Credit Card Debt and Default over the Life Cycle

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Abstract

This paper solves an empirically parameterized model of life-cycle consumption which allows for uncollaterized borrowing and the possibility of default. The simulation results show that: (i) " social *stigma*" and credit limit have a very large impact on default rates; (ii) education level also has a significant effect on the probability of default, namely through differences in the shape of lifetime labor income profiles; (iii) the response of simulated default rates to labor income shocks is determined by the nature of labor income uncertainty (temporary vs permanent). Additionally, the model generates simultaneous consumer holdings of credit card debt and liquid assets.

Keywords: consumer credit, life cycle, credit card, personal bankruptcy

JEL classification codes: D14, D91, E21

1 Introduction

Over the last two decades there has been a substantial increase in personal bankruptcy filings, from 0.35% to 1.4% per year. Such high levels in bankruptcy rates may put credit market stability at risk. As a result, politicians and economists have become increasingly concerned with the magnitude of these numbers. However, in order to take adequate measures to revert this trend, policymakers need to have a clear understanding of the factors which are more likely to have caused such an increase. Several explanation have been suggested in the literature: (i) macroeconomic factors, which may affect households' indebtness and credit supply; (ii) current bankruptcy law, which may affect the incentives to file for bankruptcy; (iii) a decrease in *stigma*, or social embarrassment associated with filing for bankrupt; among others. However, so far no consensus has been reached.

In this context, the main purpose of the present paper is to quantify the effect of these factors on households' decision to default. In order to do so, I solve and simulate an empirically parameterized model of optimal consumption, allowing for borrowing and default. Parameter sensitivity analysis allows us to quantify the effect of each parameter on simulated default rates and therefore evaluate which parameters are more likely to be the main cause of the observed increase in personal bankruptcy. Given the nature of the exercise I do not attempt to explain the actual default rate, but rather to determine what factors are more likely to generate greater changes in the default rates, and in which direction.

More precisely, I extend the classical precautionary saving model, developed in (Deaton 1991) and (Carroll 1997), to allow for unsecured borrowing and default. In this life cycle model, the representative consumer is endowed with a stochastic labor income stream, and has the possibility to borrow from the credit card market, up to a certain credit limit. One could argue that the assumption that consumers are only able to borrow through credit card debt is very restrictive. However, since my main objective is to study unsecured credit, this assumption is not unrealistic: in the 2004 Survey of Consumer Finances 46.2% of the families hold credit card balances as opposed to 7.6% of families holding other types of unsecured debt. A general equilibrium model with endogenous credit limits is presented by (Chatterjee, Corboe, Nakajima and Rios-Rull 2002). A two period life cycle model with possibility of default is presented in (Lawrence 1995). In each period, after observing his current income the agent chooses how much to consume (and save), how much to borrow, and in case of outstanding debt, whether he wants to default, where by default I mean to file for bankruptcy. However, if he chooses to default: (a) his assets (net of an exemption level) will be relinquished; (b) he will not be able to borrow in the future; and, (c) he will bear an utility cost associated with the social embarrassment of filing for bankruptcy.

The parameters of the model are carefully estimated and calibrated. The labor income process is estimated using the Panel Survey of Income Dynamics (PSID). Given the differences in the shape of labor income profiles I consider 3 education groups. For each group, income is subject to a permanent and a temporary shock. The exemption level is estimated from the different state level asset exemption, allowed by the Bankruptcy Reform Act of 1978. Interest rate premium charged by credit card companies and credit limits are calibrated to ensure zero expected profits in the credit card market. The *stigma* effect is calibrated in order to match U.S. annual bankruptcy rates with the rates from the simulated model. The discount factor and the rate of relative risk aversion are taken from consumption literature.

The model makes important contributions to the literature on personal bankruptcy. On one dimension, it allows us to quantify which factors have a higher impact on consumers' default decision, and therefore get us closer to an explanation to the observed increase in the bankruptcy rates. From the sensitivity analysis results we learn that the *stigma* effect has a substantial impact on default rates. This means that small decreases in the utility cost associated with the social embarrassment of filing for bankruptcy can lead to significant increases in the default rate. This result goes in the same direction and reenforces the empirical findings of (Gross and Souleles 2002a). They use a new and comprehensive data set, including all the information held by several credit card companies about its costumers, the authors were able to control for risk-composition and other economic fundamentals, and investigate the role of the stigma effect in explaining bankruptcy and default. They found robust evidence that the decline in the social embarrassment from defaulting has an important role in explaining the recent growth in personal bankruptcy. My simulations also show that an increase in credit limits results in a substantial increase in debt levels and default rates, consistent with (Gross and Souleles 2002b).

