

Identifying Monetary Policy Shocks via Changes in Volatility¹

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Abstract. A central issue of monetary policy analysis is the specification of monetary policy shocks. In a structural vector autoregressive setting there has been some controversy about which restrictions to use for identifying the shocks because standard theories do not provide enough information to fully identify monetary policy shocks. In fact, to compare different theories it would even be desirable to have over-identifying restrictions which would make statistical tests of different theories possible. It is pointed out that some progress towards over-identifying monetary policy shocks can be made by using specific data properties. In particular, it is shown that changes in the volatility of the shocks can be used for identification. Based on monthly US data from 1965-1996 different theories are tested and it is found that associating monetary policy shocks with shocks to nonborrowed reserves leads to a particularly strong rejection of the model whereas assuming that the Fed accommodates demand shocks to total reserves cannot be rejected.

1 Introduction

Over the last two decades, a large literature has developed which evaluates monetary policy within a structural vector autoregressive (SVAR) framework (see, e.g., Christiano, Eichenbaum and Evans 1999, henceforth CEE). A central question in evaluating monetary policy is how to identify the monetary policy shocks. Various competing economic theories have been used to formulate restrictions which help in identifying the shocks. However, often there is not enough information from economic theory to identify all shocks. In this situation it is sometimes possible to identify only the shocks of primary interest, that is, in the present case the monetary policy shocks. Unfortunately, the restrictions implied by some of the standard theoretical models do not even suffice to identify these shocks. Hence, additional restrictions have to be formulated which often do not have a convincing theoretical foundation. Even if theoretical considerations suffice to identify the monetary policy shocks, there may be no over-identifying information which could be used to test different theories against the data.

In this paper we will argue that sometimes the statistical properties of the data can be used to identify the shocks. In particular, Klein and Vella (2006), Lewbel (2006) and Rigobon (2003) show that a change in volatility in the shocks can provide an important and useful source of identifying information. Here we adapt the formulation in Rigobon to our needs. Our general model setup is that of CEE, that is, we use an SVAR model. These authors also argue that there may have been changes in the volatility of the US monetary policy shocks over their sample period from 1965-1996 but that the remaining structure of the model is found to be time invariant. This view is also expressed by other authors (e.g., Sims and Zha 2006). Thus we will also assume that the DGP is a VAR

with constant parameters apart from changes in the volatility of shocks. This assumption will be used to identify the shocks and thereby we can test theoretical assumptions that cannot be checked by formal statistical tests in the CEE framework.

In a related paper, Beyer and Farmer (2005) also use structural change to help identify parameters in a study of the monetary transmission mechanism. Their approach differs from ours in important respects, however. First, they assume that behavioral parameters other than the volatility of the disturbances have changed and, second, they use the structural change to obtain instruments for estimation. In other words, their model setup and estimation procedure is different from ours. In particular, their assumption that not only the parameters in the residual covariance matrices vary across regimes is in contrast to our assumption and, in fact, to what is assumed in a number of other studies.

We will consider a monthly VAR model for the US with six variables, real GDP, the GDP deflator, a spot commodity prices index, the federal funds rate, nonborrowed reserves and total reserves. Such a model was also considered by Bernanke and Mihov (1998a) (henceforth BM). The first three variables are viewed as nonpolicy variables whereas the monetary policy shocks are determined from the last three variables. BM consider a model for the federal funds market to find identifying restrictions for the monetary policy shocks. Unfortunately, this model does not fully identify the shocks and CEE question the additional restrictions imposed. CEE also find evidence for a change in the volatility of the monetary policy shocks. We will confirm this finding with further statistical tests and then use these data properties to over-identify the shocks. Thereby the assumptions of different models which can be embedded in this framework become testable. Our setup will enable us to perform such tests and we find that the data are at odds with some identifying schemes which have been used in previous publications whereas other

identification schemes cannot be rejected. Thus, we are able to use statistical tools for discriminating between competing models.

The structure of the paper is as follows. In the next section the model setup is presented and identification issues are discussed. In Section 3 the empirical analysis is considered. Conclusions are drawn in Section 4. A mathematical result which is useful in identifying structural shocks via changes in the volatility and details of our estimation method are presented in the Appendix.

2 Model Setup

2.1 The Statistical Model

The general setup is an SVAR model. More precisely, an **AB**-model in the terminology of Amisano and Giannini (1997) is used (see also Lütkepohl 2005, Chapter 9):

$$\mathbf{A}y_t = C_1y_{t-1} + \cdots + C_p y_{t-p} + \mathbf{B}\varepsilon_t, \quad (1)$$

where y_t is a K -dimensional vector of observable variables, ε_t is a K -dimensional vector of structural innovations with mean zero and identity covariance matrix, i.e., $\varepsilon_t \sim (0, I_K)$, and \mathbf{A}, \mathbf{B} and C_i ($i = 1, \dots, p$) are $(K \times K)$ parameter matrices. Notice that we are not assuming normality of ε_t or y_t . The model in (1) is a structural form with corresponding reduced form error term $u_t = \mathbf{A}^{-1}\mathbf{B}\varepsilon_t \sim (0, \mathbf{A}^{-1}\mathbf{B}\mathbf{B}'\mathbf{A}^{-1})$. The reduced form error terms can be estimated from the data. To obtain estimators of the structural errors ε_t , a one-to-one mapping from the reduced form error covariance matrix to \mathbf{A} and \mathbf{B} is required. Identifying restrictions have to be imposed on \mathbf{A} and \mathbf{B} to obtain a unique relation.

2.2 Economic Setup

In our empirical model the observable variables will be divided in two groups. The first one contains variables whose current values are in the monetary authority's information set and are not influenced instantaneously by policy decisions. The second group contains variables which are determined within the money market. The first set of variables is $y_{1t} = (gdp_t, p_t, pcom_t)'$, where gdp_t , p_t and $pcom_t$ denote logs of real GDP, the log implicit GDP deflator and an index of commodity prices, respectively. The money market variables are collected in $y_{2t} = (TR_t, NBR_t, FF_t)'$, where TR_t , NBR_t and FF_t denote total reserves, nonborrowed reserves and the federal funds rate, respectively. Thus, $K = 6$ and y_{1t} and y_{2t} are both three-dimensional, as in BM.

