

Pegged Exchange Rate Regimes – A Trap?*

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Abstract:

We analyze the role of an exchange rate peg as a commitment mechanism to achieve inflation stability when multiple equilibria are possible. We show that there are ex ante large gains from choosing a more conservative regime not only in order to mitigate inflation bias from time inconsistency, but also to avoid the high inflation equilibria. In these circumstances, using a pegged exchange rate as an anti-inflation commitment device can create a “trap” whereby the regime initially confers gains in anti-inflation credibility, but ultimately results in an exit occasioned by a big enough adverse real shock that creates large welfare losses to the economy.

1. Introduction

Exits from pegged exchange rate regimes have often been accompanied by crises and severe declines in economic activity. Major currency crises over the past decade have had particularly adverse effects on output growth; for example, output declined by 6% in Mexico in 1995, 7% in Thailand and Korea in 1998, and by more than 11% in Argentina in 2002. Looking at a wide sample of countries, Eichengreen and Masson (1998) and Eichengreen (1999) find that exits from pegged exchange rate regimes typically have been accompanied by output declines, with economic growth slowing in the period leading up to the exit, falling further in the year at the time the exit occurs, and recovering thereafter. Moreover, a simple regression of the magnitude of output growth following exits over the period 1980-2001 against the duration of pegged exchange rate spells yields a significantly negative coefficient, implying that more long-lived regimes result in more costly regime change.¹ Eichengreen (1999), documenting the absence of an exit strategy from fixed exchange rates for many countries, concludes: "...exits from pegged exchange rates have not occurred under favorable circumstances. They have not had happy results."

This paper provides an explanation for why pegged exchange rate regimes have tended to end so explosively. It argues that using pegged exchange rate as a commitment device for achieving inflation stability can create a "trap" whereby the regime initially confers gains in anti-inflation credibility, but ultimately results in an exit occasioned by large adverse real shocks, resulting in big welfare losses to the economy. We thus suggest that fixed exchange rate regimes can plant the seeds of their own demise.

We do so in a framework extending Obstfeld setting(1996), where the monetary authority determines the exchange rate (and hence the inflation rate) based on its degree of

aversion to inflation relative to output fluctuations, the magnitude of shocks, and the (fixed) cost of allowing discrete exchange rate changes. In accordance with Rogoff (1985)'s insight, we find that among the pool of possible candidates/regimes, each differing in its degree of anti-inflation firmness, the optimal monetary authority candidate is characterized by a "conservative bias", i.e. a relative weight on inflation stabilization greater than that of society as a whole, in order to mitigate the inflation bias arising from time inconsistency. However, we also show that multiple equilibria are possible. In this case there are ex ante gains from choosing a more conservative monetary authority not only in order to lower private sector inflation expectations and mitigate the inflation bias, but also to avoid high inflation equilibria. These gains, however, come at a cost in the form of the monetary authority's lesser responsiveness to output shocks to the economy. Hence in choosing the monetary authority there is a tradeoff between, on the one hand, the gains from greater firmness in stabilizing inflation and, on the other hand, the ex post costs associated with a lesser willingness to respond to output shocks.

This tradeoff may plant the seeds of the regime's ultimate demise: bad enough shocks ultimately lead to the costly collapse of the regime. This result follows from the observation that large negative output shocks not resulting in devaluation will induce welfare losses to the public. In these circumstances, the gap in welfare evaluation between the public and monetary authority is of a first-order magnitude, proportional to the difference between the public's and the monetary authority's degree of firmness. Ultimately, for a bad enough shock, the cost of sustaining the existing regime will rise above the cost of regime change, inducing a large devaluation accompanied by sizable disruption of the economy. This suggests that more conservative and longer-lasting pegged regimes are likely to end with severe output losses.

That is, the longer the duration of a peg before its collapse, the greater is the adverse effect on the economy when it does collapse.² However, despite the eventual collapse of the regime, a pegged exchange rate regime with a more conservative anti-inflation firmness can still be desirable ex ante because of the disinflationary credibility gain at the beginning of the regime.

These results may explain the severity of the demise of an Argentinean type of currency board: legally anchoring the currency board in the constitution increased the cost of devaluation and regime change, but also planted the seeds of a trap associated with the fixed exchange rate regime. While in the short run it led to obvious credibility gains, it also increased the duration of the peg as well as the output costs associated with the devaluation that occurred when Argentina ultimately exited from the regime.

Of course, it is important to consider why many countries have adopted exchange rate pegs in the first place. In recent years, there has been greater awareness that high and variable inflation distorts the private sector's decision-making with regards to investment, savings, and production, and ultimately leads to slower economic growth. Consequently, various institutional reforms have been suggested to lower inflation expectations through the adoption of credible monetary policy institutions. These include creating an independent central banker that places a higher weight on inflation stabilization than the private sector (Rogoff 1985), designing incentive contracts that penalize central bank governors either by loss of compensation or prestige (Walsh 1995), or adopting some credible rule of monetary policy commitment.

In an open economy, the same array of institutional reforms are available. However, pegging the exchange rate to a stable foreign currency provides another instrument for providing a credible commitment to low inflation. Indeed, as Frankel (1999) has observed, choosing the exchange rate as the nominal anchor and importing credibility from abroad has the advantage of

being relatively easy to implement and providing an easily observable commitment to monetary policy.³ Moreover, emerging economies traditionally lack the institutions and knowledge to establish an independent monetary framework that credibly delivers price stability. For these reasons, a currency peg of some sort has been perceived by many countries as an attractive approach to expressing monetary policy commitment.⁴ In these circumstances, a country may end up with a second-best choice, delegating the conduct of exchange rate policy to a conservative monetary authority and trading off the immediate disinflationary credibility gain with the downside risk that a large negative shock may trigger a future crisis.

Calvo and Vegh's analysis (1999) of stabilization programs shows that many developing countries adopted exchange rate pegs to provide a nominal anchor to reduce inflation, which in many instances were successful in stabilizing inflation from historically high levels. Giavazzi and Pagano (1988) argued that membership in the European Monetary System generally brought significant credibility gains to policymakers in inflation-prone countries, explaining why countries such as Spain, Italy, and Portugal were eager to import Germany's monetary discipline. Ravenna (2005) argues that joining the European Monetary Union currently affords new and potential accession members from Eastern Europe the same benefits.

