A long-run nonlinear approach to the Fisher effect

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Abstract: We argue that the empirical failure of the Fisher effect found in the literature may be due to the existence of nonlinearities in the long-run relationship between interest rates and inflation. We present evidence that, for the US during the 1960-2004 period, the Fisher relation presents important nonlinearities. We model the long-run nonlinear relationship and find that an ESTR model for the pre-Volcker era and an LSTR model for the post-Volcker era are able to control for nonlinearities and constitute long-run cointegration vectors. Monte Carlo evidence produce support for the hypothesis that nonlinearities may also be responsible for the less than proportional coefficients of inflation usually found in the linear specifications.

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1 Introduction

The Fisher equation that relates nominal interest rates and expected inflation forms part of the core of macroeconomic analysis. A full Fisher Effect in the long run would imply monetary super-neutrality and no money illusion. Also, the Fisher effect would imply that market interest rates are good indicators of inflationary expectations. Despite the theoretical importance of this relationship, empirically it has met with little success. On the one hand, it is difficult to establish a long-run cointegrating relationship between the two variables. On the other hand, the proportional relationship advocated by the Fisher equation usually fails to be found. The less than proportional reaction of the interest rate to changes in expected inflation found in empirical studies has come to be known as the Fisher effect puzzle.

A large body of evidence has been devoted to testing the Fisher effect with varying degrees of success.[1] Since the seminal work of Rose (1988), who finds that the real interest rate is nonstationary for a large set of countries, other studies have applied unit root tests to real interest rate series and cointegration tests between nominal interest rates and inflation (see Mishkin 1992 and Evans and Lewis 1995). Recent studies such as Rapach and Weber (2004) also find that both nominal interest rates and inflation are I(1) but they usually do not move together in the long run. Some other studies using cointegration techniques have found Fisher coefficients close to the expected after-tax adjusted values such as Crowder and Hoffman (1996). Sun and Phillips (2004) make use of a bivariate exact Whittle estimator that allows for the presence of short-memory noise in the data. They find little support for a long-run Fisher equation. Recently, the work of Bierens (2000) suggests the existence of nonlinear common trends between interest rates and inflation. Lanne (2005) also finds that interest rates and inflation share common nonlinear trends, which makes the real interest rate devoid of nonlinearities. This common nonlinear trend is a long-run phenomenon. Authors such as Kapetanios et al (2003), Maki (2005), and Koustas and Lamarche (2005) have also provided evidence that the real interest rate behaves as a stationary process with asymmetric mean reversion. This body of evidence seems to suggest that nonlinearities could explain the apparent failure of the Fisher relation.

In this paper we take this route and suggest that nonlinearities may exist in the equilibrium relationship between nominal interest rates and inflation which may explain the failure of the Fisher hypothesis found in many studies. Contrary to previous studies that assume only nonlinear mean reversion in the Fisher relationship, we argue that the long-run vector may be nonlinear itself due to various reasons discussed in the literature. Using US data for inflation and the nominal Treasury Bill rate, we first show that the linear interest rate-inflation relationship does not constitute a cointegration vector and yields less than proportional inflation coefficients. Using the approach recently proposed by Saikkonen and Choi (2004), we then estimate two different nonlinear long-run specifications based on a Gauss-Newton leads and lags estimator. These are an Exponential and a Logistic Smooth Transition models (ESTR and LSTR). We carry out our analysis for the whole sample and also the pre- and post-Volcker periods and find that the ESTR model for the former and the LSTR model for the latter constitute cointegrating vectors between interest rates and inflation. This suggests that monetary policy may have played a role in explaining nonlinearities. We then show that the linear model suffers from important nonlinearities that are corrected by these two specifications. Finally, we show, using a Monte Carlo experiment, that nonlinearities can be responsible for the findings of inflation coefficients that are less than unity.

The paper is organized as follows. The next section discusses the concept and findings about the Fisher hypothesis and the potential role of nonlinearities. Section 3 presents the econometric specification and results. Section 4 contains the Monte Carlo experiment and Section 5 concludes.

2 The Fisher effect and nonlinearities

The Fisher equation states that the nominal interest rate equals the real interest rate, determined by real factors, plus the expected rate of inflation:

$$i_t^{t+j} = r_t + E_t(\pi_t^{t+j}) . \tag{1}$$

Where i_t^{t+j} is the nominal interest rate earned on an asset between t and $t+j$, where j is the maturity of the asset; r_t is the real interest rate and $E_t(\pi_t^{t+j})$ is the expected inflation at time t for the period of maturity of the asset. Given that we cannot observe expected inflation directly, equation (1) is usually re-written by assuming rational expectations, i.e. that expected inflation equals actual inflation plus a forecast error term (ε_t) distributed as iid ~ N(0, σ^2). Hence if $E_t(\pi_t^{t+j}) = \pi_t^{t+j} + \varepsilon_t$ we have that

$$i_t^{t+j} = r_t + \pi_t^{t+j} + \varepsilon_t . \tag{2}$$

The importance of the Fisher equation for macroeconomic modeling is that it implies monetary super-neutrality as inflation is assumed not to have an impact on the real interest rate that is determined solely by the marginal productivity of capital and the rate of time preference. The equation is also important in terms of information content for monetary policy. If the Fisher relationship holds then nominal interest rates would be an important leading indicator for inflation.

