Structural Error Correction Models: A System Method for Linear Rational Expectations Models and an Application to an Exchange Rate Model*

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Abstract

This paper develops a system instrumental variable method to estimate the speed of adjustment coefficient in the long-run equilibrium of Structural Error Correction Models for a class of linear rational expectations models. This method is applied to an exchange rate model with sticky prices, in which the speed of adjustment coefficient governs the half-life of the real exchange rate. Compared to single equation methods, the system method gives smaller half-life estimates with sharper standard errors.

JEL Classification: C32, F31, F41

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1 Introduction

As Rogoff (1996) pointed out, there had been a remarkable consensus on 3 to 5 year half-life estimates of the real exchange rates obtained by single equation methods. This consensus may at first seem to support the reliability of these estimates, but Kilian and Zha (2002) and Murray and Papell (2002) showed that these estimators had very large standard errors. Therefore the remarkable consensus seems to arise from the correlation of single equation method estimators. As shown by Choi, Mark, and Sul (2005) and Murray and Papell (2005), one way to sharpen the standard error is to use panel data with the homogeneity restriction. It is hard to believe, however, that this restriction is true, because factors such as menu costs that cause gradual adjustment of real exchange rates are likely to be different across countries. In this paper, we develop an alternative method of adding information from monetary variables to a single equation method, using an economic model of exchange rate determination. Our empirical results from this system method yields half-life estimates that range from 0.14 to 0.98 year, much shorter than the established consensus of 3-5 year half-lives. Sharper standard errors from the system method, as compared to those from the single equation method, indicate that monetary variables contain information about half-lives of the real exchange rates.

The system method developed in this paper combines a single equation method with Hansen and Sargent's (1982) method, which applies Hansen's (1982) Generalized Method of Moments (GMM) to linear rational expectations models. The system method makes use of information from the real exchange rate and monetary variables. In the single equation method, either OLS or an Instrumental Variables (IV) method is applied to an autoregression of the real exchange rate. Our system method is based on relationships of the half-life of the exchange rate with monetary variables implied by an exchange rate model with sticky prices. The model is a one-good version of Mussa's (1982) model, which may be viewed as a stochastic discrete time version of Dornbusch's (1976) model.[1] This model includes a gradual adjustment equation, in which the domestic price adjusts to the long-run equilibrium level determined by Purchasing Power Parity (PPP) with rational expectations. We refer to the speed of adjustment coefficient for this equation as the structural speed of adjustment coefficient.

The system method in our paper can be applied to a class of structural Error Correction Models (ECM). This method focuses on the estimation of the speed of adjustment towards the long-run equilibrium level. As Engle and Granger (1987) shows, an ECM representation exists when the variables are cointegrated, and vice versa.[2] Davidson, Hendry, Srba, and Yeo's (1978) ECM is widely used to estimate dynamic cointegrated systems. As Urbain (1992) and Boswijk (1994, 1995) have pointed out, however, the standard ECMs are reduced form models just like VAR models.

An important issue for the structural ECM is the estimation of the speed of adjustment towards the long-run equilibrium level.[3] In a structural ECM, at least one linear combination of variables gradually adjusts to the long-run equilibrium level with a constant speed of adjustment. In general, the speed of adjustment coefficient in a structural ECM is different from the speed of adjustment coefficient in its reduced form ECM. The structural speed of adjustment coefficient in our application can be used to compute the half-life of the real exchange rate, and is a parameter of interest. However, since the reduced form speed of adjustment is a nonlinear function of the structural speed of adjustment and the interest elasticity of money demand, it cannot be directly compared with the half-life estimates of real exchange rates in the literature.

Structural ECMs have been considered by several authors. Urbain (1992) investigates sufficient conditions for weak exogeneity for structural ECMs. Boswijk (1994,1995) and Hsiao (1997) discuss the relationship between the ECM and structural simultaneous equations models. However, unlike Urbain and Hsiao, we do not assume that exogenous variables are observed by the econometrician. Papell (1997) derives a reduced form ECM from an exchange rate model. Dolado, Galbraith and Banerjee (1991) and Gregory, Pagan, and Smith (1993) derive structural ECMs from linear quadratic models. They do not combine their method with Hansen and Sargent's (1982) IV method for linear rational expectations models.

Even though Hansen and Sargent's (1982) method was developed for the stationary case, it is possible to use their method for cointegrated systems as does West (1987). Just as in the stationary case discussed by Hansen and Sargent, there are some principal virtues of the IV estimators vis-à-vis the Maximum Likelihood (ML) estimators. First, fewer parameters need to be estimated simultaneously than are required with the ML estimators. Second, precise parameterizations of the disturbances need not be specified with the estimators present here, while they must be with the ML estimators. Finally, the forcing variable in the present paper need not be strictly exogenous but it should be with the ML approach.[4]

2 Structural Models and Error Correction Models

Let $\Delta \mathbf{y}_t$ be an n-dimensional vector of first difference stationary random variables. We assume that there exist ρ linearly independent cointegrating vectors so that $\mathbf{A}'\mathbf{y}_t$ is stationary, where $\mathbf{A}'$ is a $(\rho \times n)$ matrix of real numbers whose rows are linearly independent cointegrating vectors. Consider a standard ECM

$$\boldsymbol{\Delta}\mathbf{y}_{t+1} = \mathbf{k} + \mathbf{G}\mathbf{A}'\mathbf{y}_t + \mathbf{F}_1\boldsymbol{\Delta}\mathbf{y}_t + \mathbf{F}_2\boldsymbol{\Delta}\mathbf{y}_{t-1} + ... + \ \mathbf{F}_p\mathbf{y}_{t-p+1} + \mathbf{v}_{t+1} \tag{1}$$

where $\mathbf{k}$ is an $(n \times 1)$ vector, $\mathbf{G}$ is an $(n \times \rho)$ matrix of real numbers, and $\mathbf{v}_t$ is a stationary n-dimensional vector of random variables with $\widehat{E}[\mathbf{v}_{t+1}|H_{t-\tau}] = 0$. $\widehat{E}[\cdot|H_t]$ is the linear projection operator conditional on an information set H_t which is the econometrician's information set. In many applications $\tau = 0$, but we will give examples of applications in which $\tau > 0$.[5]

A class of structural models can be written in the following form of a structural ECM:

$$\mathbf{C}_0\boldsymbol{\Delta}\mathbf{y}_{t+1} = \mathbf{d} + \mathbf{B}\mathbf{A}'\mathbf{y}_t + \mathbf{C}_1\boldsymbol{\Delta}\mathbf{y}_t + \mathbf{C}_2\boldsymbol{\Delta}\mathbf{y}_{t-1} + ... + \ \mathbf{C}_p\mathbf{y}_{t-p+1} + \mathbf{u}_{t+1} \tag{2}$$

where $\mathbf{C}_i$ is an $(n \times n)$ matrix, $\mathbf{d}$ is an $(n \times 1)$ vector, and $\mathbf{B}$ is an $(n \times \rho)$ matrix of real numbers. Here $\mathbf{C}_0$ is a nonsingular matrix of real numbers with ones along its principal diagonal, and $\mathbf{u}_t$ is a stationary n-dimensional vector of random variables with $\widehat{E}[\mathbf{u}_{t+1}|H_{t-\tau}] = 0$. For this study, there is no need to assume that $\mathbf{u}_t$ is a vector of structural shocks nor that the covariance matrix of $\mathbf{u}_t$ is diagonal. We just assume that an economic model implies (2). An example is given in Section 3.