Nevertheless, depending on its degree of firmness, pegging the exchange rate does not exempt a country from concerns about the credibility of the peg, since the public knows that the government has an incentive to renege and devalue the currency to depreciate the real exchange rate in response to a large enough economic disturbance. There is a vast literature on why pegged regimes may be crisis prone, dealing with issues that are well beyond the scope of the present paper. Some focus on how limiting exchange rate flexibility may increase risk taking by borrowers and lenders. Others focus on how liberalization and increased capital mobility have

increased exposure to shocks, particularly those creating inconsistencies between a currency peg and other macroeconomic policies (see Agenor and Montiel 1999, and Obstfeld and Rogoff 1996) for further details). Our study is more closely related to papers that have studied the impact of policy maker preferences on the conduct of discretionary policy (see Cukierman and Liviatan 1991, Lohman 1992, and the references therein).

We proceed to formulate a model that explains the stylized fact that exits from long-lasting pegs appear to be accompanied by particularly large falls in economic activity.

2. Model

Our model specification is influenced by a desire for the simplest framework within which to characterize the role of credibility in the design of an optimal monetary policy based on the exchange rate. The model is deliberately stylized in order to highlight the tension between the government and the private sector in policy design rather than the details of the transmission mechanism. The basic framework follows Flood and Marion (1999), a simplified version of Obstfeld (1996).

Output is determined by inflation surprises and real shocks:

$$y = \pi - \pi^e - u$$

where π , π^e are actual and expected inflation, respectively, and u is an adverse productivity shock, $u \sim f(0, \sigma_u^2)$.⁵ We assume purchasing power parity holds, so that domestic inflation equals the rate of domestic currency devaluation for a small open economy (assuming foreign inflation is zero), so that with a pegged exchange rate $\pi = 0$.⁶ The social loss function attaches a penalty to inflation (or deflation), deviations of output from a target, and any realignment of the exchange rate:

$$L = \beta_p \pi^2 + (y - k)^2 + \chi c \quad (1)$$

where $k (>0)$ is the target level of output associated with distortions in the economy⁷, β_p is the relative weight placed by the public on inflation/output losses, i.e. its degree of desired firmness against inflation, χ is the indicator function,

$$\chi = \begin{cases} 1 & \text{if } \pi \neq 0 \\ 0 & \text{if } \pi = 0 \end{cases}$$

and c is the fixed cost associated with any exchange rate realignment.

Policy control is delegated to a monetary authority with the following loss function

$$M = \beta \pi^2 + (y - k)^2 + \chi c, \quad (1')$$

where the authority's inflation/output loss weight or firmness β is initially greater than that of society β_p , i.e. $\beta > \beta_p$. We assume that β and β_p are publically known. We deal with the case in which the equilibrium is time invariant and omit time subscripts.

2.1 Optimal Monetary and Exchange Rate Policy

The monetary authority optimizes by setting the inflation rate -- and the corresponding devaluation or revaluation rate -- that minimizes the expectation of the loss function (1'). As in similar models, the productivity shock and public's expected inflation rate are known by the monetary authority before choosing the level of inflation, while productivity and the inflation rate are not ex ante known by the public. This leads to a trigger rule, conditional on the authority's anti-inflation firmness β :

$$\pi = \begin{cases} \frac{k + \pi^e + u}{1 + \beta} & \text{if } u > \bar{u} \text{ or } \underline{u} > u \\ 0 & \text{if } \bar{u} > u > \underline{u} \end{cases} \quad (2)$$

where $\bar{u} \equiv \sqrt{c(1 + \beta)} - k - \pi^e$, $\underline{u} \equiv -\sqrt{c(1 + \beta)} - k - \pi^e$.

The monetary authority's ex post loss from fixing or changing the exchange rate, respectively, is

$$M_{\beta}^{fix} = (k + \pi^e + u)^2$$

$$M_{\beta}^{dev} = \beta \left(\frac{k + \pi^e + u}{1 + \beta} \right)^2 + \left(\frac{k + \pi^e + u}{1 + \beta} - (k + \pi^e + u) \right)^2 + c = \frac{\beta}{1 + \beta} (k + \pi^e + u)^2 + c \quad (3)$$

where the latter is conditional on the regime's firmness β . Thus, the authority changes the exchange rate only when u is high enough, i.e. $u > \bar{u}$ (in which case the currency is devalued) or low enough, i.e. $u < \underline{u}$ (in which case the currency is revalued), to make $M_{\beta}^{dev} < M_{\beta}^{fix}$.⁸ For shock realizations $\bar{u} > u > \underline{u}$, the fixed exchange rate is maintained.

The expected social loss function corresponding to the optimal inflation rate, given public inflation expectations π^e , and public's anti-inflation weight β_p , is

$$E(L) = \int_{\underline{u}}^{\bar{u}} (k + \pi^e + u)^2 f(u) du + \int_{\bar{u}}^{\infty} \left\{ (k + \pi^e + u)^2 \frac{\beta_p + \beta^2}{(1 + \beta)^2} + c \right\} f(u) du$$

$$+ \int_{-\infty}^{\underline{u}} \left\{ (k + \pi^e + u)^2 \frac{\beta_p + \beta^2}{(1 + \beta)^2} + c \right\} f(u) du \quad (4)$$

As is well known, because expected inflation π^e enters here both in (i) determining the inflation rate the authority chooses conditional on preferring to realign and in (ii) determining the probability of realignment (through \bar{u} , \underline{u}), the possibility of multiple equilibria arises.⁹ Rational expectation equilibrium implies that public sector's expected inflation π^e equals expected inflation $E(\pi)$, where

$$E(\pi) = \int_{-\infty}^{\underline{u}} \frac{k + \pi^e + u}{1 + \beta} f(u) du + \int_{\bar{u}}^{\infty} \frac{k + \pi^e + u}{1 + \beta} f(u) du . \quad (5)$$