Given that both the interest rate and inflation appear to behave as non-stationary variables at least for small samples, equation (2) is usually tested by means of a cointegrating regression of the nominal interest rate on a constant plus inflation ($i_t^{t+j} = \alpha + \beta\pi_t^{t+j} + \varepsilon_t$). If the variables are cointegrated and the coefficient on inflation (β) equals one, there is support for (2). Another common way of testing for the Fisher effect is by applying a unit root test on the real interest rate obtained as $r_t = i_t^{t+j} - \pi_t^{t+j}$. This is equivalent to imposing β = 1 in the cointegration vector and testing for cointegration with the restricted model.

A large body of evidence has been produced in recent years since the work of Rose (1988), who found that real interest rates appear to be I(1) variables, rejecting the Fisher hypothesis. Recent evidence using unit root and cointegration tests can also be found in Rapach and Webber (2004). The usual finding of non-stationary real interest rates can be related to either a failure of the cointegration hypothesis between nominal interest rates and inflation or a failure of the proportionality hypothesis (i.e. the hypothesis that β = 1).[2]

In recent years, the development of nonlinear methods in time series econometrics has allowed for more flexible specifications of the tests of the Fisher hypothesis. Maki (2005) for the case of Japan and Bajo-Rubio et al (2004) for the case of Spain, for instance, show that the Fisher effect presents a nonlinear speed of adjustment. Taking this asymmetry into account one is able to establish a cointegrating relationship. The work of Million (2004) argues that if monetary authorities follow an opportunistic approach as in

Aksoy et al (2005), then mean reversion of the real interest rate is asymmetric. He then tests for cointegration using a threshold model for the US Treasury Bill and CPI inflation rate and finds support for the existence of long run cointegration with asymmetric adjustment. A similar argument and findings is put forward by Koustas and Lamarche (2005), who test the hypothesis for the G7 countries. These studies are able to explain the failure to find cointegration in previous studies, but cannot address the non-proportionality problem as they still assume a linear cointegration vector between the two variables. It may be the case that the nonlinearities present in the error correction mechanism arise because the true long-run relationship is nonlinear in itself, and hence estimating a linear cointegration relationship would lead to misspecification. Nevertheless, these works point towards a possible source of nonlinearity arising from the impact of monetary policy on the determination of short-term interest rates.

Given the ability of the monetary authority to determine the short run interest rate as reported, among others, by Demiralp and Jordá (2004) and Rudebusch (1995), monetary policy can influence the relationship between interest rates and inflation. With the adoption of explicit or implicit inflation targeting approaches to monetary policy, the reaction of the monetary authority may vary depending on whether inflation is below or above a particular target. This would imply nonlinearities in the monetary policy rule as reported in Kim et al (2005), Dolado et al (2005) and Bec et al (2002). If, for instance, a monetary authority is more worried about high inflation than recessions due to their impact on disinflationary credibility, then we would expect monetary policy to increase interest rates more aggressively when expected inflation is above a certain target than when it is close to or below it. If monetary policy can affect short-term rates for a sustained enough period of time, we would expect that the relation between interest rates and expected inflation contained in the Fisher hypothesis would also be highly nonlinear and coefficient β would depend on the level of inflation. We should also expect that the form of nonlinearity adopted would be affected by institutional changes in monetary policy. An example of this is the change in monetary policy of the Fed before and after 1979 with the Volcker mandate.[3]

Other sources of nonlinearity may be prevalent in the data arising from the existence of transaction costs not uniformly distributed among agents, changes in uncertainty, etc. Bierens (2000), for instance, attributes the existence of nonlinear common trends between interest rates and inflation to the impact of the two oil shocks of the 1970s. A related finding is that of Lanne (2005), who finds that a two regimes model, characterized by high and low inflation, seems to be adequate to describe the nonlinear relationship found for inflation and interest rates in the long-run.

We argue that these potential sources of nonlinearity may be responsible for the poor empirical performance of the Fisher equation. In this paper we adopt a flexible, statistical based, approach to modeling the functional form of the relationship between interest rates and inflation. Our evidence can also shed light about the underlying sources of the failure of the Fisher hypothesis.