We are interested in $\mathbf{G}$ in (1) and $\mathbf{B}$ in (2). $\mathbf{G}$ contains parameters of the speed of adjustment toward the long-run equilibrium represented by $\mathbf{A}'\mathbf{y}_t$ (see Enders (1995) and Davidson and MacKinnon (2004)). We assume that the gradual adjustment toward the long-run equilibrium in the first equation in (2) is of particular interest. Thus the first row of $\mathbf{B}$ gives the structural parameters of interest.[6] In our model, there is only one unknown parameter in the first row of $\mathbf{B}$, and this parameter is of interest because it determines the half-life of the real exchange rate.

To see the relationship between the standard and the structural ECMs, we premultiply both sides of (2) by $\mathbf{C}_0^{-1}$ to obtain (1), where $\mathbf{k} = \mathbf{C}_0^{-1}\mathbf{d}$, $\mathbf{G} = \ \mathbf{C}_0^{-1}\mathbf{B}$, $\mathbf{F}_i = \mathbf{C}_0^{-1}\mathbf{C}_i$, and $\mathbf{v}_t = \mathbf{C}_0^{-1}\mathbf{u}_t$. Thus the standard ECM estimated by Engle and Granger's two step method or Johansen's (1988) ML method is a reduced form model. Hence it cannot be used to recover structural parameters in $\mathbf{B}$, nor can the impulse-response functions based on $\mathbf{v}_t$ be interpreted in a structural way unless some restrictions are imposed on $\mathbf{C}_0$. As in a VAR, various restrictions are possible for $\mathbf{C}_0$. A sufficient condition is that all the off-diagonal elements of the first row of $\mathbf{C}_0$ are zero. If $\mathbf{C}_0$ is lower triangular without changing the order for the first equation, then this condition is satisfied. If so, the first row of $\mathbf{G}$ is equal to the first row of $\mathbf{B}$, and structural parameters in the first row of $\mathbf{B}$ are estimated by the standard methods used to estimate an ECM. However, the structural ECM from the exchange rate model in section 3.2 does not satisfy the sufficient condition for $\mathbf{C}_0$.

3 An Exchange Rate Model with Sticky Prices

3.1 The Gradual Adjustment Equation

Let p_t (p_t^*) be the log domestic (foreign) price level, and e_t be the log nominal exchange rate. We assume that these variables are first difference stationary and PPP holds in the long-run, so that the real exchange rate, $p_t - p_t^* - e_t$, is stationary, or $\mathbf{y}_t = (p_t,\, e_t,\, p_t^*)'$ is cointegrated with a cointegrating vector (1, -1, -1). Let $\mu = E[p_t - p_t^* - e_t]$, then μ can be nonzero when different units are used to measure prices in the two countries.

Using Mussa's (1982) model, the domestic price is assumed to adjust slowly to the PPP level

$$\Delta p_{t+1} = b(\mu + p_t^* + e_t - p_t) + E_t[p_{t+1}^* + e_{t+1}] - (p_t^* + e_t) \tag{3}$$

where $\Delta x_{t+1} = x_{t+1} - x_t$ for any variable x_t, $E[\cdot\,|I_t]$ is the expectation operator conditional on I_t, the information available to the economic agents at time t, and a positive constant b ($0 \leq b \leq 1$) is the adjustment coefficient. The idea behind (3) is that the domestic price slowly adjusts toward its PPP level of $p_t^* + e_t$, while it adjusts instantaneously to the expected change in its PPP level. The adjustment speed is slow (fast) when b is close to zero (one). From (3),

$$\Delta p_{t+1} = d + b(p_t^* + e_t - p_t) + \Delta p_{t+1}^* + \Delta e_{t+1} + \varepsilon_{t+1} \tag{4}$$

where $d = b\mu$, $\varepsilon_{t+1} = E_t[p_{t+1}^* + e_{t+1}] - (p_{t+1}^* + e_{t+1})$. Hence ε_{t+1} is a one-period ahead forecasting error, and $E[\varepsilon_{t+1}|I_t] = 0$. (4) can be referred to as the structural gradual adjustment equation which implies a first order AR structure for the real exchange rate. To see this, let $s_t = p_t^* + e_t - p_t$ be the log real exchange rate. Then (4) implies

$$s_{t+1} = -\,d + (1-b)s_t - \varepsilon_{t+1} \tag{5}$$

We define the half-life of the real exchange rate as the number of periods required for a unit shock to dissipate by one half in (5). Without measurement errors, b can be estimated by OLS directly from (4). In the presence of measurement errors, IV are necessary.

3.2 The Exchange Rate under Rational Expectations

Let the money demand equation and the Uncovered Interest Parity (UIP) condition be

$$m_t = \theta_m + p_t - hi_t \tag{6}$$

$$i_t = i_t^* + E[e_{t+1}|I_t] - e_t \tag{7}$$

where m_t is the log nominal money supply minus the log real national income, i_t (i_t^*) is the nominal interest rate in the domestic (foreign) country. In (6), we are assuming that the income elasticity of money is one. From (6) and (7),

$$E[e_{t+1}|I_t] - e_t = (1/h)\{\theta_m + p_t - \omega_t - hE[(p_{t+1}^* - p_t^*)|I_t]\} \tag{8}$$

where $\omega_t = m_t + hr_t^*$ and r_t^* is the foreign real interest rate, $r_t^* = i_t^* - E[p_{t+1}^*|I_t] + p_t^*$.

Following Mussa (1982), solving (3) and (8) as a system of stochastic difference equations

$$p_t = E[F_t|I_{t-1}] - \sum_{j=1}^{\infty}(1-b)^j\{E[F_{t-j}|I_{t-j}] - E[F_{t-j}|I_{t-j-1}]\} \tag{9}$$

$$e_t = \frac{bh+1}{bh}E[F_t|I_t] - p_t^* - \frac{1}{bh}p_t \tag{10}$$

where $F_t = (1-\delta)\sum_{j=0}^{\infty}\delta^j\omega_{t+j}$ and $\delta = h/(1+h)$. We assume that ω_t is first difference stationary. Since δ is a positive constant that is smaller than one, this implies that F_t is also first difference stationary. From (9) and (10), $e_t + p_t^* - p_t = \frac{bh+1}{bh}\sum_{j=0}^{\infty}(1-b)^j\{E[F_{t-j}|I_{t-j}] - E[F_{t-j}|I_{t-j-1}]\}$, which means $e_t + p_t^* - p_t$ is stationary.[7]

3.3 Hansen and Sargent's Formula

For a structural ECM representation from the exchange rate model, we use Hansen and Sargent's (1980, 1982) formula for linear rational expectations models. From (10),

$$\Delta e_{t+1} = \frac{bh+1}{bh}(1-\delta)E[\sum_{j=0}^{\infty}\delta^j\Delta\omega_{t+j+1}|I_t] - \frac{1}{bh}\Delta p_{t+1} - \Delta p_{t+1}^* + \varepsilon_{e,t+1} \tag{11}$$

where $\varepsilon_{e,t+1} = \frac{bh+1}{bh}[E(F_{t+1}|I_{t+1}) - E(F_{t+1}|I_t)]$, so that the law of iterated expectation implies $E[\varepsilon_{e,t+1}|I_t] = 0$. The system method using Hansen and Sargent's (1982) method is applicable because this equation involves a discounted sum of expected future values of $\Delta\omega_t$.