Our partitioning of $y_t = (y'_{1t}, y'_{2t})'$ implies that we can choose

$$\mathbf{A} = \begin{bmatrix} I_3 & 0 \\ -\mathbf{A}_{21} & I_3 \end{bmatrix}, \quad (2)$$

and $v_t = (v'_{1t}, v'_{2t})' = \mathbf{B}\varepsilon_t$ has a block-diagonal covariance matrix. Hence, \mathbf{B} is also block-diagonal,

$$\mathbf{B} = \begin{bmatrix} \mathbf{B}_{11} & 0 \\ 0 & \mathbf{B}_{22} \end{bmatrix}, \quad (3)$$

where the \mathbf{B}_i 's ($i = 1, 2$) are both (3×3) . In this model setup the matrix \mathbf{A}_{21} can be estimated by OLS from

$$y_{2t} = \mathbf{A}_{21}y_{1t} + C_{2,1}y_{t-1} + \cdots + C_{2,p}y_{t-p} + v_{2t}, \quad (4)$$

where $C_{2,i}$ consists of the last three rows of C_i ($i = 1, \dots, p$). Moreover, since we are just interested in identifying the monetary shocks, we just need to recover the money market innovations ε_{2t} . In other words, we need restrictions which ensure a one-to-one mapping from $E(v_{2t}v'_{2t}) = \mathbf{B}_{22}\mathbf{B}'_{22}$ to \mathbf{B}_{22} .

Following BM and CEE the demand for total reserves is specified as

$$TR_t = -\alpha FF_t + f_{TR}(\text{policy information}) + \sigma_d \varepsilon_t^d,$$

the demand for borrowed reserves is given by

$$BR_t = \beta FF_t - \gamma NBR_t + f_{BR}(\text{policy information}) + \sigma_b \varepsilon_t^b$$

and the Fed policy rule for setting nonborrowed reserves is

$$NBR_t = f_{NBR}(\text{policy information}) + \phi^d \sigma_d \varepsilon_t^d + \phi^b \sigma_b \varepsilon_t^b + \sigma_s \varepsilon_t^s,$$

where ε_t^s is the exogenous monetary policy shock. The policy information consists of all lagged variables and the current values of gdp_t , p_t and $pcom_t$. The functions $f_*(\cdot)$ are all linear functions and α , β , γ , ϕ^d , ϕ^b , σ_d , σ_b and σ_s are unknown parameters.

Using $TR = NBR + BR$, CEE derive from these relations that

$$\mathbf{B}_{22} = \begin{bmatrix} \sigma_d \frac{\beta - \phi^d \alpha \gamma + \phi^d \alpha}{\beta + \alpha} & -\alpha \sigma_s \frac{\gamma - 1}{\beta + \alpha} & -\alpha \sigma_b \frac{-1 + \phi^b \gamma - \phi^b}{\beta + \alpha} \\ \sigma_d \phi^d & \sigma_s & \sigma_b \phi^b \\ \sigma_d \frac{\phi^d \gamma - \phi^d + 1}{\beta + \alpha} & \sigma_s \frac{\gamma - 1}{\beta + \alpha} & \sigma_b \frac{-1 + \phi^b \gamma - \phi^b}{\beta + \alpha} \end{bmatrix}. \quad (5)$$

Thus, the nine elements of \mathbf{B}_{22} are determined by eight free parameters $\psi = (\alpha, \beta, \gamma, \phi^d, \phi^b, \sigma_d^2, \sigma_b^2, \sigma_s^2)'$.

These parameters are not identified in a model with time invariant covariance matrix $\Sigma_{v2} = E(v_{2t} v_{2t}')$ because this matrix is symmetric and, thus, has six distinct elements only. Hence, further restrictions are needed. CEE consider different specifications of policy shocks and derive the following restrictions (see also BM):

- *FF policy shock*: $\phi^d = 1/(1 - \gamma)$ and $\phi^b = -\phi^d$. These restrictions mean that the monetary shocks are induced through the federal funds rate and correspond to the

assumption of Bernanke and Blinder (1992) that the Fed targets the federal funds rate.

- *NBR policy shock*: $\phi^d = \phi^b = 0$. The assumption that policy shocks can be associated with the errors in the equation for nonborrowed reserves was made by Christiano and Eichenbaum (1992).
- *NBR/TR policy shock*: $\alpha = \phi^b = 0$. BM derived this restriction from the assumption made by Strongin (1995) that shocks to total reserves are demand shocks which are accommodated by the Fed.
- *BR policy shock*: $\phi^d = 1$, $\phi^b = \alpha/\beta$ and $\gamma = 0$. These restrictions are obtained if the Fed is assumed to target borrowed reserves, as e.g. in Cosimano and Sheehan (1994).

Unfortunately, the first three sets of restrictions still do not over-identify the shocks. Consequently, they are not sufficient to actually test the underlying assumptions against the data in CEE's framework. Therefore BM assume in addition that $\gamma = 0$ to obtain over-identified models. As CEE pointed out, such an approach is unsatisfactory because rejection of a particular set of restrictions may then be caused by an unjustified assumption rather than false restrictions derived from theory.

In our empirical analysis there is, however, a way out of this dilemma. Both BM and CEE find that over the sample period considered there is some change in the structure of the relations. BM actually fit models to different sample periods while CEE find that there may have been a change in the volatility of the shocks whereas the remaining structure is unaffected. Even with the kind of structural change diagnosed by CEE we are able to identify B_{22} as we will argue now.

2.3 Identification of Shocks via a Change in Volatility

Suppose there is just one change in the volatility of the shocks during the sample period, say in period T_B , so that

$$E(v_{2t}v'_{2t}) = \begin{cases} \Sigma_1 & \text{for } t = 1, \dots, T_B - 1, \\ \Sigma_2 & \text{for } t = T_B, \dots, T. \end{cases} \quad (6)$$

Here T is the sample size. A well-known result from matrix algebra states that the covariance matrices Σ_1 and Σ_2 can be diagonalized simultaneously. More precisely, there exists a (3×3) matrix W and a diagonal matrix $\Omega = \text{diag}(\omega_1, \omega_2, \omega_3)$ with positive diagonal elements ω_i such that $\Sigma_1 = WW'$ and $\Sigma_2 = W\Omega W'$ (e.g., Leamer 1978, Theorem 35, Lütkepohl 1996, Section 6.1.2). Here the diagonal elements of Ω incorporate the changes in the variances of the shocks after the possible change in volatility has occurred. If the ω_i 's are different from one, there is a change in volatility. It can be shown that W is unique except for sign changes if all ω_i 's are distinct.² In other words, if we choose $B_{22} = W$, we get uniqueness of the shocks $\varepsilon_{2t} = B_{22}^{-1}v_{2t}$ (except for changes in sign) without the need for further identifying assumptions. With this choice of B_{22} transformation the structural shocks have identity covariance matrix in the first regime, $E(\varepsilon_{2t}\varepsilon'_{2t}) = I_3$ ($t = 1, \dots, T_B - 1$), and they have covariance matrix Ω in the second regime, $E(\varepsilon_{2t}\varepsilon'_{2t}) = \Omega$ ($t = T_B, \dots, T$). Hence, they are orthogonal in both regimes and this requirement suffices to identify our shocks uniquely if all ω_i 's are distinct. Clearly, orthogonality of the shocks is a standard assumption in SVAR analysis. It does not go beyond what is assumed in other studies as well.