3 Specification, estimation and results

We carry out our analysis with quarterly data for the US from 1960:1 to 2004:4. The nominal interest rate is the 3-month Treasury Bill rate. Inflation is the rate of growth of the CPI during the period of maturity of the bond, that is, from time t to $t + 1$ given the quarterly nature of the data. We also carried out all the estimation process using monthly data to check for the robustness of the results to the periodicity of the data. Given that the results were essentially the same, we only report those from quarterly data.[4] We used the whole sample and also two sub-samples corresponding to the pre-Volcker (1960:1-1978:4) and Volcker-Greenspan (1979:1-2004:4) mandates. The data are plotted in **Figure 1**.

<INSERT FIGURE 1 ABOUT HERE>

3.1 Linear cointegration results

We first applied the Ng and Perron (2001) Modified Information Criteria unit root tests to interest rates and inflation to analyze the univariate properties of the time series. The results, not reported here to save space, indicated the presence of a unit root in both series for the three sample periods.[5] We then estimated a linear model for the Fisher equation as explained in the previous section. We tested for cointegration and estimated the cointegration vectors using the Johanssen Maximum Likelihood method.[6] The findings of the trace test for cointegration and the estimated coefficients are reported in **Table 1.** The results indicate no-cointegration and coefficients of inflation that are smaller than one. These results are in line with previous findings in the literature – confirming the Fisher effect puzzle – and also reveal that, in the second sample, the estimated elasticity of the interest rate with respect to expected inflation increased.

<INSERT TABLE 1 ABOUT HERE>

3.2 Specifying the non linear Fisher equation

The implication of the discussion in Section 2 is that there might be a long-run nonlinear equilibrium relation between these variables that would be able to yield a cointegration relationship between interest rates and inflation. As we argued before, these nonlinearities could be attributed to several sources analyzed in the monetary literature.

We assume that those nonlinearities in the Fisher equation can be described adequately by a Logistic or an Exponential Smooth Transition model (LSTR and ESTR respectively).[7] These two models have been largely discussed in the literature (see Granger and Terasvirta, 1993). The advantage of these models is that they have an intuitive appeal as nonlinearities can be attributed to the existence of different relationships between the variables involved depending on whether a transition variable is above or below a threshold. In our case, the transition variable is inflation. The transition between states is assumed to be smooth, with the speed of transition depending on an estimated parameter.

The smooth transition Fisher Equation can be written as:

$$i_t = \alpha + \beta_1 \pi_t + \beta_2 F(\pi_t; \gamma; c)\pi_t \tag{3}$$

Where $F(\bullet)$ is a transition function, γ a transition speed parameter, and c is the location parameter. In the case of an LSTAR model, we have that:

$$F(\pi_t; \gamma, c) = \frac{1}{1 + \exp[-\gamma(\pi_t - c)]} \tag{4}$$

And in the case of an ESTAR model:

$$F(\pi_t; \gamma, c) = 1 - \exp[-\gamma(\pi_t - c)^2] \tag{5}$$

In these two specifications we have that γ represents the speed of transition between the different regimes in the data, and c is a threshold parameter. The LSTR specification implies that the coefficient on inflation would take different values depending on whether inflation is below or above a certain threshold *c*. In this case, as $(\pi_t - c) \rightarrow -\infty$, the coefficient on inflation becomes *β1*; if $(\pi_t - c) \rightarrow +\infty$ then the coefficient on inflation is *β1* + *β2* and if $\pi_t = c$ it becomes $(\beta_1 + \beta_2 / 2)$.[8] This formulation would be compatible with an inflation targeting with credibility explanation if both *β1* and *β2* are positive. This would mean that whenever inflation is beyond a particular level, the reaction of the monetary authority becomes more aggressive, leading to a larger response of interest rates to inflation. When inflation is below the target, however, the monetary authority reaction is milder. In the case of the ESTR model, the coefficient changes depending on whether inflation is close or far away from the target, regardless of whether this difference is positive or negative. In this case we have that as $(\pi_t - c) \rightarrow \pm\infty$ then the coefficient on inflation becomes *β1* + *β2* and if $\pi_t = c$ it becomes *β1*. In this case we would have that the reaction of the monetary authority is equally aggressive for negative or positive deviations from the target, provided that both the estimates of *β1* and *β2* turn out to be positive. This could be the case if the dis-inflationary credibility of the Central Bank does not play such an important role.

Given these two specifications we now proceed to test if they constitute cointegration vectors. We then check our results by testing if the linear model presents nonlinearities and if these are appropriately controlled for by the nonlinear models. Finally, we present the estimated coefficients and their economic interpretation.

3.3 Testing for cointegration

Testing for cointegration in this nonlinear context is problematic since, as Choi and Saikkonen (2004b) have shown, the distribution of the tests under the null hypothesis of either cointegration or no-cointegration will depend on unknown nuisance parameters that are difficult to eliminate. To deal effectively with this issue we adopt a residual-based test for the null of cointegration due to Choi and Saikkonen (2004b).