Hansen and Sargent's (1982) method can be applied to this model by projecting the conditional expectation of the discounted sum, $E[\delta^j \Delta\omega_{t+j+1}|I_t]$, onto an econometrician's information set H_t. We take the econometrician's information set at t, H_t, to be the one generated by linear functions of current and past values of Δp_t^*. For simplicity, we follow West (1987) in that we choose a single variable to generate the information set H_t. In terms of the orthogonality condition, any variable in I_t can be used for this purpose.[8] Replacing $E[\sum_{j=0}^{\infty} \delta^j \Delta\omega_{t+j+1}|I_t]$ by the econometrician's linear forecast based on H_t in (11), we obtain

$$\Delta e_{t+1} = \frac{bh+1}{bh}(1-\delta)\widehat{E}[\sum_{j=0}^{\infty} \delta^j \Delta\omega_{t+j+1}|H_t] - \frac{1}{bh}\Delta p_{t+1} - \Delta p_{t+1}^* + u_{2,t+1} \tag{12}$$

where $u_{2,t+1} = \varepsilon_{e,t+1} + \frac{bh+1}{bh}(1-\delta)E[(\sum_{j=0}^{\infty} \delta^j \Delta\omega_{t+j+1}|I_t) - \widehat{E}(\sum_{j=0}^{\infty} \delta^j \Delta\omega_{t+j+1}|H_t)]$ and $\widehat{E}[u_{2,t+1}|H_t] = 0$. Following Hansen and Sargent (1980,1982), we obtain (See appendix A.)

$$\widehat{E}[\sum_{j=0}^{\infty} \Delta\omega_{t+j+1}|H_t] = \xi_1 \Delta p_t^* + \xi_2 \Delta p_{t-1}^* + ... + \xi_p \Delta p_{t-p+1}^* \tag{13}$$

A system of four equations will be[9]:

$$\Delta p_{t+1} = d + \Delta p_{t+1}^* + \Delta e_{t+1} - b(p_t - p_t^* - e_t) + u_{1,t+1} \tag{14}$$

$$\Delta e_{t+1} = -\frac{1}{bh}\Delta p_{t+1} - \Delta p_{t+1}^* + \alpha\xi_1 \Delta p_t^* + \alpha\xi_2 \Delta p_{t-1}^* + ... + \alpha\xi_p \Delta p_{t-p+1}^* + u_{2,t+1} \tag{15}$$

$$\Delta p_{t+1}^* = \beta_1 \Delta p_t^* + \beta_2 \Delta p_{t-1}^* + ... + \beta_p \Delta p_{t-p+1}^* + u_{3,t+1} \tag{16}$$

$$\Delta\omega_{t+1} = \gamma_1 \Delta p_t^* + \gamma_2 \Delta p_{t-1}^* + ... + \gamma_{p-1} \Delta p_{t-p+2}^* + u_{4,t+1} \tag{17}$$

where $\alpha = \frac{bh+1}{bh}(1-\delta)$ and $u_{1,t+1} = \varepsilon_{t+1}$ with a set of nonlinear restrictions imposed by (13),

$$\begin{aligned} \xi_0 &= \gamma(\delta)[1 - \delta\beta(\delta)] \\ \xi_j &= \delta\gamma(\delta)[1 - \delta\beta(\delta)]^{-1}(\beta_{j+1} + \delta\beta_{j+1} + ... + \delta^{p-j}\beta_p) + (\gamma_j + \delta\gamma_j + ... + \delta^{p-j}\gamma_p) \end{aligned} \tag{18}$$

for $j = 1, ..., p$. We call (14) the gradual adjustment equation, and (15)-(17) the Hansen and Sargent equations. Given the data for $[\Delta p_{t+1},\ \Delta e_{t+1}, \Delta p_{t+1}^*, \Delta\omega_{t+1}]'$, GMM can be applied to the system of four equations, (14)-(17).[10]

It is instructive to observe the relationship between the structural ECM and the reduced form ECM in the exchange rate model (See appendix B.). Comparing **G** and **B** shows that the speed of adjustment coefficient for the domestic price is b in the structural model, while it is $b^2h/(bh+1)$

in the reduced form model. b in the structural form is not a deep structural parameter, unlike parameters of a production function or a utility function. However, it is clearly a parameter of interest because it determines the half-life of the real exchange rate. The reduced form speed of adjustment coefficient is a nonlinear function of b, and thus cannot be directly compared with the half-life estimates in the literature.

3.4 Applying the System Method to the Exchange Rate Model

To apply the system method to (14)-(17) of the exchange rate model, we need data for $\Delta\omega_t$, which requires knowledge of h. Even though h is unknown, a cointegrating regression can be applied to money demand if money demand is stable in the long-run, as in Stock and Watson (1993). For this purpose, we augment the model as follows:

$$m_t = \theta_m + p_t - hi_t + \zeta_{m,t} \tag{19}$$

where $\zeta_{m,t}$ is assumed to be stationary so that money demand is stable. By redefining m_t as $m_t - \zeta_{m,t}$, the same equations as those in section 3.2 are obtained. For the measurement of $\Delta\omega_t$, the *ex ante* foreign real interest rate can be replaced by the *ex post* value because of the Law of Iterated Expectations. Using (19), we obtain

$$\Delta\omega_{t+1} = \Delta p_{t+1} - h\Delta i_{t+1} + h\Delta i^*_{t+1} - h(\Delta p^*_{t+2} - \Delta p^*_{t+1}) \tag{20}$$

With this expression, $\Delta\omega_t$ can be measured from price and interest rate data once h is obtained, even if data for the monetary aggregate and national income are unavailable.

We have now obtained a system of four equations, (14)-(17). Because $E[u_{i,t}|I_{t-\tau}] = 0$ and $\widehat{E}[u_{i,t}|H_t] = 0$, we obtain a vector of IV $\mathbf{z}_{1,t}$ in $I_{t-\tau}$ for $u_{1,t}$ and $\mathbf{z}_{i,t}$ in H_t for $u_{i,t}$ $(i = 2, 3, 4)$.[11] Using the moment conditions $E[z_{i,t}u_{i,t}] = 0$ for $i = 1, ..., 4$ we form a GMM estimator, imposing the Hansen-Sargent restrictions and the other cross-equation restrictions implied by the model.[12] Given estimates of cointegrating vectors from the first step, this system method provides more efficient estimators than Kim's (2004) single equation method as long as the restrictions implied by the model are true.[13] The cross-equation restrictions can be tested by Wald, Likelihood Ratio (LR) type, and Lagrange Multiplier (LM) tests in the GMM framework (see Ogaki (1993)). When restrictions are nonlinear, LR and LM tests are known to be more reliable than Wald tests.

3.5 A Measurement Model

In this paper we assume that $\mathbf{y}_t = (p_t, e_t, p_t^*)'$ is cointegrated with (1, -1, -1). This assumption may cause a problem in applications of the model to data in the post Bretton Woods period because many researchers have failed to find cointegration using similar data sets. Because more favorable evidence for the assumption is often found when a longer sample period is used, the failure to reject no cointegration may be due to the low power of the tests in small samples (see, Rogoff (1996)). Because the evidence is mixed, a sensitivity analysis for this assumption is in order. For the sensitivity analysis, we employ Cheung and Lai (1993) and Fisher and Park's (1991) model with measurement errors to allow the cointegrating vector to differ from (1, -1, -1).