Of course, we could equally well have normalized the variances in the second regime. Given this normalization, for uniqueness of B_{22} all we need to know is whether the volatility changes in different shocks are proportional. If they are not, then the ω_i 's are distinct

and, hence, \mathbf{B}_{22} is unique except for sign changes. In fact, the volatility in one of the shocks may not change at all, that is, one of the ω_i 's may be unity. The crucial condition is that they are all distinct. If there are other restrictions on \mathbf{B}_{22} , as in the present analysis, these become overidentifying restrictions which can be tested. The advantage of this setup is that changes in the variances can be investigated with statistical means, as we will see in Section 3.2, and, hence, we do not have to rely exclusively on information from economic theory to ensure identification of the structural shocks.

Notice, however, that it is always possible to reverse the signs of all elements in a single column of \mathbf{B}_{22} without affecting the likelihood. For practical purposes this is no problem, of course, because it just means that, to obtain uniqueness, we have to specify whether a shock is positive or negative. For estimation and deriving asymptotic results this kind of local uniqueness is sufficient.

In general one may actually allow for the possibility of various changes in volatility. Suppose there are $n + 1$ different regimes and the covariances in the different regimes are WW' , $W\Omega_1W'$, \dots , $W\Omega_nW'$, where the Ω_i 's are all diagonal matrices and the variances in the first regime are normalized to one for convenience. A normalization in any other regime would serve the same purpose. In this case uniqueness of W (up to sign) is ensured, for example, if the diagonal elements in only one of the Ω_i matrices are all distinct and we can again choose $\mathbf{B}_{22} = W$. This result is also analogous to a bivariate result of Rigobon (2003).

One may argue that the assumption of a time invariant \mathbf{B}_{22} is a strong one because this matrix represents the instantaneous effects of shocks and these may change as well if the volatility changes. Note, however, that if there are only two regimes, we can always find decompositions $\Sigma_1 = WW'$ and $\Sigma_2 = W\Omega W'$, where the same W matrix is used in

both products. Hence, choosing $\mathbf{B}_{22} = W$ will result in the same instantaneous effects of the shocks in both regimes. That this choice of \mathbf{B}_{22} reflects the actual instantaneous responses to the economic shocks of interest is an assumption, of course, which may or may not hold. If it holds, the instantaneous effects are time invariant. Clearly, there may also be changes in some or all of the other VAR parameters. Such changes can be checked by formal statistical tests, however. Of course, our model is useful only if it is consistent with the data. We have used the specific change in volatility here because it suffices to get identification of the shocks and it was argued by CEE and Bernanke and Mihov (1998b) that structural changes found by other authors in the data set underlying our empirical study may have been due to just this kind of change rather than a change in the whole dynamic structure. We will address this issue in the empirical analysis. Of course, identification of the shocks can also be obtained if more substantial structural changes have occurred. In that case the impulse responses may be affected, however, and this fact has to be taken into account in the evaluation of the model.

If a change in the volatility of shocks is diagnosed and uniqueness of \mathbf{B}_{22} is ensured by our setup, then all the restrictions from economic theories are over-identifying and, hence, can be tested. In other words, it can be checked whether they are compatible with orthogonality of the shocks in both regimes. Since there are only eight elements in the vector of economic parameters ψ , while \mathbf{B}_{22} has nine elements, there is in fact already one restriction implied by the overall general model which nests the others, provided the diagonal elements of Ω are distinct. If there are only two different ω_i 's, then an over-identifying restriction may not be available in the general model while the additional restrictions implied by the different theories can still be tested under suitable conditions. In the next section we present the empirical analysis and discuss these issues in the context

of our model.

3 Empirical Analysis

3.1 The Data

Monthly US data from BM for the period 1965M1-1996M12 are used in our empirical analysis. The monthly data for gdp and p are constructed from quarterly data by using a set of monthly interpolator variables. These data were also used by CEE and BM. Hence, our results are directly comparable to those of the earlier studies. Using the same sample period, although longer time series are available, has the advantage that the results are not driven by the extended sample period but differences to the other studies are a direct consequence of the alternative methods used.

3.2 Estimation and Testing

Estimation under our assumption of a change in volatility is done by a multi-step iterative procedure. In the first step equation wise OLS is applied to a model such as (4) with an additional constant term. We denote the residuals by \tilde{v}_{2t} and define

$$\tilde{\Sigma}_1 = \frac{1}{T_B - 1} \sum_{t=1}^{T_B-1} \tilde{v}_{2t} \tilde{v}'_{2t} \quad \text{and} \quad \tilde{\Sigma}_2 = \frac{1}{T - T_B + 1} \sum_{t=T_B}^T \tilde{v}_{2t} \tilde{v}'_{2t}.$$

Then the following concentrated log likelihood type function is maximized with respect to ψ , ω_1 , ω_2 and ω_3 :

$$\begin{aligned} \log l = & -\frac{T_B - 1}{2} \left(\log |\mathbf{B}_{22} \mathbf{B}'_{22}| + \text{tr} \left\{ \tilde{\Sigma}_1 (\mathbf{B}_{22} \mathbf{B}'_{22})^{-1} \right\} \right) \\ & -\frac{T - T_B + 1}{2} \left(\log |\mathbf{B}_{22} \Omega \mathbf{B}'_{22}| + \text{tr} \left\{ \tilde{\Sigma}_2 (\mathbf{B}_{22} \Omega \mathbf{B}'_{22})^{-1} \right\} \right) \end{aligned} \quad (7)$$

and thereby we obtain estimators $\tilde{\mathbf{B}}_{22}$ and $\tilde{\mathbf{\Omega}}$ of \mathbf{B}_{22} and $\mathbf{\Omega}$, respectively.