To test for cointegration in a nonlinear context Choi and Saikkonen (2004b) propose a test for the null hypothesis of nonlinear cointegration in the context of smooth transition models that follows Kwiatkowski et al (1992) (KPSS hereafter). They develop this test for the case of nonlinear least squares estimators. The Choi and Saikkonen (2004b) test is a KPSS variance ratio test for the null of cointegration that uses sub-sample residuals. They show that tests using full residuals have limiting distributions that depend on unknown parameters. They propose to use sub-residuals from the leads and lags nonlinear least squares estimation[9] with block size *b* and select the one that yields the maximum value of the cointegration test ($C_{LL}^{b,\max}$) . The selection of the block size can be done by using a fixed rule or a minimum volatility rule.

The latter consists of choosing b so as to minimize the standard deviation of $C_{LL}^{b,\max}$ for each value of b from $b_i = b_{small}$ to $b_i = b_{big}$. **Table 2** reports the p-values of the Choi-Saikkonen test using the minimum volatility rule together with the selected block size for the estimated LSTR and ESTR models. The null hypothesis is cointegration, so we would not be able to reject cointegration for high p-values. Both the LSTR and ESTR models appear to form a nonlinear cointegration equation in the whole sample and in the two sub-periods. The null hypothesis cannot be rejected at 5% or lower. Therefore we have strong evidence in favor of cointegration. Nevertheless, the evidence is stronger for the LSTR model for the whole sample and the 1979:1-2004:4 sub-sample and for the ESTR model for the 1960:1-1978:1 sub-sample. This, as we shall see later, is compatible with the interpretation of changes in US monetary policy after 1979.

<INSERT TABLE 2 ABOUT HERE>

3.4 Testing for nonlinearity

The cointegration results in the previous sections show that the linear model is not an equilibrium relationship for interest rates and inflation. However, the two nonlinear models form cointegrated relationships according to our priors. We now provide evidence to show that the lack of cointegration in the linear model is due to neglected nonlinearity and that this nonlinearity is controlled for by the LSTR and ESTR models. In order to test for nonlinearity, we used a flexible approach to nonlinear modeling as advocated in Hamilton (2001). The procedure we will follow consists of two steps. We first test for nonlinearity in the linear model. Then, we test for nonlinearities in the nonlinear models and check if the results are consistent with those of the cointegration tests.

The Hamilton (2001) approach to nonlinear modeling is appropriate in our context as it does not impose any specific nonlinear functional form. In our case, the Hamilton (2001) model can be represented by the following nonlinear regression model (we remove superscripts in what follows):

$$i_t = \mu(\pi_t) + \vartheta_t \tag{6}$$

Where $\mu(\bullet)$ is a nonlinear functional form that we treat as unknown, and ϑ is an iid (0, σ_ϑ) error term. In our particular case, given that our model is bi-variate, we do not allow any variable to enter the relationship linearly. The function $\mu(\bullet)$ is viewed as the outcome of a random field. When π_t equals an arbitrary value υ the function is treated as a Gaussian random variable with mean $\alpha + \beta\upsilon$ and with variance λ^2. Hence, when $\lambda = 0$ then the function does not vary and we have a linear model. Hence, a test of $\lambda^2 = 0$ is a test of linearity.

The parametric approach developed in Hamilton (2001) is based on the idea that, if nonlinearities are present, then the values of $\mu(\pi_t)$ and $\mu(\pi_s)$ are positively correlated when π_t and π_s are close for periods t and s. This correlation is then parameterized by making use of the distance measure $h_{st} = (1/2)\left[g(\pi_s - \pi_t)\right]$, where g is an additional parameter to estimate and determines the extent to which inflation contributes towards the nonlinear variation of $\mu(\bullet)$. In our case, a test $g = 0$ is also a test of linearity as there is only one variable that can contribute to the nonlinearity of the function.[10]

Hamilton develops a Lagrange multiplier (LM) test for the null hypothesis of nonlinearity $H_0: \lambda^2 = 0$, which we will call λ^H. Given that this test depends on a nuisance parameter, g, and a particular specification of the variance-covariance function, Dahl and González-Rivera (2003) propose a new set of LM tests for testing the null hypothesis that $\lambda^2 = 0$ and also $g = 0$. These tests overcome the shortcomings of the Hamilton test and perform better in terms of power under some general forms of nonlinear DGPs. We will call these tests λ_E^{DGR}, λ_A^{DGR}, and g^{DGR} respectively. The first one assumes full knowledge of the covariance matrix associated with the random field, whereas λ_A^{DGR} does not as it is based on a higher order Volterra series matrix of the covariance function. The g^{DGR} test is a direct test of the hypothesis $g = 0$. These tests are all distributed asymptotically as standard chi-squared statistics, although Dahl and González-Rivera (2003) recommend the use of bootstrapped critical values as they perform better in terms of size distortions.