The model with measurement errors suggests a single equation method and a system method with IV. For the single equation method, OLS is typically used as in Kim (2004). However, in the model with measurement errors, the OLS estimator is not consistent. Instead, we can use a two-step single equation method. In our system method with measurement errors, a two-step method is also employed (See appendix C.).

4 Empirical Results

We use quarterly data from 1974 Q1 to 2001 Q1 for Canada, France, Germany, Italy, Japan, the United Kingdom, and the United States. Exchange rates, CPI data, three month T-bill rates, nominal and real gross domestic products for the countries are taken from the International Financial Statistics (IFS). However, the DRI data is used for the UK, and three-month deposit rates are employed for Japan because T-bill rates are not available. To estimate interest elasticity of money demand, we use the sum of M1 and Quasi Money as the measure of the money stock, as the IFS suggests. The US dollar is the base currency.[14]

[TABLE 1 ABOUT HERE]

[TABLE 2 ABOUT HERE]

For the measurement error model, we need estimates of the coefficients in the cointegrating relationship, which is based on PPP. According to Table 1, the deterministic cointegrating restrictions are rejected for 2 out of 6 cases at the five percent significance level. For most cases, the null of stochastic cointegration is not rejected at the five percent significance level for any $H(1, q)$ test in the table.[15] Based on the CCR results for the money demand equations in Table 2[16], the null of stochastic cointegration is rejected only for Germany, regardless of the assumption of measurement errors, and the deterministic cointegrating restriction is rejected for Germany, Italy, and Japan at the five percent level when we allow for measurement errors. In all cases the signs of the estimates for the interest elasticity of money demand are negative, as predicted by the economic model. For Canada and France, the specification of measurement errors does not affect the estimates for the interest elasticities. However, most estimates from the measurement error models have smaller values than those from the models without measurement error.

[TABLE 3 ABOUT HERE]

In Table 3 we report the results of GMM estimation using the system method with an additional sample period, to see whether the creation of the German Economic and Monetary Union affects our results.[17,18] In the system method, the structural speed of adjustment coefficient b appears in both the gradual adjustment equation and the Hansen-Sargent equations. The model imposes the restriction that b in the gradual adjustment equation is the same as b in the Hansen-Sargent equations. In the case of unrestricted estimation, b_{hs} is from the Hansen-Sargent equations, and b_{ga} is from the gradual adjustment equation. The restricted estimate is denoted by b_r. The LR type test statistic denoted by LR, is used to test the restriction. In all cases this restriction is not rejected at the five percent level. Furthermore, for the test of the Hansen-Sargent restrictions in (18), we also report the LR type test statistic, denoted by LR1. For all cases the null hypothesis is not rejected at the five percent level, which is evidence in favor of the Hansen-Sargent restrictions.

The half-life estimate is based on the restricted estimate of b in (5) and is calculated as $0.25 \ln(0.5)/\ln(1-b)$. Most of them are significant at the five percent level. The half-life estimates range from 0.14 to 0.98 year and are much shorter than the established consensus of 3-5 year half-lives in single equation methods. As explained by Rogoff (1996), price adjustment within 1 or 2 years is plausible, but the consensus of 3-5 years is a puzzle. Given that Murray and Papell (2002) find that the single equation methods do not give much information about the half-lives, the fact that the system method yields more plausible estimates suggests that it extracts useful information about the half-lives.

5 Concluding Remarks

We employed Mussa's (1982) model to interpret estimates of an ECM in terms of the structural model because the model fits easily into our structural econometric framework. This paper is just an initial step toward utilizing information from an economic model to estimate the structural speed of adjustment coefficient in an ECM. The model assumes the long-run PPP for CPI-based real exchange rates and UIP for the short-term interest rate differentials and implies that the real exchange rate is a first order AR. All of these are often rejected by data. Therefore the economic model is very likely to be misspecified. In many applications of structural econometrics, restrictions imposed by economic models may be misspecified and the estimators may be inconsistent as a result. However, if restrictions are approximately true for an application, the benefit of a smaller variance by imposing restrictions may more than compensate for the cost of inconsistency.[19]

It is beyond the scope of the present paper to see how sensitive the results are when we relax these assumptions. However, Kim and Ogaki (2004), Kim (2005), and Kim (2006) have relaxed some of the assumptions, and the result that half-life estimates are shorter than Rogoff's (1996) consensus of 3 to 5 years when monetary variables are used together with the real exchange rate in system methods has been robust. The first two of these papers relax the long-run PPP assumption by modifying the system method developed in this paper to a two-good model and applied it to traded and non-traded good prices.[20] The third paper used a different monetary model based on a Taylor rule. In future work, we also plan to relax the UIP assumption. As in Lim and Ogaki's (2003) model, the UIP essentially holds for the long-term interest rate differential, but the forward premium anomaly exists for the short-term interest differential. It may be possible to develop a system method based on the UIP for the long-term interest rate differential.

A Derivation of (13)

As in Hansen and Sargent (1980, 1982), $\widehat{E}[\cdot|H_t]$ is the linear projection operator onto H_t and there exist possibly infinite order lag polynomials $\beta(L)$, $\gamma(L)$, and $\xi(L)$, such that

$$\widehat{E}[\Delta p^*_{t+1}|H_t] = \beta(L)\Delta p^*_t \tag{A1}$$

$$\widehat{E}[\Delta \omega_{t+1}|H_t] = \gamma(L)\Delta p^*_t \tag{A2}$$

$$\widehat{E}[\sum_{j=0}^{\infty} \delta^j \Delta\omega_{t+j+1}|H_t] = \xi(L)\Delta p^*_t \tag{A3}$$

Then, following Hansen and Sargent (1980), we obtain the restrictions imposed by (12) on $\xi(L)$:

$$\xi(L) = \frac{\gamma(L) - \delta L^{-1}\gamma(\delta)[1-\delta\beta(\delta)]^{-1}[1-L\beta(L)]}{1-\delta L^{-1}} \tag{A4}$$

Assume that linear projections of Δp^*_{t+1} and $\Delta\omega_{t+1}$ onto H_t have only a finite number of Δp^*_t terms:

$$\widehat{E}[\Delta p^*_{t+1}|H_t] = \beta_1\Delta p^*_t + \beta_2\Delta p^*_{t-1} + ... + \beta_p\Delta p^*_{t-p+1} \tag{A5}$$

$$\widehat{E}[\Delta\omega_{t+1}|H_t] = \gamma_1\Delta p^*_t + \gamma_2\Delta p^*_{t-1} + ... + \gamma_{p-1}\Delta p^*_{t-p+2} \tag{A6}$$

We assume $\beta(L)$ is of order p and $\gamma(L)$ is of order $p-1$ in order to simplify the exposition, but we do not lose generality because either β_i or γ_i can be zero. Then (A4) implies that $\xi(L) = \xi_0 + \xi_1 L + ... + \xi_p L^p$, where

$$\xi_0 = \gamma(\delta)[1-\delta\beta(\delta)]^{-1} \tag{A7}$$

$$\xi_j = \delta\gamma(\delta)[1-\delta\beta(\delta)]^{-1}(\beta_{j+1} + \delta\beta_{j+1} + ... + \delta^{p-j}\beta_p) + (\gamma_j + \delta\gamma_j + ... + \delta^{p-j}\gamma_p)$$

for $j = 1, ..., p$. Thus

$$\widehat{E}[\sum_{j=0}^{\infty} \Delta\omega_{t+j+1}|H_t] = \xi_1\Delta p^*_t + \xi_2\Delta p^*_{t-1} + ... + \xi_p\Delta p^*_{t-p+1} \tag{A8}$$

