On the Intertemporal Elasticity of Substitution under Nonhomothetic Utility *

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Abstract

In this note, we use a model with nonseparable and nonhomothetic preferences to estimate the intertemporal elasticity of substitution (IES). We show that, while the homothetic utility model may induce a bias that increases the elasticity of substitution between nondurables and durables, the estimated IES remains positive and significant.

Key words: Intertemporal Substitution, Consumer Durables, Nonseparable Preferences, Nonhomothetic Preferences

JEL classification: C22, E21

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1 Introduction

As Ogaki and Reinhart (1998) demonstrated, incorporating nonseparability between nondurable and durable goods into a model with homotheticity of preferences yields a more plausible estimate of the intertemporal elasticity of substitution (IES) in consumption. The IES estimates reported in their paper are now recognized as important findings in macroeconomics. However, as Deaton (1992, pp. 8–9) and others have emphasized, the homotheticity assumption contradicts virtually all household budget studies and is inconsistent with time series of expenditure patterns. Thus, the simple question that arises is: does the additional requirement of nonhomotheticity affect the IES estimates?

To answer this question, Okubo (2005) relaxed the homotheticity assumption by extending the addilog-type utility function widely applied in the cointegration literature. However, it does not incorporate completely, as a special case, Ogaki and Reinhart's (1998) homothetic constant elasticity of substitution (CES) utility function. For this reason, it is difficult to compare the empirical results of the two papers and determine the differences between them. The purpose of this note is to reexamine the effect of imposing homotheticity on the IES estimates by exploiting the nonhomothetic utility function introduced by Pakoš (2004), in order to check the robustness of Okubo's (2005) findings, and thus also those of Ogaki and Reinhart. In what follows, unless otherwise stated, the definitions of variables and the data used in this paper follow Ogaki and Reinhart (1998).

2 First-Order Conditions

In Pakoš (2004), the representative consumer's preference is specified as

$$u(C_t, S_t) = [aC_t^{1-1/\epsilon} + S_t^{1-\lambda/\epsilon}]^{1/(1-1/\epsilon)}, \tag{1}$$

where $a > 0$, C_t is the consumption of a nondurable good, S_t is the service flow from purchases of a durable good, D_t, and $\lambda > 0$ is a parameter representing the nonhomotheticity of preferences. As Pakoš (2004) shows, λ is equivalent to the ratio of the income elasticity of the nondurable good to the income elasticity of the service flow from the durable good; therefore, when $\lambda < 1$, the nondurable good is a necessity and the durable good is a luxury. The case of $\lambda = 1$ corresponds to the case of a homothetic CES utility function. In this case, ϵ represents the elasticity of substitution between the nondurable good and the service flow from the durable good.

Supposing that the consumer maximizes the lifetime utility

$$U = \mathrm{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1 - 1/\sigma} \right) u(C_t, S_t)^{1-1/\sigma} \right], \tag{2}$$

the intratemporal first-order condition (FOC) under the preferences implied by (1) is

$$Q_t = \frac{1 - \lambda/\epsilon}{a(1 - 1/\epsilon)} \frac{S_t^{-\lambda/\epsilon}}{C_t^{-1/\epsilon}}, \tag{3}$$

where the user cost for the service flow from the durable good is given by

$$Q_t = P_t - \delta \mathrm{E}_t \left[\beta \frac{mu_{t+1}}{mu_t} P_{t+1} \right], \tag{4}$$

and the marginal utility is defined as

$$mu_t = aC_t^{-1/\epsilon} [aC_t^{1-1/\epsilon} + S_t^{1-\lambda/\epsilon}]^{\frac{1/\epsilon - 1/\sigma}{1-1/\epsilon}}. \tag{5}$$

Note from (4) that the ratio of the user cost for the service flow to the relative price of the durable good, Q_t/P_t, is a function of two stationary variables (the growth rates of marginal utility and the relative price). Taking the logarithm of both sides of (3) and then applying the add-and-subtract strategy about $(\lambda/\epsilon) \ln D_t$ and $\ln P_t$ gives

$$\ln \left(\frac{1 - \lambda/\epsilon}{a(1 - 1/\epsilon)} \right) + \frac{1}{\epsilon} \ln C_t - \frac{\lambda}{\epsilon} \ln D_t - \ln P_t = \ln \left(\frac{Q_t}{P_t} \right) + \frac{\lambda}{\epsilon} \ln \left(\frac{S_t}{D_t} \right). \tag{6}$$

