi_e' = \alpha\phi_\pi < \beta$. (here we use the fact that $\beta^{-1}+\beta > 2$). Note that $\phi_e' = \alpha\phi_\pi < \beta$ necessarily implies that $\phi_e' = \alpha\phi_\pi < 1$ since $0 < \beta < 1$. Now we need to show that under these circumstances (A5) holds given $\phi_e' = \alpha\phi_\pi < \beta$. Notice that (A5) can be expressed as,

$$\kappa_\alpha\left(\phi_\pi - 1\right) + (1-\beta)\,\phi_x + \kappa_\alpha\left(1-\beta^{-1}\phi_e'\right) + \left(\beta-\phi_e'\right)\phi_x \tag{A8}$$

$$+\left(\beta^{-1}-1\right)\sigma_\alpha\left(1-\phi_e'\right)\left(\beta-\phi_e'\right) > 0.$$

The first two terms (A8) corresponds to condition (A4) which along with $\phi_e' = \alpha\phi_\pi < \beta$, guarantees that (A8) will hold. Therefore, (A4) or (26) in the main text, is a necessary and sufficient condition for determinacy.

5.2 Appendix B: Proof of Proposition 4

The characteristic polynomial of Ω (given by (35)) is $\rho(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_0 = \frac{\beta\left(\sigma_\alpha - \phi_x'\right)}{\sigma_\alpha} \tag{B1}$$

$$A_1 = \frac{\phi_x' + \kappa_\alpha\left(\phi_\pi' - 1\right) - \sigma_\alpha(\beta+1)}{\sigma_\alpha} \tag{B2}$$

with $\phi_\pi' = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi_x' = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$. Both eigenvalues of Ω are inside the unit circle if and only if both of the following conditions hold

$$|A_0| < 1 \tag{B3}$$

$$|A_1| < 1 + A_0. \tag{B4}$$

After replacing the definitions of ϕ_π' and ϕ_x', we can note that condition (B4) implies (37) and (38). The only relevant case is $\phi_\pi < 1/\alpha$, given by (36). The other case, $\phi_\pi > 1/\alpha$, can be ruled out by showing that it contradicts condition (B4).

5.3 Appendix C: Proof of Proposition 5

Using results of Evans and Honkapohja (2001), E-stability needs that the eigenvalues of $\rho\Omega$ (Ω is given by equation. (35)) to have real parts less than one. The eigenvalues of $\rho\Omega$ are given by the product of the eigenvalues of Ω and ρ, and since $0 < \rho < 1$, it suffices that eigenvalues of Ω to have real parts less than 1. On the other hand, the MSV solution will not be E-stable if any eigenvalue of Ω has a real part greater than 1. The characteristic polynomial of $\Omega - I$ given by $\rho(\mu) = \mu^2 + A_1\mu + A_0$ where

$$A_1 = \frac{\kappa_\alpha(\phi_\pi' - 1) + \phi_x' + \sigma_\alpha(1-\beta)}{\sigma_\alpha} \tag{C1}$$

$$A_0 = \frac{\phi_x'(1-\beta) + \kappa_\alpha(\phi_\pi' - 1)}{\sigma_\alpha} \tag{C2}$$

where $\phi_\pi' = \left(\frac{\phi_\pi(1-\alpha)}{1-\phi_\pi\alpha}\right)$ and $\phi_x' = \left(\frac{\phi_x}{1-\phi_\pi\alpha}\right)$.

It is necessary for both eigenvalues of $\Omega - I$ to have negative real parts. According to the Routh Theorem, that condition holds if and only if $A_1 > 0$ and $A_0 > 0$. We can note that

$$A_1 = A_0 + \frac{\sigma_\alpha(1-\beta) + \beta\phi_x'}{\sigma_\alpha} \tag{C3}$$

After replacing the definitions of ϕ_π' and ϕ_x', under the case of $\phi_\pi < 1/\alpha$, $A_0 > 0$ implies $A_1 > 0$. In this case, the second E-stability condition, given by (40), is derived from $A_0 > 0$. As in determinacy analysis, there is a second case, $\phi_\pi > 1/\alpha$. However, this case is not relevant, since it contradicts $A_0 > 0$.