B Structural ECM and reduced form ECM

We have $\Delta\mathbf{y}_{t+1} = [\Delta p_{t+1},\, \Delta e_{t+1}, \Delta p^*_{t+1}, \Delta\omega_{t+1}]'$, $\mathbf{B} = [-b, 0, 0, 0]'$, $\mathbf{A} = [1, -1, -1, 0]'$,

$$\mathbf{C}_0 = \begin{bmatrix} 1 & -1 & -1 & 0 \\ 1/bh & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{B1}$$

and

$$\mathbf{C}_j = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \alpha\xi_j & 0 \\ 0 & 0 & \beta_j & 0 \\ 0 & 0 & \gamma_j & 0 \end{bmatrix} \tag{B2}$$

for $j = 1, ..., p$. For any nonzero constant ψ, ψ(1, -1, -1)$'$ is also a cointegrating vector. However, the first row of $\mathbf{B}$ in (2) is b only when ψ is normalized to one. As discussed in the text, a sufficient condition for identification of b by the reduced form ECM is that all the off-diagonal elements of the first row of $\mathbf{C}_0$ are zero. From equation (B1), however, the structural ECM from the exchange rate model does not satisfy this condition. Even though some structural models may be written in the form that satisfies the sufficient condition, this example suggests that many structural models cannot be written in that particular form. Because

$$\mathbf{C}_0^{-1} = \begin{bmatrix} bh/(bh+1) & bh/(bh+1) & 0 & 0 \\ -1/(bh+1) & bh/(bh+1) & -1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{B3}$$

$G \ = C_0^{-1}B \ = [-b^2h/(bh+1), b/(bh+1), 0, 0]'$.

C Derivation of a system with measurement errors

Let p_t^m ($p_t^{*,m}$) be the log measured domestic (foreign) price, which are related to the true prices by

$$p_t^m = \theta + \phi p_t + v_t \tag{C1}$$

$$p_t^{*,m} = \theta^* + \phi^* p_t^* + v_t^* \tag{C2}$$

where $E_{t-1}[v_t] = 0$ and $E_{t-1}[v_t^*] \ = 0$. We assume that the model in Section 3 is correct for the true price levels, but that only measured prices that follow (C1) and (C2) are observed. In particular, we assume that true prices follow the model of section 3 and satisfy PPP in the long-run. Then

$$p_t^m - \phi e_t - (\phi/\phi^*)p_t^{*,m} = (\theta - \theta^*\phi/\phi^*) + \phi(p_t - e_t - p_t^*) + [v_t - (\phi/\phi^*)v_t^*] \tag{C3}$$

is stationary. Hence, $y_t = (p_t^m, e_t, p_t^{*,m})'$ is cointegrated with a cointegrating vector $(1, -\phi, -\phi/\phi^*)$. To obtain an estimate of the cointegrating vector, we run a cointegrating regression of the form

$$p_t^m = \psi_0 + \psi_1 e_t + \psi_2 p_t^{*,m} + \zeta_t \tag{C4}$$

where $\psi_1 = \phi$, $\psi_2 = \phi/\phi^*$, and ζ_t is stationary with mean zero.

To obtain the gradual adjustment equation with measurement errors, we use (4) and $\Delta p_{t+1}^m = \phi \Delta p_{t+1} + \Delta v_{t+1}$ to obtain

$$\Delta p_{t+1}^m = d - b[p_t^m - \phi e_t - (\phi/\phi^*)p_t^{*,m}] + (\phi/\phi^*)\Delta p_{t+1}^{*,m} + \phi \Delta e_{t+1} + w_{t+1} \tag{C5}$$

where $d = b(\mu + \theta - \theta^* \phi/\phi^*)$, and $w_{t+1} = \phi \varepsilon_{t+1} + v_{t+1} - (1-b)v_t - (b\phi/\phi^*)v_{t+1} + (1-b)(\phi/\phi^*)v_t^*$. Here, $E_{t-1}[w_{t+1}] = 0$. Since p_t^m and $p_t^{*,m}$ are observed, (C1) and (C2) are substituted into (14)-(17) to express these equations in terms of measured prices. It is also assumed that H_t is the information set generated by the current and past values of $\Delta p_t^{*,m}$ instead of Δp_t^*. For $\Delta \omega_t$, we use

$$\Delta \omega_{t+1}^m = \frac{1}{\phi} \Delta p_{t+1}^m - h \Delta i_{t+1} + h \Delta i_{t+1}^* - \frac{h}{\phi^*}(\Delta p_{t+2}^{*,m} - \Delta p_{t+1}^{*,m}) \tag{C6}$$

so that

$$\Delta e_{t+1} = d_2 + \frac{bh+1}{bh}(1-\delta)\widehat{E}[\sum_{j=0}^{\infty} \delta^j \Delta \omega_{t+j+1}^m | H_t] - \frac{1}{bh\phi}\Delta p_{t+1}^m - \frac{1}{\phi^*}\Delta p_{t+1}^{*,m} + u_{2,t+1}^m \tag{C7}$$

where $u_{2,t+1}^m = u_{2,t+1} - \frac{bh+1}{bh}(1-\delta)\widehat{E}[\frac{1}{\phi}v_t - \frac{h}{\phi^*}v_t^* | H_t] + \frac{1}{bh\phi}\Delta v_{t+1} - \frac{1}{\phi^*}\Delta v_{t+1}^*$and $\widehat{E}[u_{2,t+1}^m | H_{t-1}] = 0$. In addition, because of the measurement error as in (C1),

$$m_t = \theta_2 + (1/\phi)p_t^m - h i_t + \zeta_{2,t} \tag{C8}$$

For the two-step single equation method, in the first step we run a cointegrating regression (C4). In the second step, the estimates of $\psi_1 = \phi$, $\psi_2 = \phi/\phi^*$ can be used to estimate (C5) with IV. In our system method with measurement errors, (14) is replaced by (C5), and (C5) is estimated with instrumental variables. Thus we run two cointegrating regressions, (C4) and (C8), in the first step. In the second step, GMM is applied to the system of four equations which consist of (C5) and :

$$\Delta e_{t+1} = -\frac{1}{bh\phi}\Delta p_{t+1}^m - \frac{1}{\phi^*}\Delta p_{t+1}^{*,m} + \alpha \xi_1 \Delta p_t^{*,m} + \alpha \xi_2 \Delta p_{t-1}^{*,m} + ... + \alpha \xi_p \Delta p_{t-p+1}^{*,m} + u_{2,t+1}^m \tag{C9}$$

$$\Delta p_{t+1}^{*,m} = \beta_1 \Delta p_t^{*,m} + \beta_2 \Delta p_{t-1}^{*,m} + ... + \beta_p \Delta p_{t-p+1}^{*,m} + u_{3,t+1} \tag{C10}$$

$$\Delta \omega_{t+1}^m = \gamma_1 \Delta p_t^{*,m} + \gamma_2 \Delta p_{t-1}^{*,m} + ... + \gamma_{p-1} \Delta p_{t-p+2}^{*,m} + u_{4,t+1} \tag{C11}$$

where h is replaced by its estimate from (C8) and ϕ and ϕ^* are replaced by their estimates from (C4).