To gain further insight, let us consider the case where u follows the double exponential distribution, with zero mean and variance $2/\theta^2$:

$$f(u) = \frac{\theta}{2} \exp[-\theta|u|]. \quad (6)$$

A convenient feature of this distribution is that we can solve for the closed form of (5):¹⁰

$$E(\pi) = \begin{cases} \left[\frac{(k + \pi^e) \{1 - 0.5(\exp[\bar{u}\theta] - \exp[\underline{u}\theta])\}}{2\theta} - \frac{(\bar{u}\theta - 1)\exp[\bar{u}\theta] - (\underline{u}\theta - 1)\exp[\underline{u}\theta]}{2\theta} \right] \frac{1}{1 + \beta} & \text{if } \underline{u} < \bar{u} < 0 \\ \left[\frac{(k + \pi^e) \{0.5(\exp[-\bar{u}\theta] + \exp[\underline{u}\theta])\}}{2\theta} + \frac{(\bar{u}\theta + 1)\exp[-\bar{u}\theta] + (\underline{u}\theta - 1)\exp[\underline{u}\theta]}{2\theta} \right] \frac{1}{1 + \beta} & \text{if } \underline{u} < 0 < \bar{u} \\ \left[\frac{(k + \pi^e) \{1 + 0.5(\exp[-\bar{u}\theta] - \exp[-\underline{u}\theta])\}}{2\theta} + \frac{(\bar{u}\theta + 1)\exp[-\bar{u}\theta] - (\underline{u}\theta + 1)\exp[-\underline{u}\theta]}{2\theta} \right] \frac{1}{1 + \beta} & \text{if } 0 < \underline{u} < \bar{u} \end{cases} \quad (7)$$

The full equilibrium in which $E(\pi) = \pi^e$ is outlined by Figure 1, which graphs expression (7) as an S-shaped curve for different values of β , together with a 45° line. The firmness of the policymaker β plays a key role in determining the equilibrium. Greater firmness (greater β) shifts the S-shaped curve downward to the right. For “tough” regimes (i.e. high β), there is a unique equilibrium, associated with low expected inflation. For “soft” regimes (i.e. low β), we have a unique equilibrium associated with high inflation. For intermediate regimes, we have multiple equilibria, with 2 or 3 possible inflation rates. The Figure illustrates the case of 2 equilibria for $\beta = 0.2$ (points A and D) and 3 equilibria for $\beta = 0.1$ (points A, B, and C).

In the Appendix we show that for β close to β_p the multiple equilibria can be Pareto ranked by the level of expected inflation by proving the following claim:

Claim 1: *Expected social loss $E(L)$ rises with expected inflation π^e .*

Assuming that the multiple equilibria occur with equal probability, it follows that the association between the firmness of the regime and expected inflation is discontinuous in the intermediate range of firmness, and that the move from the multiple equilibrium range to a unique equilibrium of low inflation is associated with a large drop in expected inflation.

This discontinuity implies that there are large potential gains from eliminating the multiple equilibria and is the key for our analysis. It implies that it is desirable to bias the choice of the monetary authority's firmness towards the conservative end of the available pool, not only in order to mitigate the inflation bias arising from time inconsistency, but also to eliminate the excessive expected inflation due to multiple equilibria. Achieving these gains requires picking a monetary authority with a sufficiently high firmness level and hence conservative bias. Such a bias, however, comes also with costs in the form of a lesser responsiveness by the authority to shocks. Hence, there is a trade off between the ex-ante gains from stabilizing expectations and the ex-post costs associated with having a conservative decision maker who is unwilling to allow the exchange rate to adjust in response to very large shocks.

2.2 Costly Regime Change

We complete the model by assuming that ex-post the public has the costly option of replacing the existing monetary regime with one representing the public's preferences, i.e. a monetary authority characterized by $\beta = \beta_p$.¹¹ We denote the cost of regime change by c_{rc} and assume¹²

$$\frac{c_{rc}}{c} > \frac{(\beta - \beta_p)^2}{(1 + \beta)(1 + \beta_p)}, \quad (8)$$

implying the relative cost of regime change exceeds the relative firmness bias of the existing monetary authority. It follows that the inflation rate observed with a monetary authority of type β is

$$\pi = \begin{cases} \frac{k + \pi^e + u}{1 + \beta} & \text{if } \bar{u}_{rc} > u > \bar{u} \text{ or } \underline{u} > u > \underline{u}_{rc} \\ 0 & \text{if } \bar{u} > u > \underline{u} \\ \frac{k + \pi^e + u}{1 + \beta_p} & \text{if } u > \bar{u}_{rc} \text{ or } \underline{u}_{rc} > u \end{cases} \quad (9)$$

where

$$\begin{aligned} \bar{u}_{rc} &\equiv \left(\frac{1 + \beta}{\beta - \beta_p} \right) \sqrt{c_{rc}(1 + \beta_p)} - k - \pi^e; & \underline{u}_{rc} &\equiv - \left(\frac{1 + \beta}{\beta - \beta_p} \right) \sqrt{c_{rc}(1 + \beta_p)} - k - \pi^e \\ \bar{u} &\equiv \sqrt{c(1 + \beta)} - k - \pi^e; & \underline{u} &\equiv - \sqrt{c(1 + \beta)} - k - \pi^e \end{aligned}$$

A monetary authority of type β leaves the exchange rate unchanged for shocks in the range $\bar{u} > u > \underline{u}$. The regime remains in place when u is higher, but not “too” high, i.e. $\bar{u}_{rc} > u > \bar{u}$ (in which case the currency is devalued) or low enough, but not “too” low, i.e. $\underline{u} > u > \underline{u}_{rc}$ (in which case the currency is revalued). In these ranges the authority will devalue (or revalue) by an amount $\pi = (k + \pi^e + u)/(1 + \beta)$. For very large shocks, however, i.e. $u > \bar{u}_{rc}$ or $\underline{u}_{rc} > u$, it is socially optimal to replace the regime as well as realign the exchange rate by setting $\pi_{rc} = (k + \pi^e + u)/(1 + \beta_p)$, which is larger than π since $\beta > \beta_p$.¹³