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Notes

[1]Calvo and Reinhart (2002) coin the phrase "Fear of Floating" referring to those central banks that systematically tend to defend their exchange rates by increasing interest rates

[2]This condition suggests that if the nominal interest rate is adjusted positively, and more than one-for-one, in response to inflation movements above its target, and positively to output above target, a determinate REE is attainable; see Woodford (2003*b*). Nevertheless, some kind of Taylor-type rules can induce indeterminacy with undesirable properties even if the Taylor Principle holds; see for example Bernanke and Woodford (1997).

[3]Even when a determinate equilibrium exists, coordination at that equilibrium cannot be assured if the assumption of rational expectations is relaxed. E-stability therefore provides a robustness criterion: if agents make small mistakes in expectations relative to those consistent with the associated REE, then a policy rule that is E-stable ensures such mistakes are corrected over time.

[4]Even so learnability is a more general concept than E-stability, throughout the paper we will use both terms interchangeably.

[5]Therefore, under these rules an aggressive reaction to domestic inflation is all to the good as in the closed economy case analyzed by BM.

[6]See Lubik and Schorfheide (2007).

[7]De Fiori and Liu (2005) analyze a small open economy version of Cooley and Hansen (1989)'s model whereas Zanna (2003) analyzes a model with tradable and non tradable goods. Similarly, focusing on determinacy, Batini et. al. (2004) study forward-looking policy rules in a two-country model. Their results point out that potential local indeterminacy is exacerbated in the open economy regardless of whether CPI or domestic inflation enters in the policy rule.

[8]Besides Taylor rules, Bullard and Schaling (2006) study other forms of instrument rules such as PPP rules as well as target rules.

[9]One interesting property of GM's model is that it is isomorphic to the workhorse sticky price model of a closed economy of Woodford (2003*b*). More specifically, GM's model is identical to the closed economy model if the degree of openness collapses to zero. This feature allows us to isolate the effects of openness and study its interaction with monetary policy.

[10]Recently, Preston (2005) has proposed an interesting reformulation of intertemporal behavior under learning in which agents are assumed to incorporate a "subjective version" of their intertemporal budget constraint into their behavior under learning. In this paper, we abstract from this approach.

[11]Another case discussed by GM (2005) is when $\sigma = \eta = \gamma = 1$, which implies $\omega = 1$. Under this case, there is a balance of trade at all times.

[12]For a general discussion about this class of policy setting; see Battini and Haldane (1999). Empirical evidence also suggests that central banks indeed set their interest rate in a forward-looking fashion.

[13]Lubik and Schorfheide (2007) find robust evidence that the Bank of Canada and the Bank of England follow similar policy rules. Yet, they treat the terms of trade as an exogenous variable. Thus, they use a different model from that of the present paper.

[14]The value of 0.4 corresponds roughly to the import/GDP ratio for the Canadian economy.

[15]For details see Blanchard and Kahn (1980).

[16]GM have also found the same condition, although in our paper we explore the E-stability conditions for a broader set of policy rules.

[17]Since GM's model is isomorphic to the baseline New Keynesian closed-economy model, it is always the case that, for any policy rule, the usual Taylor principle ($\phi_\pi > 1$) implies its long-run version. The above claim is valid for all rules because the law of one price holds and the small open economy assumption. Moreover, for

models that exhibit non-separability in the utility function between consumption and money or the cost channel of monetary policy, the previous implication does not apply. For the analysis of determinacy and E-stability in those frameworks, see Kurozumi (2006) and Llosa and Tuesta (2007), respectively.

[18]Although one case corresponds to the closed-economy case, the graphic does not coincide with BM (2002) Figure 1 due to differences in calibration.

[19]In fact, the system analyzed here is similar to the one for a closed economy under policy inertia studied by Woodford (2003*a*).