Thus, the intratemporal FOC yields a cointegrating regression

$$\ln D_t = const. - \frac{\epsilon}{\lambda} \ln P_t + \frac{1}{\lambda} \ln C_t + \zeta_t, \tag{7}$$

where ζ_t is a stationary error term. Note that (7) is the same as the cointegrating regression estimated by Okubo (2005), although the interpretation of the coefficients differs between the two regressions. Hence, we can simply use both the parameter estimates and the test results for the null hypothesis of homotheticity in Okubo's (2005) Table 2. That is, the present change of the period utility does not require an additional estimation for the intratemporal FOC. On the other hand, the Euler equation is

$$\mathrm{E}_t \left[\beta \left(\frac{C_{t+1}}{C_t} \right)^{-1/\epsilon} \left(\frac{aC_{t+1}^{1-1/\epsilon} + S_{t+1}^{1-\lambda/\epsilon}}{aC_t^{1-1/\epsilon} + S_t^{1-\lambda/\epsilon}} \right)^{\frac{1/\epsilon - 1/\sigma}{1-1/\epsilon}} R_{t+1} \right] = 1, \tag{8}$$

which is the same as Ogaki and Reinhart's, except that the parameter λ is included in the marginal utility.

3 Estimation and Test Results

Because our main purpose is to make a direct comparison with Ogaki and Reinhart (1998), we concentrate on Gordon's durable goods data. Table 1 reports the implied estimates of ϵ and λ, along with the results from Okubo's (2005) Table 2. The estimates $\lambda = 0.437$ and 0.405 are smaller than Pakoš's (2004) estimate of $\lambda = 0.7056$; however, both estimates imply that durable goods have a higher income elasticity than nondurable goods. The estimates $\epsilon = 0.333$ and 0.284 are somewhat larger than Pakoš's (2004) estimate of $\epsilon = 0.2553$ but are less than one. Thus, the results we have obtained by using Ogaki and Reinhart's data confirm the recent finding that ϵ is smaller than the elasticity of substitution under homothetic preferences.[1]

Table 2 presents the GMM results. Here, we follow Ogaki and Reinhart's (1998) setting precisely: the discount factor and the weight parameter are fixed at $\beta = 0.990$ and $a = 2.691$, and the same instrumental variables and sample period of 1951:1–1983:4 are used.[2] As shown in the first row of Table 2, when $\delta = 0.94$, the estimate of the IES is $\sigma = 0.246$ with a standard error of 0.087. This is similar to Okubo's (2005) estimate of $\sigma = 0.218$ with a standard error of

0.099 but is smaller than Ogaki and Reinhart's (1998) estimate of $\sigma = 0.447$ with a standard error of 0.072. The J-test fails to reject the model even at the 10% significance level. The results for $\delta = 0.92$ and $\delta = 0.96$ are also similar.

4 Discussion

The above results show that, even when preferences are nonhomothetic, σ is estimated to be significant and positive. However, how should we interpret the difference between our estimates and those of Ogaki and Reinhart?

One way to do this may be to draw on their discussion. That is, the elasticity of substitution ϵ accounts for the role of durable goods in intertemporal substitution in that "the point estimates of σ are expected to be lower when the value of ϵ is decreased", as Ogaki and Reinhart (1998, p.1095) discuss. Because ϵ takes a lower value under our nonhomothetic preferences, the resulting estimate of σ decreases. In other words, our results suggest that imposing homotheticity a priori overestimates the downward bias, so that the estimate of σ is corrected in the positive direction more than necessary. However, when λ is not equal to one, we need to note that ϵ does not have the usual interpretation as the elasticity of substitution.

An important implication of our model and its estimation results is that the nondurable good is a necessary good and the durables service is a luxury good, and the budget share for each good can change over time. Therefore, as the consumer becomes richer, if the budget share for the nondurable good declines, then the IES also can change over time. This will cause some deviation from the IES under homothetic preferences. In order to state this, we consider a definition of the IES in terms of total consumption expenditure.