To understand this behavior, we compare the ex post social loss function (2) for the cases of no devaluation (L^{fix}), devaluation by a policymaker of firmness type β (L_{β}^{dev}), and a regime change and devaluation set by a (new) policymaker of firmness type β_p ($L_{\beta_p}^{rc}$):

$$\begin{aligned}
L^{fix} &= (k + \pi^e + u)^2 \\
L_{\beta}^{dev} &= \beta_p \left(\frac{k + \pi^e + u}{1 + \beta} \right)^2 + \left(\frac{k + \pi^e + u}{1 + \beta} - (k + \pi^e + u) \right)^2 + c \\
&= \frac{\beta_p + \beta^2}{(1 + \beta)^2} (k + \pi^e + u)^2 + c \\
L_{\beta_p}^{rc} &= \beta_p \left(\frac{k + \pi^e + u}{1 + \beta_p} \right)^2 + \left(\frac{k + \pi^e + u}{1 + \beta_p} - (k + \pi^e + u) \right)^2 + c + c_{rc} \\
&= \frac{\beta_p}{1 + \beta_p} (k + \pi^e + u)^2 + c + c_{rc}
\end{aligned} \tag{10}$$

L_{β}^{dev} , the social loss function corresponding to a devaluation by the existing regime of magnitude $\pi = (k + \pi^e + u)/(1 + \beta)$, is the sum of three terms: the inflation cost associated with devaluation (the public cost parameter, β_p , times the square of the amount of devaluation-induced inflation), the cost induced by suboptimal output level, and the direct devaluation cost, c . Analogously, $L_{\beta_p}^{rc}$, the social loss function corresponding to a regime change associated with the greater devaluation rate of $\pi_{rc} = (k + \pi^e + u)/(1 + \beta_p)$ (reflecting the public preferences for a greater devaluation, since $\beta > \beta_p$) is the sum of these three terms evaluated at the higher devaluation rate, plus the extra cost associated with regime change, c_{rc} .

Applying (3), the existing monetary authority will not devalue for u shocks in the range $\bar{u} > u > \underline{u}$, since $M_{\beta}^{dev} > M^{fix}$. We can also show for shocks in this range that the public will not prefer a regime change because the welfare loss of doing so is less than that of maintaining the regime and leaving the exchange rate unchanged, i.e. $L_{\beta_p}^{rc} > L^{fix}$.

To demonstrate this, note from (10) that $L_{\beta_p}^{rc} > L^{fix}$ implies

$$\frac{\beta_p}{1 + \beta_p} (k + \pi^e + u)^2 + c + c_{rc} > (k + \pi^e + u)^2.$$

or, equivalently, the cost of a devaluation induced by a regime change ($c + c_{rc}$) exceeds the decline of the cost of inflation plus output deviations triggered by the devaluation:

$$c + c_{rc} > \left(1 - \frac{\beta_p}{1 + \beta_p}\right) (k + \pi^e + u)^2,$$

which reduces to

$$c + c_{rc} > \frac{1}{1 + \beta_p} (k + \pi^e + u)^2. \quad (11)$$

However, the definitions of \bar{u} , \underline{u} imply

$$-c(1 + \beta) < (k + \pi^e + u)^2 < c(1 + \beta),$$

which combined with (8) implies (11) holds. Thus, for $\bar{u} > u > \underline{u}$, the incumbent monetary authority remains in place and no devaluation occurs, reflecting the “inflation taming” effect of having a relatively tough central bank manager when the cost of regime change is significant.

Suppose now that u is large enough (in absolute value) to induce the policymaker to adjust the exchange rate, but not large enough to prompt a change in regime, e.g. $\bar{u}_{rc} > u > \bar{u}$.

The resultant rate of devaluation is

$$\pi = \frac{k + \pi^e + u}{1 + \beta}, \quad (12)$$

implying a social welfare loss of $L_\beta^{dev} < L^{fix}$. In this case the public continues to support the existing regime as long as the magnitude of devaluation chosen by the policymaker, given by (12), is not viewed as “too timid” a response to the shock and/or if the cost of regime change is not too high, i.e. $L_\beta^{dev} < L_{\beta_p}^{rc}$.

To demonstrate this, note that a regime change entails a devaluation of magnitude

$$\pi_{rc} = \frac{k + \pi^e + u}{1 + \beta_p} \quad (12')$$

and welfare loss of $L_{\beta_p}^{rc}$. Thus, the existing regime is maintained if $L_{\beta_p}^{rc} > L_{\beta}^{dev}$, or applying (10), if

$$\left(\frac{\beta^2 + \beta_p}{(1 + \beta)^2} - \frac{\beta_p}{1 + \beta_p} \right) (k + \pi^e + u)^2 < c_{rc}, \quad (13)$$

or equivalently, if

$$u < \bar{u}_{rc} \equiv \frac{1 + \beta}{\beta - \beta_p} \sqrt{c_{rc}(1 + \beta_p)} - k - \pi^e. \quad (13')$$

Thus for $\bar{u}_{rc} > u > \bar{u}$, the shock u is high enough to induce the policymaker to devalue, but not high enough to prompt a regime change. While the policymaker devalues at a rate that is below the public's desired rate ($\pi_{rc} > \pi$ since $\beta > \beta_p$), the shock is not high enough to induce a regime change. This is the down side of the conservative bias: the policymaker is too timid in the use of discretionary policy. Yet, for this range of shocks, the social cost of regime change still exceeds the marginal benefit to implementing a higher devaluation rate. Only when $u > \bar{u}_{rc}$ is a regime change and a greater magnitude of devaluation desirable. A similar analysis holds for $\underline{u} > u > \underline{u}_{rc}$ and $\underline{u}_{rc} > u$.

Figure 1 illustrates the ex-ante gains associated with conservative bias. It graphs $E(\pi) = \pi^e$ and illustrates the determination of equilibrium expected inflation, for the case where $c = 0.1, k = 0.1, \theta = 7, c_{rc} = 0.06, \beta_p = 0.1$ and the firmness of the monetary authority corresponding to the two S-shaped curves, from top to bottom, is $\beta = 0.1, 0.2$. It is easy to confirm that condition (8) is satisfied for these parameters.