[20]This finding is an analytically novel result and can also be useful in analyzing determinacy and learnability in a two-sector closed economy model. As emphasized by Aoki (2001), there is a parallel between a small open economy model like the one we use and a two-sector closed economy model. In a small open economy, the domestic sector is analogous to a sector showing price stickiness, whereas the foreign sector is analogous to the one showing flexible prices. Carlstrom et. al. (2006) show that the Taylor principle is a necessary and sufficient condition for determinacy in a two-sector closed economy model regardless which price index the central bank is targeting.

[21]In Llosa and Tuesta (2006) we also include a sketch of the link between determinacy and E-stability under CPITR, which is analogous to McCallum's result.

[22]Bullard and Schaling (2005) study a similar environment. Their results coincides with ours in the case of small open economy. The authors also found that when small open economy assumption is dropped, interaction with the rest of the world is important in the sense that it modifies the conditions for a determinate and learnable equilibrium in the domestic economy.

[23]In the numerical evaluation of E-stability conditions, matrices Ω, Φ and Θ change accordingly. For more details, see Llosa and Tuesta (2006).

[24]In contrast to the previous analyses, in this section we have focused on managed exchange rate rather than openness. Nevertheless, it is worth to emphasize that our numerical results, not shown, confirm that the impact of the size of openness ambiguously alters both determinacy and E-stability. Again, the impact on determinacy and expectational stability mostly depends on the degree of substitutability between foreign and domestic produced goods and the coefficient of risk aversion.

[25]Matrices Ω, Φ and Θ change accordingly. See Llosa and Tuesta (2006) for details.

[26]Contrary to this, Bullard and Schaling (2006) found that the interaction with the rest of the world is important in the sense that it modifies the conditions for a determinate and learnable equilibrium in the domestic economy.

[27]Numerical results are available upon request from the authors.

[28]Notice that in the present model, an increase in RER is a real depreciation whereas in Taylor (2001) is a real appreciation. Thus, we change the signs of $\phi_{q,0}$ and $\phi_{q,1}$ accordingly.

[29]We obtain the same results for a rule that targets CPI inflation. Details are available upon request from the authors.

[30]Notice that BM (2002) declare three conditions for determinacy. However, the third condition (not shown in the present paper) is redundant because it is implied by the other two conditions, (31) and (32).

[31]Indeterminacy and instability coexist when ϕ_π is too large. This area is not shown in the graph because the value of ϕ_π in this case so far exceeds the limit for this parameter in the calibration.

[32]Notice that as $\alpha \to 0$, when CPI inflation coincides with domestic inflation, determinacy and E-stability conditions for FB-CPITR converge to the conditions found by BM (2002) for the closed economy counterpart.

[33]Moreover, the parameterization of Taylor (1993), $\phi_\pi = 1.5$ and $\phi_x = 0.5$, implies that a degree of openness of roughly more than 0.66 could easily induce both indeterminacy and E-instability.

[34]Details of the toolbox are available at http://www.econ.cam.ac.uk/research/learning/

[35]In Llosa and Tuesta (2006) we also find analytical conditions for the REE conditional to each shock.

[36]We follow GM (2005) in this discussion and we find reasonable to stay closer to that analysis given that we are focusing in understanding variants on standard policy prescriptions that would apply in small open economy settings. An alternative would be to follow Evans and Honkapohja (2003, RES) and find optimal policy rules in the linear class that will also be consistent with determinacy and learnability.

Table 1: Parameterization

γ	Elasticity of substitution between imported goods	1
θ	Probability of not adjusting prices	0.75
β	Discount factor	0.99
α	Degree of openness	0 or 0.4
η	Elasticity of substitution between foreign and domestic goods	1.5
σ	Coefficient of risk aversion	5
φ	Inverse of the elasticity of labor supply	0.47
ϕ_π	Reaction to inflation	$0 \leq \phi_\pi \leq 4$
ϕ_x	Reaction to output gap	$0 \leq \phi_x \leq 4$
ϕ_e	Reaction to changes in the nominal exchange rate	$\phi_e \geq 0$