The results of the nonlinearity tests are reported in **Table 3.** All the tests, save for the g^{DGR}, show that for the period 1960-2004 the null hypothesis of linearity can be rejected in favor of the alternative of nonlinearity at conventional significance levels. The results remain qualitatively similar when we break up the sample into two sub-periods namely 1960:01-1978:04 and 1979:01-2004:04. In that case all the tests reject the null of linearity at the 10% level, and all but one reject the null at the 5% level.

<INSERT TABLE 3 ABOUT HERE>

In order to complement these results, we also carried out nonlinearity tests that are especially designed for relationships in which the regressors involved are I(1). We used the Choi and Saikkonen (2004a) LM test using inflation as transition variable. The results, in the last row of **Table 3**, confirm the results from the random fields parametric tests. Therefore we can conclude that the evidence strongly supports the idea that the long-run Fisher equation presents important nonlinearities.

We then applied the Hamilton (2001) and Dahl and González-Rivera (2003) nonlinearity tests on the two nonlinear specifications. The null hypothesis can now be interpreted as testing if the functional form chosen successfully removes nonlinearity. It is straightforward to adapt this test for the case of our three functional forms as suggested by Hamilton (2001).[11] The results in **Table**

4 reveal that the ESTR model is only able to remove nonlinearities in the first sub-sample, but clearly not in the other two periods. This is in line with the Choi-Saikkonen cointegration tests. The LSTR model removes nonlinearities in both the whole sample and the second sub-period, which also supports the results of the Choi-Saikkonen cointegration test. According to this criterion, the LSTR model for the whole and second samples and the ESTR model for the first sub-sample seem to be the preferred choices. Thus, we can conclude that the lack of linear cointegration may be due to neglected nonlinearity that is appropriately modeled by an ESTR model in the pre-Volcker period and an LSTR model in the post-Volcker period. These results support the evidence from the cointegration tests.

<INSERT TABLE 4 ABOUT HERE>

3.5 Estimating the nonlinear cointegrating vector

Having established the existence of nonlinear cointegration among the variables involved, we now proceed to present the results of the estimation of the nonlinear version of the Fisher equation. The presence of I(1) variables poses non-trivial problems for estimating the nonlinear equilibrium vector. For the case of smooth transition models, Saikkonen and Choi (2004) develop methods for estimating nonlinear cointegrating vectors. In particular, Saikkonen and Choi (2004) show that traditional Nonlinear Least Squares (NLLS) estimators are consistent but their asymptotic distribution is biased when there is regression-error dependence and hence is inefficient and not suitable for hypothesis testing. For this reason, Saikkonen and Choi (2004) propose using a Gauss-Newton estimator that uses the NLLS estimator as an initial estimator and is based on regressions augmented by leads and lags.[12] The advantage of this estimation method is that it is more efficient than the NLLS estimator and is suitable for hypothesis testing.

The estimation results are reported in **Table 5.** Although it appears that several of the estimated parameters are not significant, as Saikkonen and Choi (2004) have pointed out, we cannot use conventional hypothesis testing for the parameters inducing nonlinearity due to identification issues. The estimates of the LSTR and ESTR models show a variety of results. First, the models seem to behave quite differently depending on the sub-sample of estimation. The implied inflation coefficients for different values of inflation seem to be implausible in the case of the LSTR model for the whole sample and the first sub-sample. This is because it would imply that as inflation increases the coefficient becomes smaller. A similar case happens with all the ESTR models. As explained in Section 3.2., the hypothesis that monetary policy drives the nonlinearities requires positive β_1 and β_2 coefficients. The transition speed for the ESTR model in the first sub-sample is also very low. Nevertheless, the estimated inflation threshold falls substantially from the first to the second sub-sample. This is consistent with the fact that monetary policy became more concerned with price stability and settled for a lower implicit inflation target.

<INSERT TABLE 5 ABOUT HERE>

The results of the LSTR model for the 1979:1-2004:4 sub-sample reveal a different picture. The estimates imply that, for this period, as inflation grows beyond a threshold of 3.12% per year, the coefficient on inflation becomes larger and it is 1.1 for high values of the inflation rate. When inflation equals 3.12%, the coefficient is 0.731, and when inflation is well below the threshold the coefficient is 0.35. These results would be supportive of the explanation of nonlinearity based on the argument that monetary policy targets inflation and cares more about inflation than recessions for the Volcker-Greenspan era. The speed of transition is quite high, which explains why a threshold model behaves quite similarly to the LSTR.

Given that the cointegration and nonlinearity results favored the ESTR model for the first sample and the LSTR model for the second, the picture that emerges from our estimation results can be revealing an important change in the behavior of the monetary authority. In the pre-Volcker period monetary policy is less aggressive against inflation and only changes very slowly with the level of inflation. It is not clear, however, that the estimated nonlinear Fisher effect for this period reflects the influence of monetary policy. In the post-1979 period the Fisher effect seems to hold only when inflation surpasses a threshold of 3.12%. Below this target, interest rates respond less than one-to-one to changes in inflation. This evidence is supportive of the inflation targeting with credibility argument explaining the poor performance of the linear Fisher equation. **Figure 2** plots the implied β coefficient of the Fisher effect against inflation for the ESTR model for the first sub-sample and the LSTR model for the second sub-sample. As commented above, it is clear that for the Volcker-Greenspan era the Fisher coefficient appears to be very sensitive to the inflation rate. Once the 3.12% threshold is surpassed, the coefficient increases rapidly to around 1.1. The ESTR specification, however, shows a slower speed of adjustment around the 5.6% threshold. However, the nonlinearity shows the opposite direction to the one expected if monetary policy was responsible for this pattern.