Let $V_t = V(P_{ct}, P_{st}, E_t)$ be the consumer's indirect utility function, defined as the maximized value of the static optimization problem that maximizes the intraperiod utility $U_t = u(C_t, S_t)^{1-1/\sigma}/(1-1/\sigma)$ subject to the budget constraint $E_t = P_{ct}C_t + P_{st}S_t$, where P_{ct} is the price of the nondurable good, P_{st} is the price of the service flow for the durable good (not in terms of the nondurable good), and E_t is total consumption expenditure. Note that because our model is within the class of intertemporally additive preferences by defining the service flow for the durable good, a two-stage budgeting procedure is applicable; therefore, the consumer's intertemporal optimization problem is equivalent to the maximization problem of lifetime indirect utility $\mathrm{E}_0[\sum_{t=0}^{\infty} \beta^t V_t]$ subject to an intertemporal budget constraint. Then the IES is $-V_E/E_t V_{EE}$, as defined by Blundell, Browning, and Meghir (1994), Attanasio and Weber (1995), and Atkeson and Ogaki (1996), where V_E is the partial derivative of V_t with respect to E_t, and V_{EE} is the partial derivative of V_E with respect to E_t (where we drop the t subscripts from V_E and V_{EE} for simplicity).

The IES implied by our model takes the form

$$\sigma_t = -\left\{\omega_{ct}\frac{U_c - U_s U_{sc} U_{ss}^{-1}}{C_t(U_{cc} - U_{cs} U_{sc} U_{ss}^{-1})} + \omega_{st}\frac{U_s - U_c U_{cs} U_{cc}^{-1}}{S_t(U_{ss} - U_{cs} U_{sc} U_{cc}^{-1})}\right\}, \tag{9}$$

where ω_{ct} and ω_{st} are the budget share for the nondurable good and the durables service, respectively, and expressions of the form U_x and U_{xy} are defined as the partial and cross-partial derivatives of U_t in the same fashion as V_E and V_{EE} (where the t subscripts were dropped again for simplicity). See the Appendix for details of the derivation. When preferences are nonhomothetic but separable over goods, the cross-partial derivatives of the form U_{cs} and U_{sc} are zero (or equivalently, $\sigma = \epsilon$ in that case). Then $\sigma_t = \omega_{ct}(-U_c/C_t U_{cc}) + \omega_{st}(-U_s/S_t U_{ss})$; that is, it becomes the budget share-weighted sum of the IES for each good. Atkeson and Ogaki (1996, equation (7)) provide one such example, which is derived from the addilog utility function with subsistence levels of consumption expenditure of the two goods. Browning and Crossley (2000, Corollary 2) consider a much simpler example with a power utility function.

Table 3 reports estimates of the IES for the total consumption expenditure in (9). The estimates of σ for three cases are used. Because $P_{st}/P_{ct} = Q_t$ by definition, P_{st} is calculated as $P_{ct}Q_t$, where Q_t is estimated by applying a vector autoregression with three lags to an approximation of the user cost, as in Ogaki and Reinhart (1998, p.1086). With this series, the budget share is calculated as the ratio of expenditure on each good to total expenditure. As shown in Panel A of Table 3, the result indicates that the value of the IES is stable over the sample period 1951–1983; for example, in the benchmark case of $\delta = 0.94$, it rises from 0.337 in 1951:1 to 0.420 in 1983:1, but its change is not large. This property is not sensitive to changes in the value of δ. In other words, while changes in the budget shares, which are ignored by the homotheticity assumption, can affect the magnitude of the IES, its impact does not appear to be substantial.

As a consequence of using our more general nonhomothetic utility function, evidence for nonseparability across goods became somewhat weak, as mentioned in the previous section. To see this effect, we also calculated the IES under nonhomothetic but separable preferences. The result is reported in Panel B of Table 3, where the IES for each good is given by σ and σ/λ, respectively. In the class of nonhomothetic utility, the difference in the IES between nonseparable and separable preferences appears to be insignificant.