Table 2: Standard Deviations and Adaptive Learning and REE

	FB-DITR ($\phi_e = 0$)			FB-DI-METR ($\phi_e = 0.6$)		
	Domestic	Foreign	Both	Domestic	Foreign	Both
	$(\overline{rr}_t)$	(r_t^*)	$(\overline{rr}_t, r_t^*)$	$(\overline{rr}_t)$	(r_t^*)	$(\overline{rr}_t, r_t^*)$
Adaptive Learning						
Output Gap (x_t)	8.25	0.00	8.25	5.14	7.65	9.12
Domestic Inflation $(\pi_{H,t})$	2.27	0.00	2.27	1.19	1.78	2.16
Rational Expectations						
Output Gap (x_t)	8.65	0.00	8.65	4.78	7.12	8.59
Domestic Inflation $(\pi_{H,t})$	2.16	0.00	2.16	1.20	1.78	2.15

Notes: Parameters ϕ_π and ϕ_x take the values of 1.5 and 0.5, respectively.

Autoregressive coefficients $\rho_{\overline{rr}} = \rho_{r^*} = 0.5; \sigma\left(\epsilon_t^{r^*}\right) = \sigma\left(\epsilon_t^{\overline{rr}}\right) = 0.07$.

The initial conditions for the algorithm are centered around the MSV solution of the REE.

All the experiments are done for a time horizon of 50 periods. We perform 500 experiments.

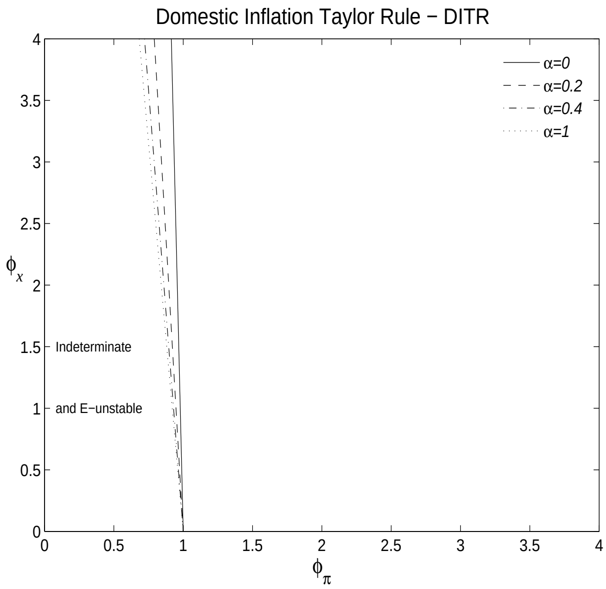

Figure 1: **Regions of Determinacy and E-stability for Contemporaneous DITR under Different Degrees of Openness.**