Footnotes

(1) Kim (2006) used the system method with a different monetary model based on a Taylor rule, and obtained half-life estimates that are shorter than 3-5 years and longer than our estimates. Hence our conclusion that using monetary variables lead to half-life estimates that are shorter than the consensus of 3-5 years from the single equation methods seem robust to the choice of the monetary model.

(2) There exist economic models in which the regularity conditions of the theorem do not apply as shown in Ogaki (1998). However, the model in this paper is not subject to this criticism.

(3) Another important issue for structural ECM is identification of structural shocks. However, it is beyond the scope of this paper. The term Vector ECM (VECM) is usually used for this. See King, Plosser, Stock, and Watson (1991), Jang (2000), Jang and Ogaki (2004), Eichenbaum and Evans (1995) and Kim and Roubini (2000) for details.

(4) This does not mean that our estimators dominate the ML estimators in all applications, but means that our estimators are more appropriate in some applications such as ours.

(5) We will treat more general cases in which the expectation of $\mathbf{v}_{t+1}$conditional on the economic agents' information is not zero, but the linear projection of $\mathbf{v}_{t+1}$onto an econometrician's information set is zero.

(6) Even though cointegrating vectors are not unique, we assume that there is a normalization that uniquely determines $\mathbf{A}$ so that parameters in $\mathbf{B}$ have structural meanings, as in Section 3.

(7) This implies that $(p_t, e_t,\ p_t^*)'$ is cointegrated with (1,-1,-1). This assumes that $E_t(F_t) - E_{t-1}(F_t)$ is stationary, which is true for a large class of first difference stationary variable F_t and information sets. This shows that the assumption of cointegration made for equation (3) is consistent with the other assumptions for the model.

(8) There is a sense in which ω_t is a more natural variable to choose. However, the measurement of ω_t is subject to h, which is estimated by a cointegrating regression. Because of super consistency, h can be treated as the true value for our GMM estimation. However, it seems

prudent to avoid using ω_t as instruments, so that empirical results are more likely to be robust with respect to h.

(9) We use (4), (12), (A1), (A2), and (13) to obtain this system of four equations.

(10) We do not use the GMM estimators that use all available orthogonality conditions discussed in Hansen and Sargent (1982). Assuming that there is no drift term in each of the first difference stationary variable in the model, we do not include the constant terms in (15)-(17). For this reason, we do not have the additional restrictions on the constant terms derived in West (1989). As a sensitivity analysis, we included constant terms in (15)-(17) without imposing West's restrictions, and obtain qualitatively similar results as reported in this paper. The results are available upon request.

(11) The list of instruments we used are as follows, for the case without measurement errors, $(1, s_t)$ for (14), $(1, \Delta p_t^*, \Delta p_{t-1}^*)$ for (15), and $(\Delta p_t^*, \Delta p_{t-1}^*)$ for (16) and (17). For the cases with measurement errors, $(1, \Delta p_{t-3}^*)$ for (14), $(1, \Delta p_{t-3}^*, \Delta p_{t-4}^*)$ for (15), and $(\Delta p_{t-3}^*, \Delta p_{t-4}^*)$ for (16) and (17).

(12) Only p_t adjusts gradually, but b affects the dynamics of other variables because of interactions between p_t and those variables. Due to this, there will be cross-equation restrictions involving b, in addition to the restrictions (A7).

(13) In de Jong (2001), the first step estimation can affect the asymptotic distributions of the second step estimator because of the nonlinear restrictions in the system method. However, since the equations are linear, the regularity conditions are likely to hold.

(14) As a sensitivity analysis, we have used all the currencies as the numeraire, and the results are available in our working paper.

(15) For the definitions of stochastic and the deterministic cointegration restriction, see Ogaki and Park (1998).

(16) For measurement errors, one estimate is from (3) and the other is from (19). If the two estimates are significantly different, it might imply misspecification of the model. Nevertheless, we use the estimates in Table 1 because we are more interested in PPP. Park and

Ogaki (1991) suggest Seemingly Unrelated CCR, but since the small sample properties of their estimator are not better than CCR, we use the estimates from CCR.

(17) For the order of AR processes, the AIC recommended $p = 2$ for all data set. The QS kernel with automatic bandwidth is used for serial correlation.

(18) To test for the quality of the instruments, we employed the Stock and Yogo (2004) test. Based on the actual size of the 5% TSLS t test, we reject the null of weak instruments for all cases. The results of the Stock and Yogo test are also available upon requests.

(19) Some evidence against the first order AR structure has been found for higher order AR structure for real exchange rates. However, Murray and Papell's (2002) estimates for half-lives based on the first order AR are strikingly similar to the corresponding estimates based on higher order AR impulse responses.

(20) See Kim (2005) and Kim and Ogaki (2004) for details.

References

[1] Boswijk, H. Peter. (1994) "Conditional and Structural Error Correction Models." *Journal of Econometrics,* 69, 159-71.

[2] Boswijk, H. Peter. (1995) "Efficient Inference on Cointegration Parameters in Structural Error Correction Models." *Journal of Econometrics,* 69, 133-58.

[3] Cheung, Yin-Wong and Kon S. Lai. (1993) "Long-Run Purchasing Power Parity During The Recent Float." *Journal of International Economics,* 34, 181-92.

[4] Choi, Chi-Young, Nelson C. Mark, and Donggyu Sul. (2006) "Unbiased Estimation of the Half-Life to PPP Convergence in Panel Data." *Journal of Money, Credit, and Banking*, 38, 921-38.

[5] Davidson, James E. H., David F. Hendry, Frank Srba, and Stephen Yeo. (1978) "Econometric Modelling of the Aggregate Time-Series Relationship between Consumers' Expenditure and Income in the United Kingdom." *Economic Journal,* 88, 661-92.

[6] Davidson, Russell and James MacKinnon. (2004) *Econometric Theory and Methods*, New York, NY: Oxford University Press, .

[7] de Jong, Robert M. (2001) "Nonlinear Estimation Using Estimated Cointegrating Relations." *Journal of Econometrics,* 101, 109-22.

[8] Dolado, Juan, John W. Galbraith, and Anindya Banerjee. (1991) "Estimating Intertemporal Quadratic Adjustment Cost Models with Integrated Series." *International Economic Review,* 32, 919-37.

[9] Dornbusch, Rudiger. (1976) "Expectations and Exchange Rate Dynamics." *Journal of Political Economy,* 84, 1161-76.

[10] Eichenbaum, Martin and Charles L. Evans. (1995) "Some Empirical Evidence on the Effects of Shocks to Monetary Policy on Exchange Rates." *Quarterly Journal of Economics*, 110, 975-1009.

[11] Enders, Walter. (1995) *Applied Econometric Time Series*, New York, NY: John Wiley & Sons, Inc.

[12] Engle, Robert. F. and Clive W. J. Granger. (1987) "Co-Integration and Error Correction: Representation, Estimation, and Testing." *Econometrica*, 55, 251-76.

[13] Fisher, Eric O'N and Joon Y. Park. (1991) "Testing Purchasing Power Parity under the Null Hypothesis of Co-Integration." *Economic Journal,* 101, 1476-84.

[14] Gregory, Allan W., Adrian R. Pagan, and Gregor W. Smith. (1993) "Estimating Linear Quadratic Models with Integrated Processes." In *Models, Methods, and Applications of Econometrics: Essays in Honour of A.R. Bergstrom*, edited by Peter C. B. Phillips, pp. 220-39. Cambridge, MA and Oxford: Blackwell.