Although (7) looks like a Gaussian log likelihood function, the OLS estimators of the VAR coefficients from (4) are not maximum likelihood (ML) estimators due to the assumed heteroskedasticity. Therefore, in the next step the estimators $\tilde{\mathbf{B}}_{22}$ and $\tilde{\mathbf{\Omega}}$ obtained in this way are used to perform a feasible multivariate GLS estimation of the VAR coefficients in (4). These are then used again in (7) to obtain new estimates of the structural parameters and this procedure is iterated. Gaussian ML estimators are obtained upon convergence. Details of this estimation procedure are provided in the Appendix. Because we are not assuming that the variables are actually normally distributed, the procedure might be called more precisely a Gaussian quasi ML procedure. We still use the shorter terminology here for simplicity and emphasize that our inference procedures remain valid asymptotically under more general than normal distributional assumptions.

Although we have presented the estimation method for two different regimes only for convenience, it is straightforward to apply it when there are more than two regimes. In our empirical analysis we have used models with up to three different regimes. Moreover, Rigobon (2003) shows that the time invariant parameters can be estimated consistently under usual assumptions even if the break times are fixed incorrectly.³

Having the ML estimators opens up the possibility to perform likelihood ratio (LR) tests. Of course, these are only quasi LR tests if the data are not actually normally distributed. We call them LR tests for simplicity even in this case. Some tests are of particular importance in the present context. Assuming again two different regimes for illustrative purposes and denoting the reduced form residual covariance matrices in the

two regimes by Σ_1 and Σ_2 , respectively, a test of interest is, for example,

$$H_0 : \Sigma_1 = \Sigma_2 \quad \text{vs.} \quad H_1 : \Sigma_1 \neq \Sigma_2. \quad (8)$$

In other words, the null hypothesis specifies that there is no regime change. Since we consider reduced form parameters here, there is no identification problem. For our three-equation model (4) the asymptotic null distribution of the corresponding LR statistic is $\chi^2(6)$, provided that LR tests have standard asymptotic properties. Given that the data generation process may have unit roots and may be cointegrated, the asymptotic properties of LR tests are in general not necessarily standard. For the present case, standard asymptotic properties are obtained, however, because the cointegration properties do not affect the estimator of the residual covariance matrix asymptotically (see, e.g., Lütkepohl 2005, Chapter 7). The test is in fact a Chow type test. Its small sample properties may not be ideal, as pointed out by Candelon and Lütkepohl (2001). According to these results the test may reject a true null hypothesis too often in small samples. However, as we will find in the empirical section, the p -values for our tests are far too small to attribute our rejection findings to such small sample bias.

If more than two regimes are considered, one may want to check that a decomposition with the same \mathbf{B}_{22} matrix in all regimes exists. This could also be tested by an LR test. In our empirical example the case of two regimes will be of primary interest and in this case the issue does not arise because a decomposition with the same \mathbf{B}_{22} matrix in both regimes always exists, as explained earlier. Therefore we do not pursue tests for a regime invariant \mathbf{B}_{22} matrix here.

3.3 Results

We have estimated a set of different VAR models for the levels variables by the ML procedure described in the previous subsection. All models have 13 lags as in BM's study. A critical issue is, of course, whether there have been changes in the volatility of the disturbances which can be used for identifying the structural shocks. In fact, the literature on the monetary transmission mechanism in the US presents evidence for a substantial number of possible break dates during the period under study. Which break dates are found in different studies depends on the sample period used, the methods applied for break date determination and the models considered. A variety of different statistical procedures has been applied in this context and also detailed investigations of periods with different operating regimes of the Fed have been used to pin down possible dates of regime changes. In the following we will only consider those two break dates on which BM and CEE agree, that is, we consider structural breaks in 1979M10 and 1984M2. The choice of these two break dates is based on a "combination of historical and statistical evidence" according to BM (p. 880). There is actually some consensus in the literature that the Volcker era differs from the pre- and post-Volcker periods, at least as far as monetary policy is concerned, although the actual break dates found in different studies vary.

We have checked the break dates and present the results in Table 1. They confirm that using models with regime changes in 1979M10 and 1984M2 is reasonable. On the basis of the p -values we clearly reject constant reduced form covariance matrices throughout the full sample period at any common significance level. Notice that in the table, Σ_1 , Σ_2 and Σ_3 denote the residual covariance matrices corresponding to the periods until 1979M9,

1979M10 – 1984M1 and from 1984M2 – 1996M12, respectively. Even though the tests may be biased in small samples and reject too often, the p -values are too small to defend constant covariance matrices.

There is some disagreement in the literature regarding the type of structural break. Using the same setup as BM and CEE in this respect again ensures comparability with their results. It also ensures that our findings are driven by using our different methods and not by differences in the assumptions regarding the data properties. We emphasize, however, that assuming changes only in the disturbance covariance matrices and, hence, in the volatility of the structural shocks is not uncommon in the related literature. For example, Sims and Zha use a related model but quite different statistical procedures and they find

“that the version of our model that fits best is one that shows no change at all in coefficients, either of the policy rule or of the private sector block of the model. What changes across ‘regimes’ is only the variances of structural disturbances.” (Sims and Zha 2006, p. 54).

Thus, our assumptions regarding the timing and type of breaks are not unconventional and have been confirmed by a variety of methods. We will now use our statistical setup to investigate how far these conventional assumptions lead in identifying structural shocks.

For our purposes it is, of course, enough that there is one break point of the sort discussed in Section 2.3. Since the break in 1979M10 is the least significant one, we assume in the following that at least $\Sigma_2 \neq \Sigma_3$ holds while we leave it open whether Σ_1 is also different from Σ_2 and Σ_3 . Thus, we consider a model where $\Sigma_2 = \mathbf{B}_{22}\mathbf{B}'_{22}$ and $\Sigma_3 = \mathbf{B}_{22}\Omega\mathbf{B}'_{22}$ with $\Omega = \text{diag}(\omega_1, \omega_2, \omega_3)$ while Σ_1 is left unrestricted and is not

decomposed. The estimates of the parameters of primary interest for our purposes for the unrestricted and several restricted models are shown in Table 2.

One question of particular interest is whether the Ω matrix has distinct diagonal elements because this identifies the shocks and opens up the possibility to test the alternative structural restrictions from the economic models discussed in Section 2.2. Clearly, the estimates and their standard errors in the last subperiod are such that one may suspect that they are different. Notice that one-standard error intervals around the estimates for the unrestricted model do not overlap. Clearly, one may feel that this criterion is not strong enough to conclude that all ω_i 's are distinct. After all, this assumption is the basis for our identification of the shocks and, thus, the validity of our subsequent tests rests on it. Therefore it may be worth pointing out that the evidence for at least two different diagonal elements of Ω is quite strong in all models. In the following we will also consider the possibility that only two of the three ω_i 's may be distinct. Even then we have over-identifying restrictions which can be tested.