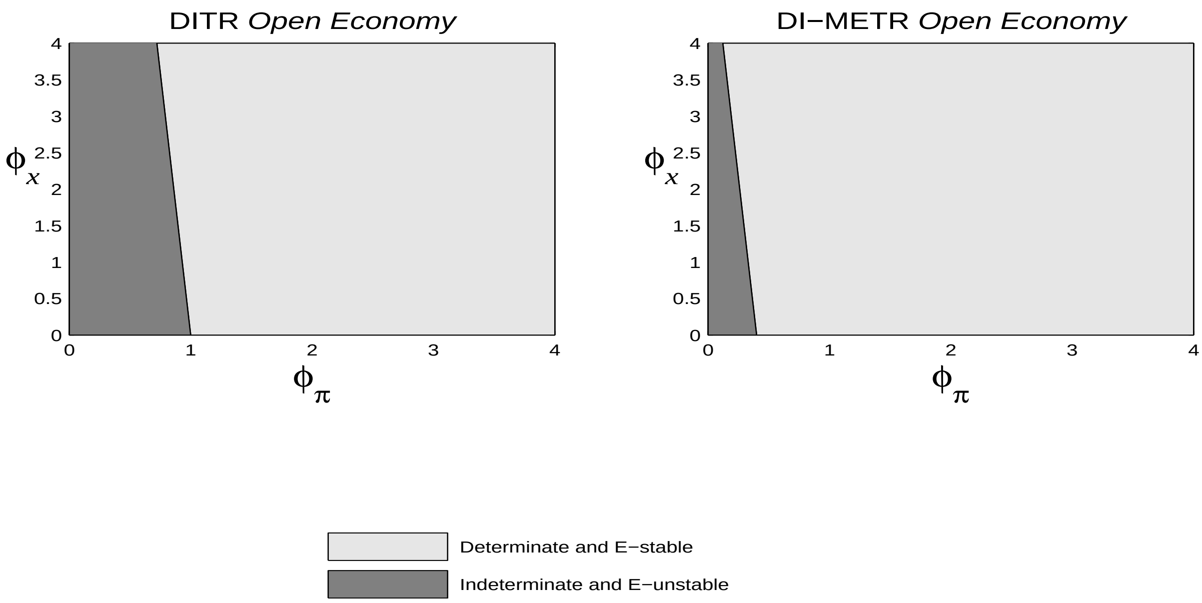

Figure 2: **Regions of Determinacy and E-stability for DITR and DI-METR.** Note: Both graphics correspond to open economies ($\alpha = 0.4$). The graphic on the left shows the case of DITR or no managed exchange rate ($\phi_e = 0$) and the graphic on the right shows the case of DI-METR ($\phi_e = 0.6$).

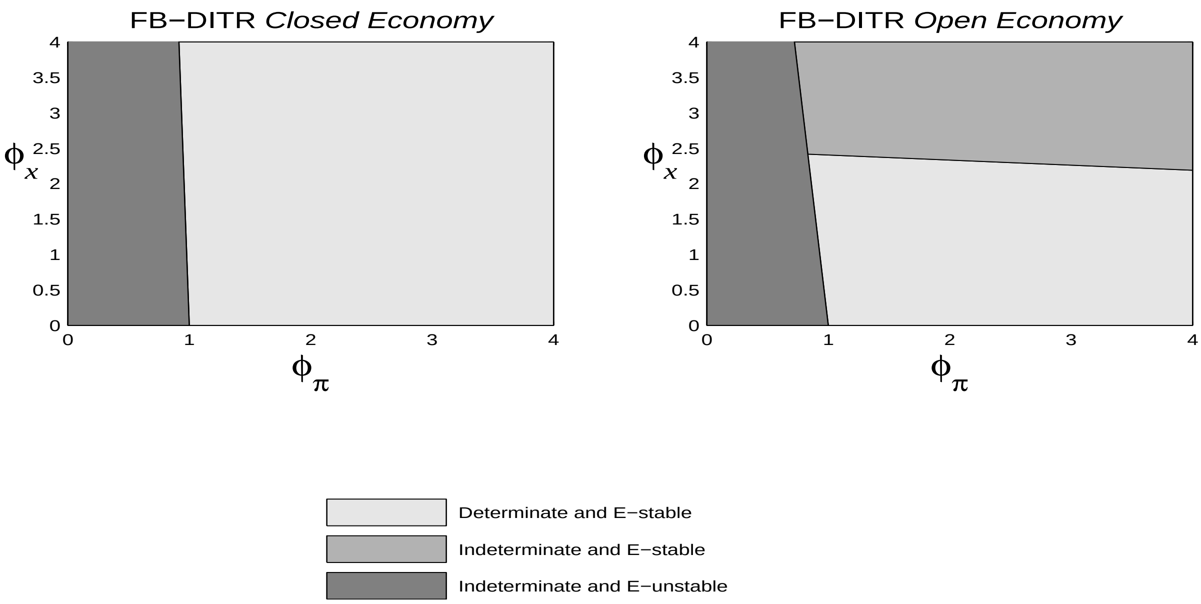

Figure 3: **Regions of Determinacy and E-stability for FB-DITR.** Note: Closed ($\alpha = 0$) and open ($\alpha = 0.4$) economies.