<INSERT FIGURE 2 ABOUT HERE>

4 Can we account for the puzzle? Simulation evidence

The results in the previous section clearly indicate that the Fisher equation hides important nonlinearities and that these can be successfully modeled with the nonlinear specifications proposed. Some of these models remove nonlinearities and appear to form cointegration relations in the long-run. This can account for the failure of the linear model to find cointegration. Another important aspect to address is whether the less than proportional impact of inflation on interest rates found in the literature may be explained by the existence of these nonlinearities. Hence, we propose to analyze, using a simulation exercise, whether the nonlinear models estimated in the previous section would generate linear estimates of the inflation coefficient that are less than one. If this is the case, this would be evidence in favor of the hypothesis that nonlinearities are responsible for this empirical puzzle.

For each one of the sample periods we generated a random walk variable x_t with zero starting value of sample size T + 100. We then obtained a variable y_t from the following DGP:

$$y_t = h(x_t) + \omega_t \tag{7}$$

where $h(x_t)$ represents one of the nonlinear functional forms obtained from the estimates in **Table 5** and ω_t is an iid random error with zero mean and constant variance. This implies that x_t and y_t are cointegrated through a nonlinear cointegration vector. We then discarded the first 100 observations to eliminate the effect of the initial condition and estimated by OLS a linear equation as

$$y_t = \alpha + \beta x_t + \omega_t\,. \tag{8}$$

We repeated this process in 10,000 Monte Carlo draws and obtained the mean value of β, its standard deviation, and the 5th and 95th percentiles of the distribution of the estimated β values.

The results of the simulation appear in **Table 6.** For the whole sample the ESTR model is the one that is able to reproduce more accurately the results from the linear estimation in **Table 1**. The point estimate was 0.792 and the mean value of β in the simulation is 0.746. For the first sub-period both the LSTR and ESTR models reproduce very closely the results of the linear model. The point estimate was 0.400 and the average β is 0.450 for the LSTR model and 0.457 for the ESTR. In both cases the point estimate falls within the 5th and 95th percentiles of the β distribution.

<INSERT TABLE 6 ABOUT HERE>

For the Volcker-Greenspan period the LSTR model produces linear estimates of β that are very close to those obtained in the linear specification. The linear estimate was 0.778, whereas the mean β from the LSTR model is 0.716. The ESTR model, however, produces β coefficients that are on average larger than those of the linear model for this sub-period. This result lends further support for the fact that, especially for the Volcker-Greenspan era, nonlinearities can explain the failure to find proportional Fisher coefficients.

5 Conclusions

The Fisher effect has found only limited support from empirical studies. Many studies fail to find a long-run relationship between nominal interest rates and inflation. Others find no support for the proportionality hypothesis. This fact would imply a violation of long-run super-neutrality and the existence of money illusion that has come to be known as the Fisher Effect puzzle. We hypothesize that this empirical failure may be due to the existence of nonlinearities in the Fisher equation. There are both, theoretical arguments and statistical evidence that, a priori, point towards the existence of these nonlinearities.

Our results clearly reject the hypothesis that the long-run relationship between nominal interest rates and expected inflation is linear for the US economy between 1960 and 2004. Accordingly, we estimated two long-run nonlinear functional forms based on the approach advocated by Choi and Saikkonen (2004b) and Saikkonen and Choi (2004). We specified a logistic and an exponential smooth transition models (LSTR and ESTR). We show that the ESTR model for the pre-Volcker period and the LSTR model for the post-Volcker period are able to account for nonlinearities and constitute long-run cointegration vectors. The success of the LSTR model in the Volcker-Greenspan period indicates that the Fisher effect almost fully holds whenever inflation increases beyond 3%. This sheds some support for the hypothesis that this behavior is due to the impact of the monetary authority targeting inflation in an asymmetric way to keep dis-inflationary credibility. The results for the pre-Volcker period are, however, not supportive of the monetary policy hypothesis.

The results from a Monte Carlo experiment also produce support for the hypothesis that nonlinearities may also be responsible for the less than proportional coefficients of inflation usually found in the linear specifications of the Fisher equation. The ESTR model explains the puzzle well for the whole sample and the first sub-sample. The LSTR model appears to explain the puzzle better in the second period. This evidence may be interpreted as further support for the argument of monetary policy influencing estimates of the Fisher effect. However, this is just a conjecture, as further work is necessary to discriminate between different theoretical explanations underlying the nonlinearities present in the Fisher effect.