Since our previous results suggest that all the identification schemes presented in Section 2.2 can be tested against the data, we present the corresponding LR tests in Table 3. In the table p -values for two alternative degrees of freedom (d.f.) of the corresponding χ^2 distributions are reported. The first one is obtained under the assumption that all ω_i 's are distinct and the d.f. correspond to the number of additional restrictions imposed by the assumed economic scheme. For example, for the FF scheme we have two d.f. in this case. The second column of p -values for a χ^2 distribution with one d.f. represents the worst case situation if only two ω_i 's differ. In this case, there is at least one restriction and possibly more. Thus, the first column of p -values in Table 3 considers the most favorable case for the models whereas the last column of p -values considers the most

difficult scenario for the models to conform with the data.

Based on the asymptotic p -values in Table 3 it turns out that both the NBR and BR schemes can be strongly rejected at common significance levels even in the most favorable situation for the models (d.f. = 2 and 3 for NBR and BR , respectively). In contrast, the FF and NBR/TR schemes cannot be rejected at the 5% level even under the least favorable scenario for the models (d.f. = 1).

In Table 2 also the estimates of the structural parameters obtained under the different sets of restrictions are given. Clearly, the NBR scheme produces some very different parameter estimates from the other identification schemes even if sampling uncertainty is taken into account. In particular, restricting the parameter ϕ^d to zero seems to have a strong effect. This parameter is clearly different from zero in all the other identification schemes. In other words, eliminating the innovations ε_t^d from the equation for nonborrowed reserves and thereby imposing that the ε_t^d shocks have a delayed impact on nonborrowed reserves only is problematic.

It may also be worth pointing out that in the unrestricted model the parameter γ is not significantly different from zero judged on the basis of its t -ratio. On the other hand, it becomes significant in the FF scheme where other restrictions are imposed. Recall that BM used the restriction $\gamma = 0$ to obtain over-identified models and thus a possibility for statistical model comparison. Given that this parameter becomes significant in the FF scheme sheds doubt either on the restriction or on the identification scheme. Other reasons why the restriction $\gamma = 0$ may be problematic were discussed by CEE. We estimated an additional FF model with $\gamma = 0$ (hence, $\phi^d = 1$ and $\phi^b = -1$). This restriction produced a p -value of 0.0003 and, hence, was clearly rejected by the LR test. This result reinforces the conclusion that ϕ^d to some extent drives the results. The parameter ϕ^d is freely

estimated only in the *NBR/TR* scheme (with an estimate very close to the estimated value in the unrestricted model), and this scheme cannot be rejected. Likewise, in the *FF* scheme the estimated value is close to the unrestricted estimate, but once γ is forced to equal zero and, hence, ϕ^d is set to unity, this model is rejected. In the *BR* model ϕ^d also equals unity and this model is rejected. These results suggest that the assumption that the Fed fully accommodates reserves demand shocks ($\phi^d = 1$) is not supported by the data. On the other hand, the *NBR* identification scheme with $\phi^d = 0$ is also rejected. Hence, the results seem to be most sensitive to the value of ϕ^d which is also estimated with a very small standard error in the unrestricted model. The *FF* scheme has the drawback that γ has to be smaller than zero for ϕ^d to be smaller than one. Thus, in the *FF* scheme γ has the wrong sign if $\phi^d < 1$ as indicated by our estimation results. In this respect the *NBR/TR* scheme is preferable because it is acceptable even without taking the effect of nonborrowed reserves on banks' borrowing into account (i.e., γ is not significantly different from zero).

In summary, the standard errors indicate that some of the parameters are not estimated very precisely, e.g., β . An identification scheme for which some of the imprecise estimates are not close to the corresponding estimates of the unrestricted model may not be rejected because the data information is insufficient to pin down the precise parameter value. On the other hand, ϕ^d is a critical parameter which is estimated very precisely. The *NBR* scheme which sets this parameter to zero is clearly rejected. Likewise the *BR* scheme is rejected which sets $\phi^d = 1$. Furthermore, the *FF* scheme which links ϕ^d to γ produces an implausible estimate of the latter parameter although it is not formally rejected. In contrast, the *NBR/TR* scheme leaves ϕ^d unrestricted and is fully compatible with the data.

In Figure 1 we present the monetary policy shocks implied by the different models. Notice that these shocks are only identified from 1979M10 onwards because our identification scheme applies only after the first subperiod. Therefore only the shocks from 1979M10 – 1996M12 are displayed in Figure 1. The shocks associated with the *NBR* scheme are quite different from the monetary shocks implied by the other identification schemes. In particular, the *NBR* shocks are considerably more volatile after the mid-1980s. On the other hand, the unrestricted and the *NBR/TR* and *BR* shocks appear to be quite similar at first glance. To some extent also the *FF* shocks fall roughly in this group although they do not display the spikes in 1984 which can be seen in the *NBR/TR* and *BR* shocks.

We have also determined the impulse responses induced by a monetary policy shock and present the graphs in Figures 2 and 3. These are the responses to a 25 basis points reduction in the federal funds rate on impact. Thus, an expansionary monetary policy shock is considered. On the left-hand side of Figure 2 the impulse responses from a model where the parameter vector ψ is unrestricted are shown with bootstrapped 95% confidence intervals.⁴ Given the estimation uncertainty reflected in these intervals, the impulse responses of the *FF*, *NBR/TR* and *BR* schemes which are shown on the right-hand side of Figure 2 are quite similar. The *FF* impulse responses are overall closest to those obtained from the unrestricted model. The *NBR/TR* and *BR* impulse responses are almost identical because both models restrict the first two elements in the last column of the B_{22} matrix to zero (see (5)). In Figure 2 they are so close together that they are almost indistinguishable. Most of these impulse responses are plausible reactions to an expansionary monetary policy shock. In particular, there is a significant increase in GDP and the commodity price index. Moreover, the GDP deflator increases, although

not significantly in the unrestricted model. The insignificant or even negative response of total reserves may be viewed as a “liquidity puzzle” which has been observed in other studies as well. However, Carpenter and Demiralp (2007) argue that this is not a puzzle at all because reserves are not the appropriate measure of money balances if the federal funds rate is the relevant interest rate in the model. In any case, the reactions to a monetary policy shock seen in Figure 2 are not unlike those observed in similar studies and, hence, are plausible in this sense.