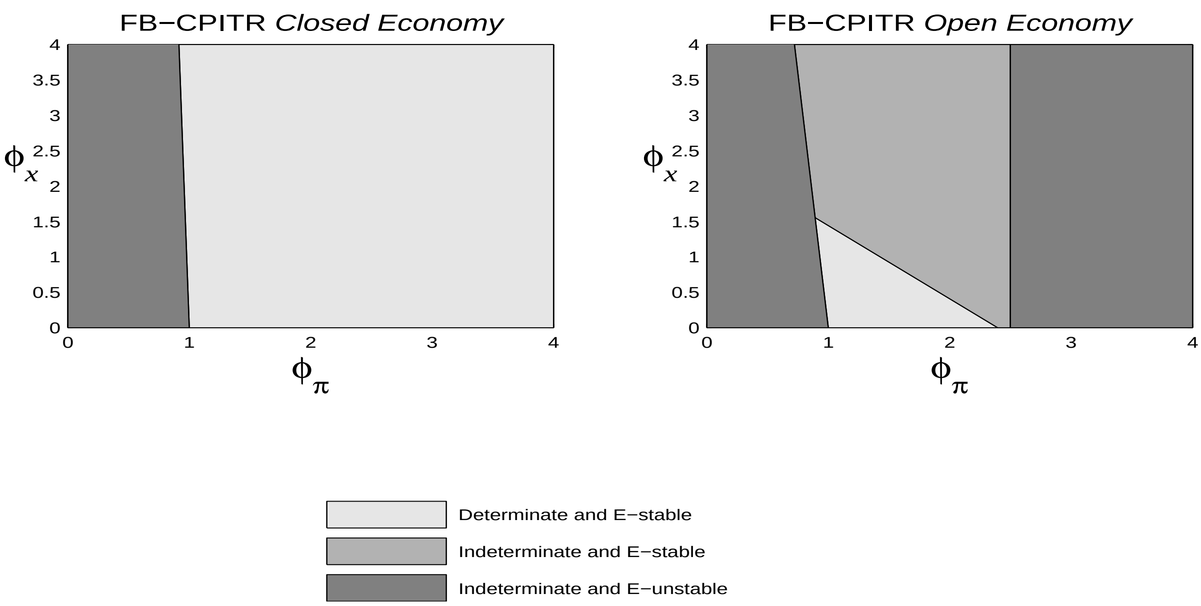

Figure 4: **Regions of Determinacy and E-stability for FB-CPITR.** Note: Closed ($\alpha = 0$) and open ($\alpha = 0.4$) economies.