The top S-shaped curve corresponds to the case where the monetary authority anti-inflation firmness equals the public's anti-inflation preference, i.e. $\beta = \beta_p = 0.1$. The rational expectations inflation rate is determined by the intersection of the 45° line and this curve. The outcome in this case is one of multiple equilibria, one of which is a high inflation equilibrium (at point B), with $\pi^e = 100\%$. The simulation reveals that the threshold level associated with the switch from multiple equilibria to a unique equilibrium case is (about) $\beta = 0.2$ (where the S-curve "kisses" the 45° ray from below, as at point C). Note that this switch is associated with a large discontinuous drop in expected inflation. It can be verified that the net effect of switching from a policymaker with firmness level $\beta = \beta_p = 0.1$ (matching the public's desired firmness level) to a policymaker with $\beta = 0.2$ is a large drop in expected social loss, $E[L]$. It is also the case that, as drawn with the S-curve tangent to the 45° ray, choosing the threshold $\beta = 0.2$ with associated inflation at point A is optimal: further increases in the conservative bias would reduce welfare.

To understand the determinants of the optimal level of bias, we note that in general, for a given public anti-inflation preference level β_p , the welfare effect of choosing a more conservative monetary authority type can be expressed as

$$\frac{dE[L]}{d\beta} = \frac{\partial E[L]}{\partial \beta} + \frac{\partial E[L]}{\partial \pi^e} \frac{\partial \pi^e}{\partial \beta} \quad (14)$$

The first term of (14) corresponds to the direct welfare effect of greater conservative bias, holding the expected inflation constant. The second term measures the indirect effect of greater conservative bias through changing expected inflation. In the Appendix we also show that

Claim 2: For a given level of β_p , (i) $\partial E[L]/\partial \beta > 0$, i.e. greater conservative bias increases the expected social loss, holding expected inflation given; (ii) $(\partial E[L]/\partial \pi^e)(\partial \pi^e / \partial \beta) < 0$, i.e. greater conservative bias reduces the expected social loss by reducing expected inflation.

That the expected loss rises with a more conservative regime follows from the property that in circumstances leading to exchange rate adjustment, a more conservative decision maker uses discretion more timidly. A more conservative monetary authority has an opposing effect on expected loss by reducing expected inflation. Consequently, the welfare effect of the conservative bias is ambiguous, being the sum of two opposing effects. Choosing the candidate with the conservative bias sufficiently high enough to eliminate the multiple equilibria would be optimal if the drop in expected inflation (captured by the second term) dominates the first. This will be the case if the discrete drop in expected inflation is large enough, as is the situation in Figure 1. The conservative bias comes, however, with potential ex-post costs: the policy maker may be “too conservative” when bad shocks hit the economy. In these circumstances, a very bad state of nature would induce a regime change, and a large discretionary devaluation.

The optimal degree of policy firmness, β^* , corresponds to the point of tangency between the lower S-shaped curve and the 45° ray (e.g. point C in Figure 1), and is associated with the relatively low expected inflation rate (corresponding to point A).¹⁵ The location of the $E(\pi) = \pi^e$ curves and this equilibrium are perturbed by variations in the costs of devaluation c and of regime change, c_{rc} . From (7) it follows that a higher cost of devaluation, reduces the range where discretionary devaluations take place, thereby shifting the expected inflation curves downward. A similar result applies for higher costs of regime change, c_{rc} . Consequently, a

higher cost of regime change or higher cost associated with devaluation each reduce the optimal conservative bias needed to prevent multiple equilibria. That is,

$$d \pi^e / dc_{rc} < 0, d \beta^* / dc_{rc} < 0; \quad d \pi^e / dc < 0, d \beta^* / dc < 0. \quad (15)$$

We can express this relation in reduced form as: $\beta^* = \beta^*[c, c_{rc}]$; $\partial \beta^* / \partial c < 0, \partial \beta^* / \partial c_{rc} < 0$.

We next turn to the effects of the exit from an exchange rate regime on output. The association between the duration of the regime and the magnitude of the output drop following a devaluation triggered by a regime change is summarized by the following:

Claim 3: *For a pegged exchange rate regime that is maintained until a large enough adverse shock induces a regime change and devaluation, the output decline associated with the change is larger the longer is the duration of the prior pegged regime.*

We prove this claim in several stages. First, we evaluate the factors determining the output decline associated with a shock large enough to lead to regime change. Next, we characterize the factors determining the duration of the peg, and identify the factors impacting both the duration of the peg, and the ultimate cost of the exiting the peg.

Denote by u_{rc} the adverse productivity shock which is large enough to cause a regime change, i.e. $u_{rc} > \bar{u}_{rc}$ (see the discussion after (12)). Applying (12') and the output equation, the resulting regime change is associated with a negative output gap of

$$y_{rc} - k = \pi_{rc} - (k + \pi^e + u_{rc}) = \frac{k + \pi^e + u_{rc}}{1 + \beta_p} - (k + \pi^e + u_{rc}) = -\frac{\beta_p}{1 + \beta_p} (k + \pi^e + u_{rc}) \quad (16)$$

Since $u_{rc} > \bar{u}_{rc}$, it follows that

$$y_{rc} - k \leq -\frac{\beta_p}{1 + \beta_p} (k + \pi^e + \bar{u}_{rc}).$$

Substituting with the definition of \bar{u}_{rc} from (13') implies that¹⁶

$$y_{rc} - k \leq -\frac{\beta_p}{1 + \beta_p} \phi, \text{ where } \phi \equiv \frac{1 + \beta}{\beta - \beta_p} \sqrt{c_{rc}(1 + \beta_p)}. \quad (16')$$

It can be verified readily that $d\phi/dc_{rc} > 0$ and $d\phi/d\beta < 0$, implying that a higher cost of regime change (c_{rc}), or a lower conservative bias ($\beta - \beta_p$), increase the output gap at the time of the regime change.¹⁷