The impulse responses obtained from the *NBR* scheme are quite different from the other ones. Therefore they are shown separately in Figure 3 together with the 95% confidence intervals obtained for the unrestricted model. This way the substantial difference of the *NBR* scheme to the other identification schemes becomes apparent (notice the change in the scales of the graphs).

The overall message from our analysis is that the data resist both the *NBR* and *BR* schemes proposed by Christiano and Eichenbaum (1992) and Cosimano and Sheehan (1994), respectively. While the monetary policy shocks implied by the *BR* scheme are still close to those from models which are not rejected by the data, using the *NBR* scheme for monetary policy analysis is clearly problematic. In some respects, the preferred specification is Strongin’s (1995) *NBR/TR* identification scheme. It is not rejected by the data, produces sensible estimates of the structural parameters and delivers plausible impulse responses.

4 Conclusions

A large body of literature has discussed the question how to identify monetary policy shocks for the US. Clearly, this is an important problem for assessing monetary policy. Different money market models have been proposed and the corresponding shocks have been derived and estimated. The fact that there is still disagreement as to which shocks actually reflect the effects of monetary policy is a consequence of the problem that the different theories do not provide sufficient restrictions for an empirical model to be able to check them by statistical tests.

Given this state of affairs we have proposed a setup where identifying information from changes in the volatility of the shocks can be used to obtain unique specifications of the shocks. Using monthly data for the US from 1965 to 1996 as in BM and CEE we find with statistical tools that the Volcker period displays larger volatility of the shocks than the post-Volcker period and we have used this statistical information in specifying monetary policy shocks. The fact that there was a decrease in volatility after the Volcker period was also found by other authors and actually seems to be a widely accepted view in the related literature.

Using the statistical information on the volatility of the shocks opens up the possibility to test different theoretical assumptions against the data. In particular, we have tested four different identification schemes proposed by Bernanke and Blinder (1992) (*FF*), Christiano and Eichenbaum (1992) (*NBR*), Strongin (1995) (*NBR/TR*) and Cosimano and Sheehan (1994) (*BR*) which have also been considered and further investigated by other authors. In these identification schemes monetary policy shocks enter via the federal funds rate, nonborrowed reserves, total reserves or borrowed reserves, respectively. So

far the empirical results have been inconclusive or otherwise not fully satisfactory. In our framework it turns out that the *NBR* and *BR* schemes are clearly rejected by the data whereas *FF* and *NBR/TR* cannot be rejected at common significance levels. Even though *BR* is overall rejected by our formal statistical test, the implied impulse responses associated with a monetary policy shock are very similar to those implied by the *NBR/TR* identification scheme. In contrast, the *FF* scheme results in slightly different impulse responses and it also produces an implausible value for at least one of the structural parameters.

In summary, the *NBR* scheme is clearly problematic from the point of view of monetary policy analysis. At least for the time period under consideration in this study, the *NBR* scheme cannot be recommended for policy analysis. On the other hand, the *NBR/TR* scheme is overall the most plausible one. It is not rejected by the data, produces structural parameters of expected sign and results in plausible responses to monetary shocks.

Finally, in this paper we have proposed a statistical setup which allows us to compare different models for monetary policy analysis. Although we have checked our setup carefully against the data, it is of course possible that none of the four models considered describes the ongoings in the money market well. In fact, critique has been raised by Carpenter and Demiralp (2007) against using borrowed, nonborrowed or total reserves as a measure for money. They argue that Federal Reserve balances should be used as a measure for money because it is the variable that is actually affected by changes in the federal funds rate. This suggests that a more detailed modelling of the money market may be a fruitful avenue for further research in this context.

Appendix

In this section we provide details on our estimation procedure. The point of departure is the Gaussian log likelihood function (apart from additive constants)

$$\log l = -\frac{1}{2} \sum_{t=1}^T \log |\Sigma_t| - \frac{1}{2} \sum_{t=1}^T \text{tr}(v_{2t} v_{2t}' \Sigma_t^{-1}),$$

where

$$\Sigma_t = E(v_{2t} v_{2t}') = \begin{cases} \Sigma_1 = \mathbf{B}_{22} \mathbf{B}_{22}' & \text{for } t = 1, \dots, T_B - 1, \\ \Sigma_2 = \mathbf{B}_{22} \Omega \mathbf{B}_{22}' & \text{for } t = T_B, \dots, T, \end{cases} \quad (9)$$

and $v_{2t} = y_{2t} - CZ_t$. Here C is the matrix of all VAR parameters in (4) and a constant and Z_t contains all the regressors from (4) plus the deterministic term used in our empirical analysis. The normal equations for C are

$$\sum_{t=1}^T \Sigma_t^{-1} (y_{2t} - CZ_t) Z_t' = 0 \quad (10)$$

(Lütkepohl 2005, Sec. 17.2.2). Using (9) and standard rules for the column vectorization operator vec , it follows that the ML estimator for C satisfies

$$\begin{aligned} \text{vec}(\tilde{C}) = & \left[\sum_{t=1}^{T_B-1} (Z_t Z_t' \otimes \tilde{\Sigma}_1^{-1}) + \sum_{t=T_B}^T (Z_t Z_t' \otimes \tilde{\Sigma}_2^{-1}) \right]^{-1} \\ & \times \left[\sum_{t=1}^{T_B-1} (Z_t \otimes \tilde{\Sigma}_1^{-1}) y_{2t} + \sum_{t=T_B}^T (Z_t \otimes \tilde{\Sigma}_2^{-1}) y_{2t} \right], \end{aligned} \quad (11)$$

where $\tilde{\Sigma}_i$ denotes the ML estimator of Σ_i ($i = 1, 2$). If some other estimators $\tilde{\Sigma}_i$ are used instead, \tilde{C} is a feasible multivariate GLS estimator of C . Using such an estimator of C and plugging the resulting $\tilde{v}_{2t} = y_{2t} - \tilde{C}Z_t$ into (7), we obtain estimators of the structural parameters in \mathbf{B}_{22} and Ω by maximizing the resulting “concentrated” log likelihood (7) in the usual way. From these estimators new estimators of Σ_1 and Σ_2 may be obtained and used again in (11) and so on. The procedure can be used to compute the Gaussian ML

estimators by continuing the iterations until convergence. If the actual distribution of the ε_t 's is nonnormal, the estimators are, of course, just Gaussian quasi ML estimators. This method was used in Section 3.

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Footnotes

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² For a formal proof see the discussion paper version of this paper (Lanne and Lütkepohl 2006). For bivariate systems the result was given by Rigobon (2003).

³ If the variances differ and the misspecification of the break points is only slight, the variances will still be different in the misspecified regimes. Thus, the time-invariant structural parameters can still be identified. Rigobon uses this property to show consistency of the estimates.