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[1] See Cooray (2002) for a survey of the empirical literature.

[2] The results from cointegration tests of the Fisher hypothesis are more mixed. Crowder and Hoffman (1996) and Atkins and Coe (2002), amongst others, find supporting evidence in favor of the Fisher effect, whereas Koustas and Serletis (1999) and Sun and Phillips (2004) find little support.

[3] See Dolado et al (2005) for a discussion of the two monetary policy regimes in the US.

[4] Results available from the authors on request.

[5] The results for inflation in the 1960-1978 sample are not as clear-cut as the DF and GLS-DF tests did not reject the null of a unit root, but the modified Phillips-Perron tests did. Results available.

[6] Results from using a two-step Engle-Granger method yielded similar results.

[7] We also used a threshold model where the transition between states is immediate. The results of this model were very similar to those of the LSTAR model and we do not report them here.

[8] The standard STR model would use π_{t-1} as the pre-determined transition variable. However, in our context, since we are concerned with long-run equilibrium nonlinear relationships, a

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lagged variable cannot appear in the (potentially) cointegrating space. Hence, we follow Saikkonen and Choi (2004) and specify the transition variable as π_t.

[9] This estimator is explained in Section 3.5.

[10] We refer the reader to Hamilton (2001) for further details and the extension to a context with more than 2 variables.

[11] Hamilton (2001) refers to this strategy as a three-step procedure of first testing for nonlinearity in the linear model, then suggesting a functional form and finally testing if this selected form eliminates the nonlinearity.

[12] This is similar to the popular Dynamic OLS estimator of Stock and Watson (1993) for linear models.

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TABLES

Table 1: Linear cointegration model

	α	β	Trace test {lag}
1960:1-2004:4	2.21 (1.98)	0.792 (4.56)	11.010 [0.351] {9}
1960:1-1978:4	3.23 (7.47)	0.400 (4.76)	7.221 [0.693] {16}
1979:1-2004:4	2.00 (1.83)	0.778 (3.50)	11.891 [0.281] {15}

Notes: Number of lags was selected using the AIC criterion. T-ratios in parenthesis, p-values in brackets and the lag order of the Johansen cointegration tests in {}'s.

Table 2: Choi-Saikkonen $C_{LL}^{b,\max}$ test. Minimum volatility rule.

	b	p-value
	LSTR	
1960:1-2004:4	64	0.291
1960:1-1978:4	38	0.091
1979:1-2004:4	58	0.341
	ESTR	
1960:1-2004:4	62	0.093
1960:1-1978:4	47	0.316
1979:1-2004:4	29	0.072

Notes: the null hypothesis is cointegration. b is the block size of the residuals obtained using the minimum volatility rule.

Table 3: Hamilton – DGR and Choi and Saikkonen tests for nonlinearity.

	Hamilton – DGR test		
	1960:1-2004:4	1960:1-1978:1	1979:1-2004:4
λ_A^H	16.06 [0.005]	14.91 [0.005]	52.06 [0.001]
λ_A^{DGR}	15.17 [0.008]	39.98 [0.001]	74.64 [0.001]
λ_E^{DGR}	6.29 [0.007]	3.63 [0.077]	13.09 [0.001]
g^{DGR}	0.44 [0.907]	3.141 [0.002]	4.81 [0.001]
	Choi and Saikkonen test		
	1960:1-2004:12	1960:1-1978:12	1979:1-2004:12
LM	12.25 (0.001)	37.23 (0.001)	29.30 (0.001)

Notes: Asymptotic p-values in parenthesis, bootstrapped p-values in brackets. The bootstrapped p-values were obtained using 2,500 bootstrap samples. LM denotes the Lagrange Multiplier test. This test is distributed as a chi-square statistic under the null.

Table 4: Nonlinearity tests for the nonlinear models

	1960:1-2004:4	1960:1-1978:1	1978:1-2004:4
	LSTR Model		
λ_A^H	0.229 [0.598]	1.249 [0.138]	0.127 [0.701]
λ_A^{DGR}	1.666 [0.651]	3.686 [0.034]	1.498 [0.656]
λ_E^{DGR}	1.741 [0.588]	4.279 [0.037]	1.587 [0.553]
g^{DGR}	1.632 [0.540]	1.817 [0.069]	1.295 [0.921]
	ESTR Model		
λ_A^H	2.485 [0.083]	0.022 [0.889]	14.93 [0.003]
λ_A^{DGR}	2.761 [0.095]	2.155 [0.610]	8.406 [0.001]
λ_E^{DGR}	3.914 [0.009]	2.045 [0.531]	0.049 [0.980]
g^{DGR}	1.950 [0.099]	1.609 [0.453]	1.899 [0.337]

Notes: Asymptotic p-values in parenthesis, bootstrapped p-values in brackets. The bootstrapped p-values were obtained using 2,500 bootstrap samples.