Importantly, in either case, the values of the IES are similar to Ogaki and Reinhart's IES estimates obtained from the homothetic CES utility function. Given that Ogaki and Reinhart's data, used in this note, yields a negative IES estimate under the one-good model (see Table 4 of Ogaki and Reinhart (1998)), our results overall show that incorporating the role of durable goods, regardless of whether preferences are homothetic or nonhomothetic, remains important in explaining the zero and negative IES estimates obtained by Hall (1988).

Appendix

Derivation of equation (9)

From the static optimization problem stated in the text, optimal consumption of the two goods requires that

$$\frac{U_c}{P_c} = \frac{U_s}{P_s}. \tag{A1}$$

This first-order condition implies that the budget shares can be expressed as

$$\begin{aligned} \omega_c &\equiv \frac{P_c C}{E} = \frac{P_c C}{P_c C + P_s S} = \frac{C U_c}{C U_c + S U_s}, \\ \omega_s &\equiv \frac{P_s S}{E} = \frac{P_s S}{P_c C + P_s S} = \frac{S U_s}{C U_c + S U_s}. \end{aligned} \tag{A2}$$

Let $\mathbf{x} = (C, S)'$ be a vector of the two goods, let $\mathbf{U_x} = (U_c, U_s)'$ be a vector of marginal utilities, let $\mathbf{U_{xx}} = \partial \mathbf{U_x} / \partial \mathbf{x}' = \{U_{ij}\}$ be the Hessian matrix, and let $\mathbf{U_{xx}^{-1}}$ be its inverse. By Theorem 1 of Hanoch (1977) and (A2), the consumer's relative risk aversion with respect to E is given by

$$\begin{aligned} \text{RRA} &\equiv -\frac{E V_{EE}}{V_E}, \\ &= -\frac{\mathbf{U_x' x}}{\mathbf{U_x' U_{xx}^{-1} U_x}}, \\ &= -\frac{C U_c + S U_s}{\frac{U_c(U_c U_{ss} - U_s U_{sc}) + U_s(U_s U_{cc} - U_c U_{cs})}{U_{cc} U_{ss} - U_{cs} U_{sc}}}, \\ &= -\frac{1}{\frac{U_c(U_c U_{ss} - U_s U_{sc}) + U_s(U_s U_{cc} - U_c U_{cs})}{(C U_c + S U_s)(U_{cc} U_{ss} - U_{cs} U_{sc})}}, \\ &= -\frac{1}{\omega_c \frac{U_c U_{ss} - U_s U_{sc}}{C(U_{cc} U_{ss} - U_{cs} U_{sc})} + \omega_s \frac{U_s U_{cc} - U_c U_{cs}}{S(U_{cc} U_{ss} - U_{cs} U_{sc})}}, \\ &= -\frac{1}{\omega_c \frac{U_c - U_s U_{sc} U_{ss}^{-1}}{C(U_{cc} - U_{cs} U_{sc} U_{ss}^{-1})} + \omega_s \frac{U_s - U_c U_{cs} U_{cc}^{-1}}{S(U_{ss} - U_{cs} U_{sc} U_{cc}^{-1})}}. \end{aligned} \tag{A3}$$

By definition, the IES is the reciprocal of this relative risk aversion. Finally, the IES under the nonseparable and nonhomothetic utility (1) is obtained by substituting marginal utilities:

$$\begin{aligned}
U_c &= aC^{-1/\epsilon}X(C,S)^{\frac{1-1/\sigma}{1-1/\epsilon}-1}, \\
U_s &= \frac{1-\lambda/\epsilon}{1-1/\epsilon}S^{-\lambda/\epsilon}X(C,S)^{\frac{1-1/\sigma}{1-1/\epsilon}-1}, \\
U_{cc} &= U_c\left[\left(-\frac{1}{\epsilon}\right)C^{-1}+\left(\frac{1-1/\sigma}{1-1/\epsilon}-1\right)(1-1/\epsilon)aC^{-1/\epsilon}X(C,S)^{-1}\right], \qquad \text{(A4)} \\
U_{ss} &= U_s\left[\left(-\frac{\lambda}{\epsilon}\right)S^{-1}+\left(\frac{1-1/\sigma}{1-1/\epsilon}-1\right)(1-\lambda/\epsilon)S^{-\lambda/\epsilon}X(C,S)^{-1}\right], \\
U_{sc} &= U_{cs}=\left(\frac{1-1/\sigma}{1-1/\epsilon}-1\right)(1-\lambda/\epsilon)aC^{-1/\epsilon}S^{-\lambda/\epsilon}X(C,S)^{\frac{1-1/\sigma}{1-1/\epsilon}-2},
\end{aligned}$$