⁴ The intervals are determined by Hall's percentile method as proposed by Benkwitz, Lütkepohl and Wolters (2001) using 2000 bootstrap replications.

Table 1: LR tests for regime changes

H_0 (type (8))	Break(s)	Test statistic	p -value
$\Sigma_1 = \Sigma_2 = \Sigma_3$	1979M10, 1984M2	334.617	2.457e-64
$\Sigma_1 = \Sigma_2$	1979M10	26.026	0.0002
$\Sigma_2 = \Sigma_3$	1984M2	220.535	8.003e-45
$\Sigma_1 = \Sigma_3$	1979M10, 1984M2	260.596	2.227e-53

Table 2: Estimates of structural parameters with standard errors in parentheses

Parameter	unrestricted	FF	NBR	NBR/TR	BR
α	0.3165 (0.1639)	0.1967 (0.1539)	68.4312 (72.8482)	0.0000	0.1912 (0.1504)
β	3.8785 (1.9464)	0.3730 (0.1828)	0.4403 (0.1786)	14.2856 (6.2248)	13.1717 (5.6372)
γ	-0.0966 (0.0821)	-0.1819 (0.0425)	0.0812 (0.0369)	-0.0375 (0.2039)	0.0000
ϕ^b	-0.1674 (0.1999)	-0.8461 (0.0304)	0.0000	0.0000	0.0145 (0.0132)
ϕ^d	0.8313 (0.0283)	0.8461 (0.0304)	0.0000	0.8420 (0.0275)	1.0000
σ_b	0.0266 (0.0102)	0.0184 (0.0019)	0.0134 (0.0014)	0.0926 (0.0420)	0.0856 (0.0382)
σ_d	0.0077 (0.0008)	0.0078 (0.0008)	0.4651 (0.4970)	0.0080 (0.0008)	0.0077 (0.0008)
σ_s	0.0164 (0.0023)	0.0032 (0.0014)	0.0163 (0.0016)	0.0161 (0.0016)	0.0166 (0.0016)
ω_1	1.7395 (0.3965)	1.6743 (0.3819)	0.0559 (0.0128)	1.5969 (0.3638)	1.7377 (0.3962)
ω_2	0.0375 (0.0085)	0.0566 (0.0129)	0.2930 (0.0668)	0.0425 (0.0097)	0.0458 (0.0104)
ω_3	0.0673 (0.0153)	0.0455 (0.0104)	0.0804 (0.0195)	0.0597 (0.0137)	0.0603 (0.0138)

Table 3: LR tests of over-identifying restrictions

Identification		<i>p</i> -values		
scheme	H_0	LR statistic	d.f.= no. of restr.	d.f.=1
<i>FF</i>	$\phi^d = 1/(1 - \gamma)$ and $\phi^b = -\phi^d$	3.116	0.211	0.078
<i>NBR</i>	$\phi^d = \phi^b = 0$	62.476	2.713e-14	2.697e-15
<i>NBR/TR</i>	$\alpha = \phi^b = 0$	3.191	0.203	0.074
<i>BR</i>	$\phi^d = 1, \phi^b = \alpha/\beta$ and $\gamma = 0$	18.476	0.0004	1.721e-5

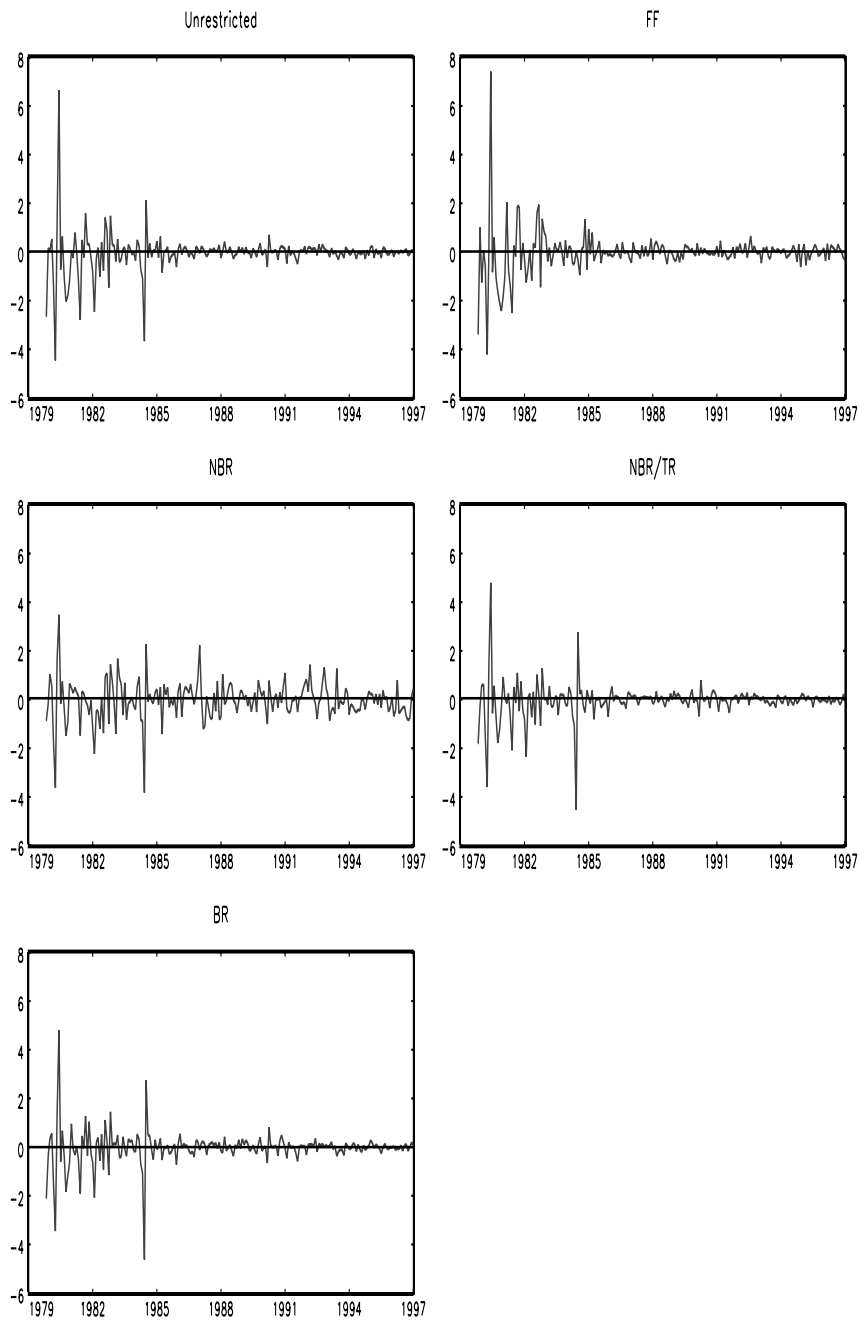


Figure 1: Estimated monetary policy shocks.