Table 5: Smooth transition model estimates. Two-step Gauss-Newton estimator.

	α	β_1	β_2	γ	c
	LSTR				
1960:1-2004:4	1.430 (1.755)	1.182 (1.677)	-0.402 (-0.716)	2.855 (0.269)	5.567 (3.954)
1960:1-1978:4	2.081 (1.691)	0.768 (0.335)	-0.376 (-0.126)	0.434 (0.245)	7.310 (0.436)
1979:1-2004:4	2.701 (1.411)	0.350 (2.243)	0.762 (1.950)	5.070 (2.601)	3.121 (3.231)
	ESTR				
1960:1-2004:4	2.113 (2.141)	1.202 (2.717)	-0.461 (-1.147)	1.564 (0.400)	4.062 (7.140)
1960:1-1978:4	2.735 (7.188)	0.582 (3.983)	-0.147 (-1.102)	0.037 (0.003)	5.857 (10.808)
1979:1-2004:4	1.765 (1.472)	1.482 (3.053)	-0.574 (-1.302)	1.331 (0.321)	4.168 (7.495)

Notes: The estimator used was the two-step Gauss-Newton estimator of Saikkonen and Choi (2004). Number of lags and leads were selected using the AIC criterion. t-ratios in parenthesis.

Table 6: Simulation results: linear estimation with nonlinear DGP

	LSTR model		ESTR model	
	$\bar{\beta}$ (s.e.)	$\bar{\beta}$ percentiles {5%, 95%}	$\bar{\beta}$ (s.e.)	$\bar{\beta}$ percentiles {5%, 95%}
1960:1-2004:4	0.999 (0.172)	{0.785, 1.202}	0.746 (0.035)	{0.709, 0.787}
1960:1-1978:4	0.450 (0.146)	{0.356, 0.620}	0.457 (0.070)	{0.375, 0.549}
1978:4-2004:4	0.716 (0.138)	{0.440, 0.911}	0.917 (0.068)	{0.848, 0.993}

Notes: $\bar{\beta}$ is the mean value of the estimated β. Standard errors in parenthesis. The results were obtained by estimating the linear model with data simulated from a DGP that follows the nonlinear models (3)-(5) assuming that inflation is a non-stationary series with initial value set to zero. The number of observations was set to 100 + T (the sample size) and we then discarded the first 100 observations so we eliminated the impact of the assumption on the initial observation. We used 10,000 Monte Carlo draws in each experiment.

FIGURES

Figure 1: Treasury-Bill Rates and Inflation

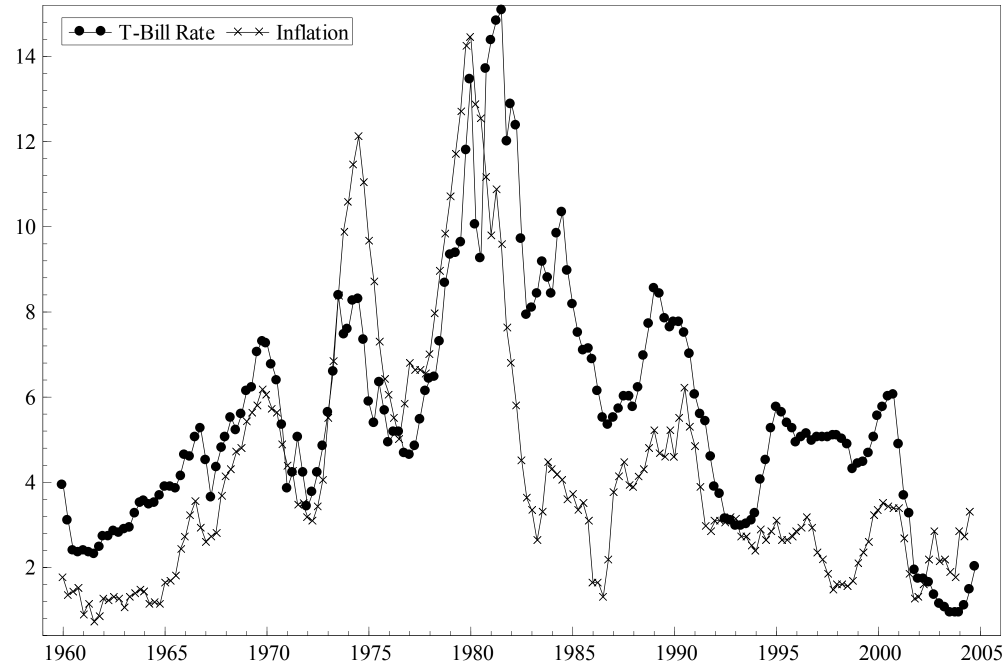

Figure 2: Beta (β) coefficients: LSTR and ESTR models