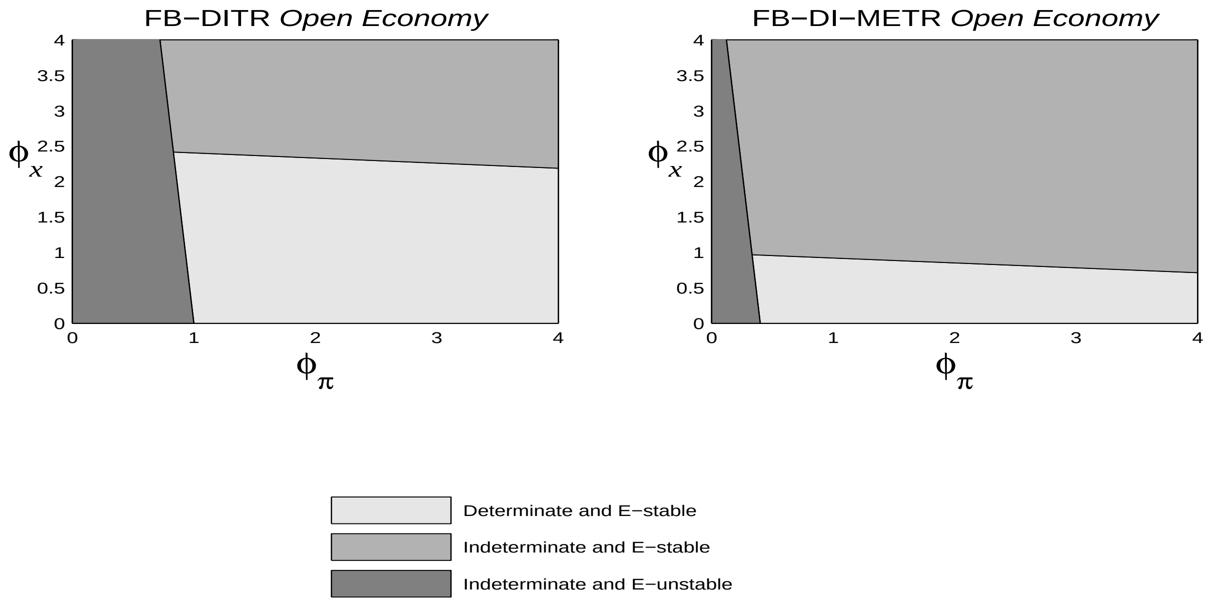

Figure 5: **Regions of Determinacy and E-stability for FB-DITR and FB-DI-METR.** Note: Both graphics correspond to open economies ($\alpha = 0.4$). The graphic on the left shows the case of FB-DITR or no managed exchange rate ($\phi_e = 0$) and the graphic on the right shows the case of FB-DI-METR ($\phi_e = 0.6$).

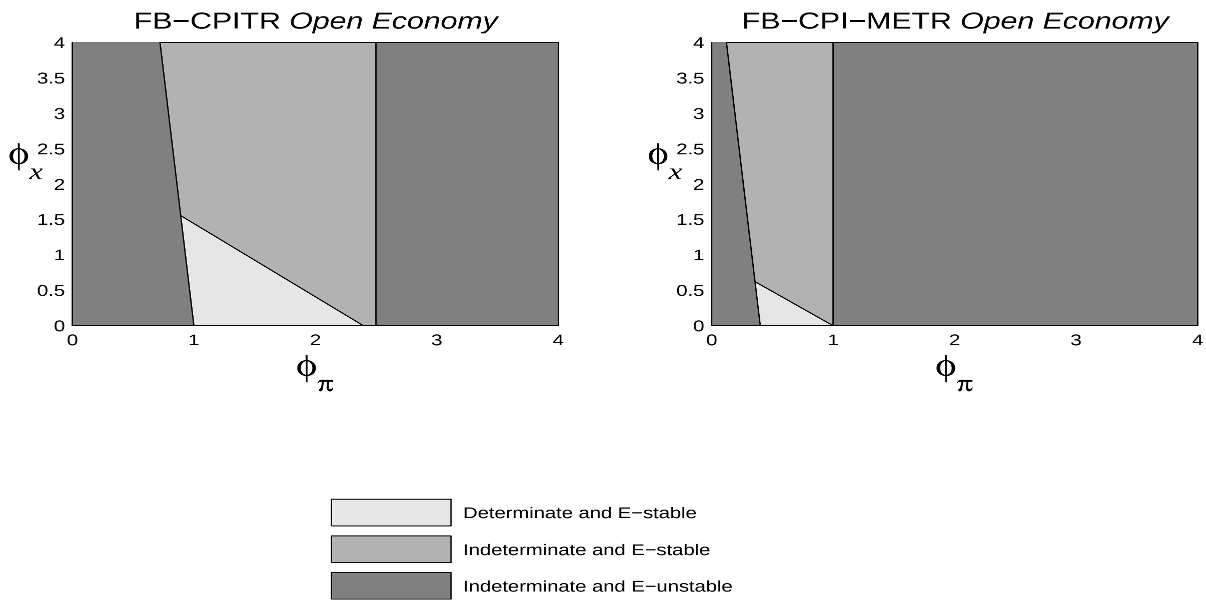

Figure 6: **Regions of Determinacy and E-stability for FB-CPITR and FB–CPI-METR.** Note: Both graphics correspond to open economies ($\alpha = 0.4$). The graphic of the left shows the case of FB-CPITR or no managed exchange rate ($\phi_e = 0$) and the graphic of the right shows the case of FB-CPI-METR ($\phi_e = 0.6$).