This paper also makes an important contribution in that it proposes and solves a theoretical model of consumer choices which generates prediction that are consistent with several important empirical regularities namely:

(i) The fact that the probability of default is decreasing in education, as reported in (Pavan 2005). In my model this results from the fact that consumers' decision to default depends on the relative importance of the benefits (waived liabilities) versus the costs of default (no future access to credit, relinquished assets and *stigma*). This balance is, in turn, heavily affected by level and shape of labor income profile. For consumers with flat and low labor income profiles (no high school education), neither the access to credit nor the stigma effect is sufficiently important to avoid default, making the probability to default very high;

(ii) The fact that most of the post war upward trend in personal bankruptcy has taken place in periods of economic expansion, with unemployment rates falling, see (Kowalewski 2000). For precautionary saving consumers, it is in periods of low temporary income uncertainty for example, period of low unemployment rates when they wish to consume and borrow more (Carroll 1992). In addition, if we allow these consumers to default, as I do here, they are more likely to do so when they expect low unemployment rates. The procyclicality of default is also found in general equilibrium models, where optimal contractual arrangements are made in the presence of commitment problems (Kocherlakota 1996) and (Kehoe and Levine 2001). However, in these models default never happens because of the threat of autarky.

(iii) The fact that I allow for a level of asset exemption, in the case of bankruptcy, generates the necessary edge that leads our consumers to hold simultaneously high cost debt and low return assets. (Gross and Souleles 2002b), (Bertraud and Haliassos 2001), among others, report empirical evidence for this portfolio puzzle. (Lehnert and Maki 2002) also explain the puzzle through exemption levels. They combine state level bankruptcy laws with data from the Consumer Expenditure Survey, to find supporting evidence for the fact that households are more likely to hold simultaneously low return liquid assets and owe high cost unsecured debt in states where exemption levels are higher. In addition, using the Survey of Consumer Finances, I have found empirical evidence supporting this result.

Finally, this paper also contributes to our understanding of the challenging task lawmakers face of distinguishing between consumers who intentionally cheat on their creditors and those who were unlucky and deserve a fresh start (Sullivan, Warren and Westbrook 1989). In my model, the option to default leads to the result that the decision to borrow is not only driven by consumption smoothing but also by strategic behavior. Namely, I show that consumers may choose to borrow with the intention of defaulting in the near future. The circumstances that trigger such strategic behaviour are clearly drawn from the policy functions, and depend to a large extent on observable variables: age; education level; and credit limit.

The rest of the paper is organized as follows. Section 2 and 3 discuss the model's specification and solution method. The parameterization is presented in section 4. In sections 5 and 6 policy functions and the resulting simulation profiles are presented and discussed. In section 7, I match some of the main model's predictions with available data. Finally, section 8 concludes.

2 The Model

2.1 Time parameters and preferences

The representative consumer lives and works for T periods. For simplicity, T is assumed to be exogenous and deterministic. Life-time preferences are described by the time-separable power utility function:

$$E_1\left\{\sum_{t=1}^{T}\beta^{t-1}\left[\frac{C_t^{1-\gamma}}{1-\gamma}-stigmaD_t^c\right]+\beta^{T+1}V_{T+1}(A_{T+1})\right\} \tag{1}$$

where C_t is the level of date t consumption, $\gamma > 0$ is the coefficient of relative risk aversion, and $\beta < 1$ is the discount factor. *stigma* is the stigma effect, or the social embarrassment caused by the decision to default, which affects current utility in the period when the consumer chooses to default (in which case D_t^c=1, where the subscript c refers to the choice of defaulting). V_{T+1} represents the value to the consumer of any assets A_{T+1} left at the time of death/retirement. The functional form assumed for the salvage function is the following:

$$V_{T+1}(A_{T+1}) = \frac{A_{T+1}^{1-\gamma}}{1-\gamma}. \tag{2}$$

2.2 The labor income process

Consumer's age t labor income, Y_t, is exogenously given by:

$$\log(Y_t) = f(t, Z_t) + \upsilon_t \tag{3}$$

where $f(t, Z_t)$ is a deterministic function of age, t, and a vector of other individual characteristics, Z_t. Following Carroll and Samwick (1995) I allow for permanent, μ_t, and transitory income, ε_t, shocks to labor income. Thus υ_t can be described by:

$$\upsilon_t = \mu_t + \varepsilon_t \tag{4}$$

where ε_t is i.i.d and distributed as $N(0, \sigma_\varepsilon^2)$ and μ_t is a unit root given by:

$$\mu_t = \mu_{t-1} + \eta_t \tag{5}$$

where η_t is i.i.d. and $N(0, \sigma_\eta^2)$.