where

$$X(C,S)=aC^{1-1/\epsilon}+S^{1-\lambda/\epsilon}. \qquad \text{(A5)}$$

Footnotes

[1] The finding of relatively low values of ϵ is also consistent with the recent finding of Yogo (2005), who assumes recursive utility and uses a different estimation method.

[2] In principle, the weight parameter a can differ depending on the choice of the period utility. However, because our interest is in how the IES estimates respond to the introduction of nonhomotheticity with other conditions being equal, it is rather convenient to fix this parameter for ease of comparison with the IES estimates from the homothetic CES utility.

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Table 1

Cointegrating Regression Results

ϵ/λ	$1/\lambda$	H(0,1)	H(1,2)	H(1,3)	H(1,4)	K	Implied Estimates ϵ	Implied Estimates λ
A. Sample Period: 1947:2–1983:4								
0.763	2.289	0.185	0.056	0.083	2.489	18.048	0.333	0.437
(0.127)	(0.303)	[0.667]	[0.813]	[0.959]	[0.477]	[0.000]		
B. Sample Period: 1951:1–1983:4								
0.701	2.469	0.041	0.279	0.791	0.993	50.419	0.284	0.405
(0.084)	(0.207)	[0.840]	[0.598]	[0.673]	[0.803]	[0.000]		

Notes: Standard errors are in parentheses. H(0,1) is a χ^2 test statistic for the null hypothesis of the deterministic cointegration restriction. H(1,2), H(1,3), and H(1,4) are χ^2 test statistics for the null hypothesis of stochastic cointegration. K is a χ^2 test statistic for the null hypothesis of homotheticity, $1/\lambda = 1$. P-values are in square brackets. The implied estimates of ϵ and λ are calculated from the values of ϵ/λ and $1/\lambda$: for the 1947:2–1983:4 period, $\lambda = 1/2.289 = 0.437$; $\epsilon = 0.763 \times 0.437 = 0.333$. For the 1951:1–1983:4 period, $\lambda = 1/2.469 = 0.405$; $\epsilon = 0.701 \times 0.405 = 0.284$.

Table 2

Generalized Method of Moments Results

	Sample Period	δ	a	ϵ	λ	β	σ	J_T
(1)	1951:1–1983:4	0.94	2.691	0.284	0.405	0.990	0.246	6.350
							(0.087)	[0.175]
(2)	1951:1–1983:4	0.92	2.691	0.284	0.405	0.990	0.242	6.327
							(0.087)	[0.176]
(3)	1951:1–1983:4	0.96	2.691	0.284	0.405	0.990	0.251	6.379
							(0.083)	[0.173]

Notes: The instrumental variables used are a constant, the realized real interest rate, the growth rate of C_t, the growth rate of C_t/D_t, and the growth rate of real defense expenditures. All instruments are lagged two periods to control for the time aggregation problem. J_T denotes the J-test statistic of the overidentifying restrictions with four degrees of freedom. Standard errors are in parentheses, and p-values are in square brackets.

Table 3

Intertemporal Elasticity of Substitution under Nonhomothetic Utility

	1951:1	1961:1	1971:1	1981:1	1983:1	Mean over 1951–83
A. Nonseparable Preferences						
(1)	0.337	0.382	0.403	0.429	0.420	0.387
(2)	0.344	0.376	0.395	0.415	0.407	0.382
(3)	0.326	0.389	0.413	0.449	0.439	0.393
B. Separable Preferences						
(1)	0.311	0.352	0.371	0.395	0.387	0.357
(2)	0.314	0.344	0.361	0.379	0.372	0.349
(3)	0.304	0.363	0.385	0.419	0.410	0.367

Notes: The intertemporal elasticity of substitution is for total consumption expenditure. The estimates of the curvature parameters used in the calculation are from Table 2, and line numbers (1)–(3) correspond to those of Table 2.