[15] Hansen, Lars P. (1982) "Large Sample Properties of Generalized Method of Moments Estimators." *Econometrica,* 50, 1029-54.

[16] Hansen, Lars P. and Thomas J. Sargent. (1980) "Formulating and Estimating Dynamic Linear Rational Expectations Models." *Journal of Economic Dynamics and Control,* 2, 7-46.

[17] Hansen, Lars P. and Thomas J. Sargent. (1982) "Instrumental Variables Procedures for Estimating Linear Rational Expectations Models." *Journal of Monetary Economics,* 9, 263-96.

[18] Hsiao, Cheng. (1997) "Cointegration and Dynamic Simultaneous Equations Model." *Econometrica,* 65 (3): 647-70.

[19] Jang, Kyungho. (2000) "Impulse Response Analysis with Long Run Restrictions on Error Correction Models." manuscript, Ohio State University.

[20] Jang, Kyungho and Masao Ogaki. (2003) "The Effects of Monetary Policy Shocks on Exchange Rates: A Structural Vector Error Correction Model Approach." *Journal of the Japanese and International Economies*, 18, 99-114.

[21] Johansen, Soren. (1988) "Statistical Analysis of Cointegrating Vectors." *Journal of Economic Dynamics and Control,* 12, 231-54.

[22] Kilian, Lutz and Tao Zha. (2002) "Quantifying the Uncertainty about the Half-Life of Deviations from PPP." *Journal of Applied Econometrics,* 17, 107-25.

[23] Kim, Hyeongwoo. (2006) "Half-Life Estimation under the Taylor Rule." manuscript, Ohio State University.

[24] Kim, Jaebeom. (2004) "Half-lives of Deviations from PPP: Contrasting Traded and Non-traded Components of Consumption Baskets." *Review of International Economics*, 12, 162-68.

[25] Kim, Jaebeom. (2005) "Convergence Rates to PPP for Traded and Non-traded Goods: A Structural Error Correction Model Approach." *Journal of Business & Economic Statistics*, 23, 76-86.

[26] Kim, Jaebeom and Masao Ogaki. (2004) "Purchasing Power Parity for Traded and Non-traded Goods: A Structural Error Correction Model Approach." *Monetary and Economic Studies,* 22, 1-25.

[27] Kim, Soyoung and Nouriel Roubini. (2000) "Exchange Rate Anomalies in the Industrial Countries: A Solution with a Structural VAR Approach." *Journal of Monetary Economics*, 45, 561-86.

[28] King, Robert G., Charles I. Plosser, James H. Stock, and Mark W. Watson. (1991) "Stochastic Trends and Economic Fluctuations." *American Economic Review,* 81, 810-40.

[29] Lim, Hyoung-Seok and Masao Ogaki. (2003) "A Theory of Exchange Rate and The Term Structure of Interest Rates." manuscript, the Ohio State University.

[30] Mussa, Michael. (1982) "A Model of Exchange Rate Dynamics." *Journal of Political Economy,* 90, 74-104.

[31] Murray, Christian J. and David H. Papell. (2002) "The Purchasing Power Parity Persistence Paradigm." *Journal of International Economics,* 56, 1-19.

[32] Murray, Christian J. and David H. Papell. (2005) "Do Panels Help Solve the Purchasing Power Parity Puzzle?" *Journal of Business and Economic Statistics*, 23, 410-15.

[33] Ogaki, Masao. (1993) "Generalized Method of Moments: Econometric Applications." In *Handbook of Statistics, Vol.11: Econometrics*, edited by G.S. Maddala, C.R. Rao, and H.D. Vinod, pp. 455-88. Amsterdam: North-Holland.

[34] Ogaki, Masao. (1998) "On the Granger Representation Theorem: A Counter Example?" *Economics Letters,* 60, 19-21.

[35] Ogaki, Masao and Joon Y. Park. (1998) "A Cointegration Approach to Estimating Preference Parameters." *Journal of Econometrics,* 82, 107-34.

[36] Papell, David. (1997) "Cointegration and Exchange Rate Dynamics." *Journal of International Money and Finance,* 445-59.

[37] Park, Joon Y. (1992) "Canonical Cointegrating Regressions." *Econometrica*, 60, 119-43.

[38] Park, Joon Y. and Masao Ogaki. (1991) "Seemingly Unrelated Canonical Cointegrating Regressions." RCER Working Papers 280, University of Rochester, Center for Economic Research.

[39] Rogoff, Kenneth. (1996) "The Purchasing Power Parity Puzzle." *Journal of Economic Literature,* 34, 647-68.

[40] Stock, James H. and Watson, Mark W. (1993) "A Simple Estimator of Cointegrating Vectors in Higher Order Integrated Systems." *Econometrica*, 61, 783-820.

[41] Stock, James H. and Motohiro Yogo. (2004) "Testing for Weak Instruments in Linear IV Regression." NBER Working Paper 284.

[42] Urbain, Jean-Pierre. (1992) "On Weak Exogeneity in Error Correction Models." *Oxford Bulletin of Economics and Statistics,* 54, 187-207.

[43] West, Kenneth D. (1987) "A Specification Test for Speculative Bubbles." *Quarterly Journal of Economics* XII, 553-80.

[44] West, Kenneth D. (1989) "Estimation of Linear Rational Expectations Models in the Presence of Deterministic Terms." *Journal of Monetary Economics,* 24, 437-42.

Table 1. Purchasing Power Parity

Currency[(1)]	$\psi_0^{(2)}$	$\phi^{(3)}$	$\phi/\phi^{*(4)}$	$H(0,1)^{(5)}$	$H(1,2)^{(6)}$	$H(1,3)^{(7)}$
CA/U.S.	0.019	0.109	0.991	138.61	0.426	1.472
	(0.244)	(0.177)	(0.063)	(0.000)	(0.513)	(0.478)
FR/U.S.	0.006	0.237	0.906	0.401	0.231	0.921
	(0.312)	(0.112)	(0.069)	(0.526)	(0.630)	(0.631)
GE/U.S.	-0.053	0.195	0.957	5.237	2.436	4.357
	(0.980)	(0.429)	(0.114)	(0.022)	(0.118)	(0.113)
IT/U.S.	-1.045	0.185	0.910	0.488	0.327	0.472
	(0.849)	(0.154)	(0.138)	(0.484)	(0.567)	(0.789)
JP/U.S.	-2.469	0.282	1.252	0.858	10.121	10.512
	(1.212)	(0.126)	(0.143)	(0.354)	(0.001)	(0.005)
U.K./U.S.	0.438	0.067	0.899	3.054	6.566	7.201
	(0.350)	(0.161)	(0.071)	(0.080)	(0.010)	(0.027)

Note: Results for $p_t^m = \psi_0 + \phi e_t + (\phi/\phi^*) p_t^{*,m} + \zeta_t$. Column (1): currencies. Columns (2)∼(4): Standard errors are in parentheses. $H(p,q)$ is a Wald test statistic with an estimate of the long-run variance of a regression with a q-th order polynomial for the hypothesis that the last *q-p* coefficients are zero. $H(0,1)$ tests null hypothesis of the deterministic cointegrating restriction and $H(1,q)$ tests the null hypothesis of stochastic cointegration. Columns (5)∼(7): P-values are in parentheses.