Further insight is gained by evaluating the duration of a peg.¹⁸ Denote the probability of sustaining the peg in each period by

$$\Gamma = \int_{\underline{u}}^{\bar{u}} f(u) du, \quad (17)$$

where, recall from (2), the upper and lower bounds of productivity shocks inducing a devaluation are given by

$$\bar{u} \equiv \sqrt{c(1 + \beta)} - k - \pi^e; \quad \underline{u} \equiv -\sqrt{c(1 + \beta)} - k - \pi^e.$$

The probability of sustaining the peg for exactly j periods is the probability that during the first j periods the real shock lies within the no devaluation zone, $\underline{u} < u < \bar{u}$, and falls outside this zone in period $j+1$: $\Gamma^j(1 - \Gamma)$. Consequently, the expected peg duration, $E[\text{Peg Duration}]$, is the weighted average of possible peg durations of length j ($= 0, 1, 2, 3, \dots$), times the probability of duration j :

$$E[\text{Peg Duration}] \equiv \sum_{j=1}^{\infty} j \Gamma^j (1 - \Gamma) = (1 - \Gamma) \Gamma \left(\sum_{j=1}^{\infty} j \Gamma^{j-1} \right).$$

Note that $\sum_{j=1}^{\infty} j \Gamma^{j-1} = 1/(1 - \Gamma)^2$.¹⁹ Hence, the expected duration of a fixed exchange rate equals

the probability of sustaining the peg, relative to the probability of exiting the peg:

$$E[\text{Peg Duration}] = \Gamma / (1 - \Gamma). \quad (18)$$

Recall from (15) that a higher cost of regime change has the effect of reducing the expected inflation π^e and the optimal conservative bias $\beta^* - \beta_p$. This implies a higher probability of sustaining the peg (since the range $\bar{u} - \underline{u}$ widens), as well as higher level of ϕ (since $d\phi/dc_{rc} > 0$ and $d\phi/d\beta < 0$).²⁰ Hence, a higher cost of regime change implies, on average, a longer duration of the peg, and greater output costs associated with exchange rate change, when a big enough adverse shock occurs.

3. Concluding Remarks

We have presented a model in which the conservative bias of an exchange rate/monetary regime as well the cost of changing an exchange rate regime affects the tradeoff between anti-inflation credibility gains and the ultimate welfare losses costs incurred when exiting the peg. In particular, we have shown that greater conservative bias or higher costs of regime change each reduce expected inflation as long as the regime remains in place. However, the output costs are correspondingly higher once a sufficiently large adverse shock prompts an exit from the regime.

This analysis helps understand the explosive ending of many recent pegged exchange rate regimes, such as that of Argentina in 2001. In our framework the legal anchoring of Argentina's currency board regime through the country's constitution can be interpreted as an effort to raise the cost of devaluation and regime change. While this effort provided obvious anti-inflation credibility gains, it also raised the eventual costs of exiting the peg. It apparently prolonged the duration of the peg, but at a cost of greater loss of output upon the ultimate exit from the peg via a regime change.

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Appendix

A. Claim 1: Expected social welfare loss increases with expected inflation, for $-k < \pi^e$:

Proof: Equation (4) and the envelope theorem imply that for β close to β_p

$$\frac{\partial E(L)}{\partial \pi^e} \cong 2 \int_{-\infty}^{\infty} y(u) f(u) du , \quad (\text{A1})$$

where

$$y(u) = \begin{cases} (k + \pi^e + u) \frac{\beta_p + \beta^2}{(1 + \beta)^2} & \text{for } \bar{u} < u \\ k + \pi^e + u & \text{for } \underline{u} < u < \bar{u} . \\ (k + \pi^e + u) \frac{\beta_p + \beta^2}{(1 + \beta)^2} & \text{for } u < \underline{u} \end{cases} \quad (\text{A2})$$

and \underline{u}, \bar{u} are defined by (2). Inspection of (A2) reveals that $y(u)$ is a piece-wise linear function;

and $|y(u)|$ is a symmetric function of u around $\pi^e = -k$. Recall that $f(u)$ is symmetric around

$u = 0$. Hence,

$$\text{for } \pi^e = -k, \quad \frac{\partial E(L)}{\partial \pi^e} = 0 . \quad (\text{A3})$$

Note also that $\partial^2 E(L) / \partial [\pi^e]^2 > 0$. Consequently, the expected loss function is minimized at

$\pi^e = -k$, and for $\pi^e > -k$, higher expected inflation increases the expected loss function.

B. Claim 2: (i) $\partial E[L] / \partial \beta > 0$, (ii) $(\partial E[L] / \partial \pi^e)(\partial \pi^e / \partial \beta) < 0$

Proof: Recall the definitions of L^{fix} , L_{β}^{dev} , $L_{\beta_p}^{rc}$ in (10) for the social loss in the absence of no devaluation, devaluation by a policy maker of type β , and regime change followed by devaluation set by a policymaker of type β_p , respectively. The expected social loss is

$$E[L] = \int_{-\infty}^{\bar{u}_{rc}} L_{\beta_p}^{rc} f(u) du + \int_{\bar{u}_{rc}}^{\underline{u}} L_{\beta}^{dev} f(u) du + \int_{\underline{u}}^{\bar{u}} L^{fix} f(u) du + \int_{\bar{u}}^{\bar{u}_{rc}} L_{\beta}^{dev} f(u) du + \int_{\bar{u}_{rc}}^{\infty} L_{\beta_p}^{rc} f(u) du \quad (B1)$$

Applying the envelope theorem, it follows that increasing the conservative bias affects the expected loss function by the sum of two terms, reflecting the direct effect of greater bias and the indirect effect through inflation expectations, respectively:

$$\frac{dE[L]}{d\beta} = \frac{\partial E[L]}{\partial \beta} + \frac{\partial E[L]}{\partial \pi^e} \frac{\partial \pi^e}{\partial \beta} . \quad (B2)$$

The first term is given by

$$\begin{aligned} \frac{\partial E[L]}{\partial \beta} = & 2 \left(\int_{\bar{u}}^{\bar{u}_{rc}} (k + \pi^e + u)^2 f(u) du + \int_{\underline{u}_{rc}}^{\underline{u}} (k + \pi^e + u)^2 f(u) du \right) \frac{\beta - \beta_p}{(1 + \beta)^3} \\ & + \frac{1}{2} \sqrt{c(1 + \beta)} c \frac{\beta - \beta_p}{(1 + \beta)^2} (f(\bar{u}) + f(\underline{u})) > 0 \end{aligned} \quad (B3)$$

Hence $\partial E[L]/\partial \beta > 0$, implying that increasing the conservative bias reduces the actual exchange rate adjustment in the range where the policymaker would devalue (or revalue), reducing social welfare.