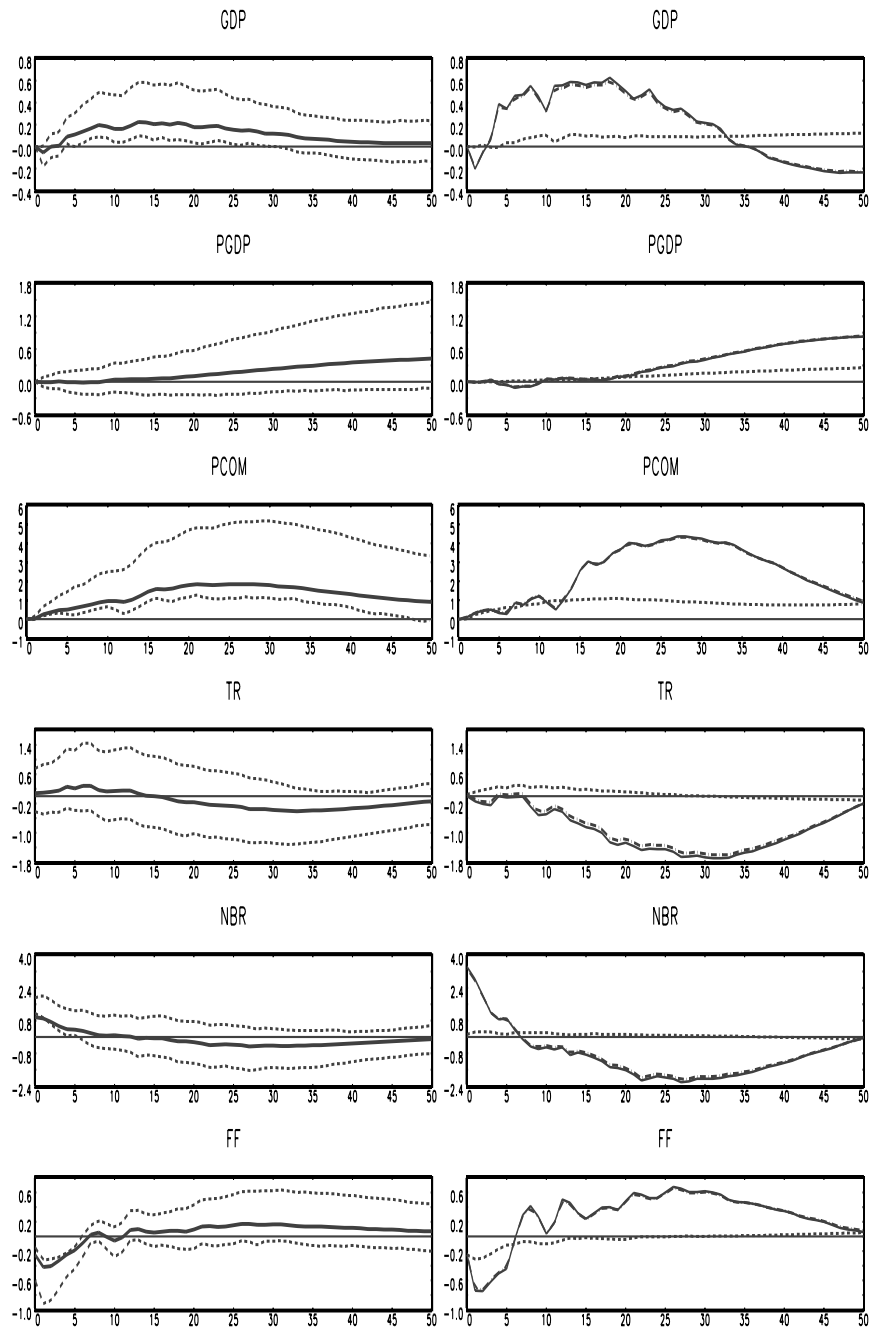


Figure 2: Responses to different monetary policy shocks (left-hand column: impulse responses from unrestricted model with 95% confidence intervals; right-hand column: — NBR/TR , - - - BR , FF impulse responses).

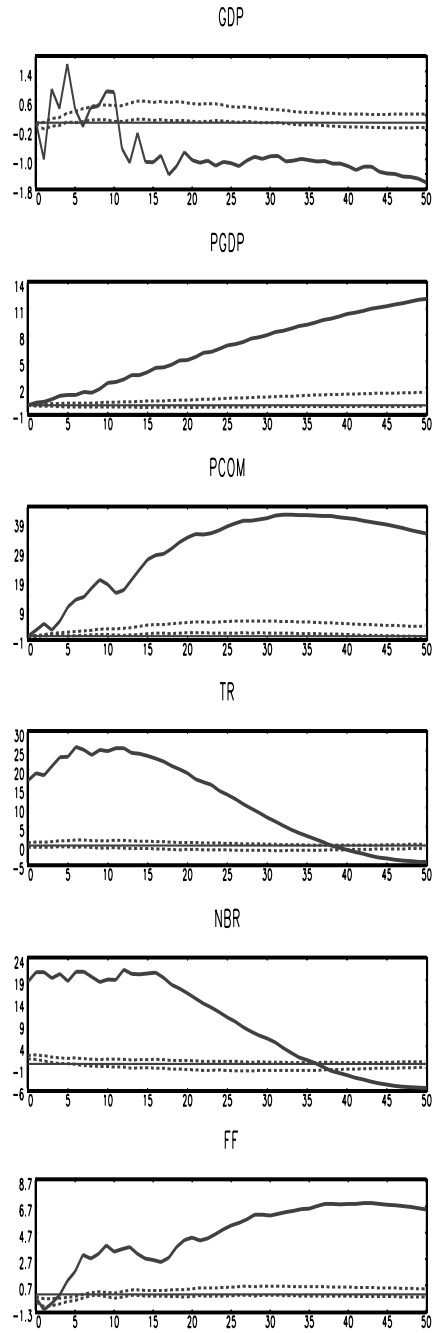


Figure 3: Responses to *NBR* monetary policy shocks with 95% confidence bands from unrestricted model.