Table 2. Money Demand Equation

Country(1)	$\theta_2^{(2)}$	$1/\phi^{(3)}$	$h^{(4)}$	$H(0,1)^{(5)}$	$H(1,2)^{(6)}$	$H(1,3)^{(7)}$
CA	-0.046	1	30.031	1.019	0.445	1.098
	(0.247)		(9.875)	(0.313)	(0.505)	(0.578)
	0.464	1.899	40.791	0.523	4.681	4.792
	(0.456)	(0.240)	(16.200)	(0.469)	(0.030)	(0.091)
FR	-0.341	1	5.661	4.823	0.963	1.183
	(0.071)		(3.182)	(0.028)	(0.326)	(0.554)
	-0.337	0.253	5.560	2.036	0.097	0.321
	(0.014)	(0.038)	(0.636)	(0.153)	(0.755)	(0.851)
GE	-0.272	1	17.882	1.537	1.600	4.575
	(0.165)		(9.856)	(0.215)	(0.206)	(0.102)
	-0.454	1.597	3.003	11.500	18.610	18.650
	(0.022)	(0.050)	(1.307)	(0.001)	(0.000)	(0.000)
IT	-0.205	1	7.847	3.755	0.753	2.424
	(0.172)		(4.807)	(0.053)	(0.386)	(0.298)
	-0.508	0.078	1.994	9.247	1.560	5.979
	(0.041)	(0.013)	(1.073)	(0.002)	(0.212)	(0.050)
JP	-11.312	1	39.661	3.274	3.229	5.353
	(0.059)		(5.827)	(0.070)	(0.072)	(0.069)
	-13.951	1.520	8.089	13.020	0.035	0.674
	(0.784)	(0.171)	(5.228)	(0.000)	(0.852)	(0.714)
U.K.	10.662	1	111.312	1.853	0.088	0.597
	(3.410)		(128.3)	(0.173)	(0.766)	(0.742)
	8.982	2.265	27.560	2.942	0.129	5.562
	(0.273)	(0.116)	(9.306)	(0.086)	(0.720)	(0.062)
U.S.	-5.151	1	12.779	0.001	1.678	17.232
	(0.156)		(6.875)	(0.965)	(0.195)	(0.000)
	-4.951	0.960	14.208	1.631	2.735	24.362
	(0.781)	(0.168)	(7.507)	(0.201)	(0.098)	(0.000)

Note: Results for $m_t = \theta_2 + (1/\phi)p_t^m - hi_t + \zeta_{2,t}$. Column (1): domestic countries. Columns (2)∼(4): Standard errors are in parentheses. $H(p,q)$ is a Wald test statistic with an estimate of the long-run variance of a regression with a q-th order polynomial for the hypothesis that the last *q-p* coefficients are zero. $H(0,1)$ tests the null hypothesis of the deterministic cointegrating restriction and $H(1,q)$ tests the null hypothesis of stochastic cointegration. Columns (5)∼(7): P-values are in parentheses.

Table 3. The System Method Results for CPI-based Real Exchange Rates

Country$^{(1)}$	$\phi^{(2)}$	$\phi/\phi^{*(3)}$	Half-Life$^{(4)}$	$b_r^{(5)}$	$J_r^{(6)}$	$b_{u,hs}^{(7)}$	$b_{u,ga}^{(8)}$	$J_u^{(9)}$	LR$^{(10)}$	LR1$^{(11)}$
CA/U.S.	1	1	0.18	0.611	5.507	2.422	1.087	5.418	0.089	0.309
			(0.036)	(0.177)	(0.239)	(1.624)	(0.646)	(0.143)	(0.765)	(0.856)
	0.109	0.991	0.28	0.464	5.503	5.960	0.516	4.012	1.491	1.667
			(0.080)	(0.112)	(0.239)	(25.191)	(0.387)	(0.260)	(0.222)	(0.434)
FR/U.S.	1	1	0.68	0.224	5.551	1.366	0.601	4.779	0.772	0.077
			(0.764)	(0.072)	(0.235)	(1.454)	(0.177)	(0.188)	(0.379)	(0.962)
	0.237	0.906	0.16	0.663	4.466	0.467	2.338	3.351	1.115	0.935
			(0.008)	(0.066)	(0.346)	(0.116)	(0.534)	(0.340)	(0.291)	(0.626)
GE/U.S.	1	1	0.21	0.557	5.669	1.779	0.766	3.768	1.901	0.317
			(0.061)	(0.190)	(0.225)	(1.541)	(0.218)	(0.287)	(0.168)	(0.853)
73:I∼90:II			0.16	0.651	3.409	2.046	0.849	2.921	0.488	0.216
			(0.029)	(0.201)	(0.491)	(1.568)	(0.222)	(0.403)	(0.484)	(0.897)
	0.195	0.957	0.98	0.162	3.182	-1.043	0.782	1.301	1.881	1.744
			(1.475)	(0.047)	(0.527)	(0.863)	(0.234)	(0.728)	(0.170)	(0.417)
73:I∼90:II			0.95	0.167	3.201	1.021	0.784	1.712	1.489	0.242
			(1.591)	(0.056)	(0.524)	(0.963)	(0.229)	(0.634)	(0.222)	(0.885)
IT/U.S.	1	1	0.27	0.475	7.098	-8.767	0.355	6.723	0.375	1.017
			(0.105)	(0.162)	(0.131)	(7.150)	(0.177)	(0.081)	(0.540)	(0.601)
	0.185	0.901	0.16	0.661	2.903	0.812	0.104	2.559	0.344	1.451
			(0.006)	(0.044)	(0.574)	(0.505)	(1.179)	(0.464)	(0.557)	(0.484)
JP/U.S.	1	1	0.92	0.172	7.113	25.060	0.132	6.162	0.951	0.618
			(0.489)	(0.019)	(0.130)	(21.918)	(0.058)	(0.104)	(0.329)	(0.734)
	0.282	1.252	0.54	0.275	2.444	0.334	0.829	2.268	0.176	0.075
			(0.307)	(0.059)	(0.654)	(0.190)	(1.359)	(0.518)	(0.674)	(0.962)
U.K./U.S.	1	1	0.20	0.576	6.380	5.896	0.635	5.716	0.664	0.969
			(0.055)	(0.201)	(0.172)	(18.299)	(0.165)	(0.126)	(0.415)	(0.615)
	0.067	0.899	0.14	0.701	3.821	0.266	0.484	3.078	0.743	0.898
			(0.022)	(0.225)	(0.430)	(0.164)	(1.362)	(0.379)	(0.388)	(0.638)

Note; For the unrestricted estimation, $b_{u,hs}$ is the estimate for the speed of adjustment coefficient obtained from Hansen and Sargent equations (15)-(17) and $b_{u,ga}$ is the estimate for the coefficient obtained from the price adjustment equation (14). LR is the likelihood ratio type test comparing the J-statistics from restricted and unrestricted GMM with respect to the restriction that b in the gradual adjustment equation is the same as b in the Hansen-Sargent equations. LR1 is the likelihood ratio type test comparing the J-statistics from restricted and unrestricted GMM with respect to the Hansen-Sargent restrictions. Thus LR1 tests the restrictions in equation (18). Column (1): currencies. Columns (2) and (3) are from Table 1. Columns (4), (5), (7) & (8): Standard errors are in parentheses. Columns (6), (9) , (10) & (11): P-values are in parentheses. Columns (4): Half-life in years.