In order to sign the second term, the impact of β on welfare through changing expectations, note that for β close to β_p

$$\begin{aligned}
\frac{\partial E[L]}{\partial \pi^e} &= 2 \int_u^{\bar{u}} (k + \pi^e + u) f(u) du + 2 \int_u^{\bar{u}_{rc}} \frac{\beta^2 + \beta_p}{(1 + \beta)^2} (k + \pi^e + u) f(u) du \\
&\quad + 2 \int_{\bar{u}_{rc}}^u \frac{\beta^2 + \beta_p}{(1 + \beta)^2} (k + \pi^e + u) f(u) du + 2 \int_{\bar{u}_{rc}}^{\infty} \frac{\beta_p}{1 + \beta_p} (k + \pi^e + u) f(u) du \\
&\quad + 2 \int_{-\infty}^{\bar{u}_{rc}} \frac{\beta_p}{1 + \beta_p} (k + \pi^e + u) f(u) du > 2 \frac{\beta_p}{1 + \beta_p} \int_{-\infty}^{\infty} (k + \pi^e + u) f(u) du \\
&= 2 \frac{\beta_p}{1 + \beta_p} (k + \pi^e) > 0
\end{aligned} \tag{B4}$$

Hence, higher expected inflation is welfare reducing:

$$\frac{\partial E[L]}{\partial \pi^e} > 0. \tag{B5}$$

Consequently, since higher β reduces expected inflation (i.e. $\partial \pi^e / \partial \beta < 0$), the lower expected inflation induced by a greater conservative bias is welfare enhancing:

$$\frac{\partial E[L]}{\partial \pi^e} \frac{\partial \pi^e}{\partial \beta} < 0. \tag{B6}$$

The effect of greater bias depends on the sum of the effects reported by (B3) and (B4) [see (B2)].

Footnotes

¹ For more details, see Aizenman and Glick (2005).

² Klein and Marion (1997) analyze the duration of exchange rate regimes in Latin America. Husain, Mody and Rogoff (2005) study exchange rate regime durability and performance in developing versus advanced economies.

³ Atkeson and Kehoe (2001) show formally that the greater transparency of an exchange rate peg relative to a monetary aggregate target helps reduce credibility problems that arise when the government cannot commit to monetary policies. Ravenna (2005) shows in a dynamic stochastic general equilibrium model that the credibility gain from permanently committing to a fixed exchange rate can outweigh the loss from maintaining an independent monetary policy if the domestic monetary authority does not enjoy full credibility. A threshold level of credibility must be achieved for a central bank to adopt inflation targeting over an exchange rate targeting regime.

⁴ Implementation of an inflation targeting framework requires reasonably well-understood channels between policy instruments and inflation, the relative effectiveness of different monetary instruments, and the lags involved; a methodology to produce inflation forecasts; and a forward-looking operating procedure for the central bank that captures how much the operating instrument (usually some domestic interest rate) should be adjusted in response to deviations of

the inflation forecast (the intermediate target) from the inflation target. These requirements are particularly demanding for emerging market countries.

⁵ The potential or natural level of output is implicitly set to zero.

⁶ The assumption of purchasing power parity (and the absence of nontraded goods) creates a rigid link between the domestic price level and the central bank's policy instrument -- the exchange rate. This allows us to abstract from a more complicated policy transmission mechanism.

⁷ Various reasons have been suggested for the existence of distortions that place the central bank's output target above the market-clearing natural output level: a difference in social and private returns to additional labor supply because of the presence of labor unions, minimum wage laws, or income taxes (e.g. Barro and Gordon 1983); a difference in firm returns to additional output because of imperfect competition; or political pressure on the central bank from the rest of the government to expand economic activity (e.g. Cukierman and Gerlach 2003).

⁸ The assumption that the costs of realignment c are the same for devaluations or revaluations implies that adjustment is symmetric.

⁹ In the absence of realignment costs, i.e. $c = 0$, the analysis reduces to the Barro-Gordon case of a unique equilibrium under policy discretion in which the inflation bias, $\pi = k / \beta$, is proportional to the gap between the central bank's target output rate (k) and the natural rate

(assumed 0), presumed to be positive. The assumption that the target level of output exceeds the natural potential rate has been subject to challenge. For example, McCallum (1995) argues that since, in equilibrium, output equals the natural rate but inflation is greater than optimal, central banks eventually should understand that the output target is unobtainable and revise their output target downward. Blinder (1998) maintains that central banks actually target the natural rate of output, implying that overly expansionary policymakers cannot be the cause of any inflation bias.

While there may be reasons to believe this critique applies to “enlightened” policymakers in industrial countries like the U.S., it seems less likely to apply to developing countries, to which our analysis is most relevant. As Cukierman and Gerlach (2003) argue, as long as monetary policymaking is dominated by political authorities with short horizons and a strong concern for employment and economic activity, the standard inflation bias story seems reasonably realistic.

¹⁰ Obstfeld (1996) assumes u is uniformly distributed; Flood and Marion (1999) show how the results are affected by using a normal distribution, with fatter tails than the uniform, implying extreme shocks are more likely to occur. The uniform distribution has been frequently used due to its analytical tractability in comparison to the normal distribution, a distribution that has more obvious empirical relevance, but requires using numerical simulations. Our assumption of a double exponential distribution may be viewed as a compromise between these two – it permits a tractable analytical solution for a (relatively) fat-tailed distribution.

¹¹ For simplicity, we assume that the new regime is also characterized by exchange rate target bands and rule out the possibility of a purely flexible exchange rate regime with an inflation expectations target.

¹² It will be shown that this condition implies a non-empty range of discretionary devaluations by the conservative policymaker.

¹³ The expected depreciation rate that prevails in the event of regime change is the same as under a completely flexible exchange rate.

¹⁴ Note that $(\beta^2 + \beta_p)/(1 + \beta)^2 - (\beta_p/(1 + \beta_p)) = (\beta - \beta_p)^2 / ((1 + \beta)^2(1 + \beta_p))$ and condition (8), $c_{rc}/c > (\beta - \beta_p)^2 / ((1 + \beta)(1 + \beta_p))$, implies $c(1 + \beta) < c_{rc}(1 + \beta)^2(1 + \beta_p)/(\beta - \beta_p)^2$; hence $\bar{u}_{rc} > \bar{u}$, i.e., there is a non-empty range of discretionary devaluations by the conservative policymaker. Note also that the gap between the public's and the monetary authorities' welfare losses in the event of a devaluation not accompanied by a regime change is proportional to the difference between their respective degree of firmness, since

$$L_{\beta}^{dev} - M_{\beta}^{dev} = \left(\frac{\beta^2 + \beta_p}{(1 + \beta)^2} - \frac{\beta}{1 + \beta} \right) (k + \pi_i^e + u_i)^2 = \frac{\beta - \beta_p}{1 + \beta} (k + \pi_i^e + u_i)^2.$$

¹⁵ The discontinuity of expected inflation associated with this equilibrium implies that the optimal bias must actually exceed β^* marginally in order to induce the low inflation equilibrium. More specifically, recall that because of the knife-edge discontinuity in the effect of β on

expected inflation, for β just below the threshold β^* there are multiple equilibria for expected inflation, while for β marginally above β^* a unique equilibrium with lower expected inflation occurs. This discontinuity also applies to the sign of (14) and the impact of β on expected welfare: for $\beta > \beta^*$ ($\beta < \beta^*$), the impact of greater conservative bias ($dE[L]/d\beta$) is to reduce (increase) the public loss.

¹⁶ Note that the response of output and inflation to productivity shocks is discontinuous around the magnitude of u that triggers a regime change. That is, for $\bar{u} < \bar{u}_{rc}$

$y - k \leq -(\beta/(1+\beta))(k + \pi^e + u)$, while for $u > \bar{u}_{rc}$ the output gap is given by (16).

¹⁷ To determine $d\phi/d\beta < 0$, note that $\partial\{(1+\beta)/(\beta-\beta_p)\}/\partial\beta = -(1+\beta_p)/((\beta-\beta_p)^2) < 0$.

¹⁸ There are four possible cases for exiting from the current currency peg: (i) devaluation without regime change, (ii) revaluation without regime change, (iii) devaluation with regime change, and (iv) revaluation with regime change. The following discussion focuses only on the first two cases; analysis of the other cases is analogous.

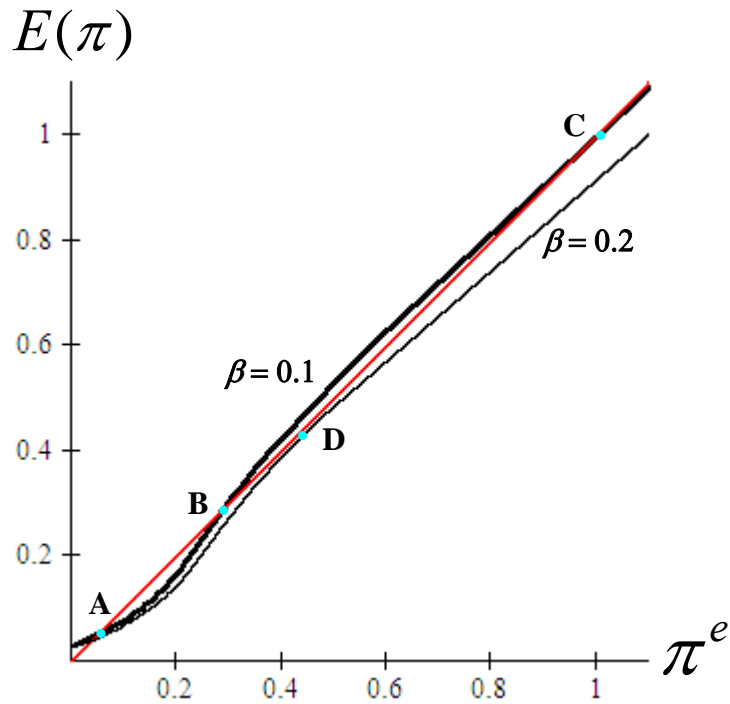
¹⁹ This follows from the observation that

$$\sum_{j=1}^{\infty} j\Gamma^{j-1} = \sum_{j=1}^{\infty} \frac{d\Gamma^j}{d\Gamma} = \frac{d[\sum_{j=1}^{\infty} \Gamma^j]}{d\Gamma} = \frac{d[1/(1-\Gamma)]}{d\Gamma} = \frac{1}{(1-\Gamma)^2}.$$

²⁰ The impact of higher devaluation cost c on the duration of the peg is the sum of two opposing forces: (i) a positive effect through lower expected inflation and an increase in \bar{u} , and (ii) a

negative effect through a lower optimal conservative bias that reduces \bar{u} . It can be shown that the first effect dominates, i.e. a higher devaluation cost c increases the peg duration. A similar result applies for a higher regime change cost.

Figure 1. Expected Inflation and Multiple Equilibria



Notes: Plotted curves assume $\beta_p = 0.1$, $c_{rc} = 0.06$, $c = 0.1$, $k = 0.1$, $\theta = 7$. The top bolded curve determines expected inflation for a monetary authority with firmness $\beta = \beta_p = 0.1$. The bottom curve determines expected inflation for an authority with firmness $\beta = 0.2$ that exceeds the public's preference $\beta_p = 0.1$. The solid straight line from the origin is the 45 degree ray.