In period $T+1$ the consumer receives a deterministic income, which could be interpreted as pension income. It is equal to fraction α of the permanent income of the last working year:

$$\log\left(Y_{T+1}\right) = \log(\alpha) + f(T, Z_T). \tag{6}$$

2.3 Borrowing constraints and credit history

In each period t, the consumer is allowed to borrow B_t, up to a certain limit λ, as long as he has a good credit history. In other words, I am assuming that there is a credit card market, willing to lend money against no collateral, whenever the consumer has not defaulted on his debt in the past. Thus, in the case of no previous default (in which case $D_t^s = 0$, where the subscript s refers to the state associated with the previous default), the borrowing constraint is given by:

$$B_t \leq \lambda \; if \;\; D_t^s = 0. \tag{7}$$

If at any time the consumer decides to default, his assets will be relinquished up to an exemption level e, and he will not be able to borrow in the future. That is, let A_t denote period t financial assets, then in periods where default occurs the credit cart company is entitled to receive:

$$\pi_t = Min[Max[A_t - e, 0], B_t] \; if \; D_t^c = 1, \tag{8}$$

and the borrowing constraint in subsequent periods is given by:

$$B_t = 0 \; if \; D_t^s = 1. \tag{9}$$

Finally, I assume that whenever a consumer files for bankruptcy, there is a stigma effect which lowers current utility by *stigma*. Note that when *lambda* is equal to zero borrowing is ruled out as in Deaton (1991).

2.4 Interest rate

In any given period t, the consumer earns a fixed real interest rate r on his assets. The second interest rate that is relevant for consumers is the interest rate on debt, denoted by i. I assume that it is equal to the interest rate on financial savings plus a constant debt premium θ such that:

$$i = r + \theta. \tag{10}$$

In reality the debt premium may vary over time, with the level of interest rates or the level of aggregate income. However, for the time being, I abstract from this issue.

2.5 The household's optimization problem

In my model the consumer may simultaneously hold positive savings (A_t) and credit card debt (B_t). In each period, after labor income Y_t is realized, the individual chooses how much to consume C_t, how much to save in financial assets A_t, and how much to borrow ($b_t > 0$) or to repay of the outstanding debt ($b_t < 0$). In the case of positive outstanding debt, he also chooses whether he wishes to honor ($D_t^c = 0$) or to default ($D_t^c = 1$) on his liabilities.

The equations describing the laws of motion for outstanding debt and financial assets are:

$$B_{t+1} = (B_t + b_t)(1+i)\left(1 - D_{t+1}^c\right) \tag{11}$$

$$\begin{aligned} A_{t+1} &= [(A_t + Y_t - C_t + b_t)(1+r)](1 - D_t^c) + \\ &\quad (A_t - Min[Max[A_t - e, 0], B_t] + Y_t - C_t)(1+r)D_t^c \end{aligned} \tag{12}$$

While the first equation is straightforward, the law of motion for financial assets deserves some explanation. The first part of this equation is the law of motion in case the consumer chooses not to default ($D_t^c = 0$). In case he chooses to default current financial assets are decreased by the amount that the credit card company is entitled to receive, which is current assets up to an exemption level, or the amount of current outstanding debt, whichever is smaller.

The consumer's optimization problem is to:

$$\max_{C_t, D_t^c, b_t} E_1 \left\{ \sum_{t=1}^{T} \beta^{t-1} \left[\frac{C_t^{1-\gamma}}{1-\gamma} - stigma D_t^c \right] + \beta^{T+1} V_{T+1}(A_{T+1}) \right\} \tag{13}$$

subject to constraints (2) through (12) plus non-negativity constrains for financial assets, $A_t \geq 0$, and consumption, $C_t \geq 0$.

The control variables of the problem are $\{C_t, b_t, D_t^c\}_{t=1}^T$. The state variables are $\{t, \mu_t, \varepsilon_t, A_t, B_t, D_t^s\}_{t=1}^T$. The level of current income is a state variable since the decision of whether to default may depend on current income.

3 Solution Method

There is no analytical solution for the consumer's problem. Hence, the policy functions are derived numerically by discretizing the state-space and variables over which the choices are made. In any period t, there are two cases, depending on whether the consumer has defaulted in the past or not. In the case of previous default, the Bellman equation is given by:

$$V_t(A) = \max_C \{u(C) + \beta E_t V_{t+1}(A')\} \tag{14}$$

where the prime refers to date $t+1$ variables. In the case of no previous default, we have the following equation:

$$V_t(\mu, \varepsilon, A, B) = \max_{C,b,D^c} \{u(C, D^c) + \beta E_t V_{t+1}(\mu', \varepsilon', A', B')\} \tag{15}$$

The problem is solved using backward induction. At the time of death, the value function V_{T+1} is given by the power utility function on assets. This value function is fed in to the last period's problem. For each combination of the state variables, I compute the utility associated with admissible values for the choice variables. This utility is equal to instantaneous utility plus the continuation value. I optimize over these different choices using grid search. This procedure is then iterated backwards.

In order to compute the value function corresponding to values of the state variables that do not lie on the grid I use cubic spline interpolation. Since full numerical integration is extremely slow, the distributions of the labor income shocks are approximated using Gaussian quadrature. To better capture the curvature in the value function at low levels of the state variables, the logarithmic function of the grid was used. The combination of the choice variables ruled out by the constraints of the problem were given a very large negative utility so that they will never be optimal. Optimization is done over the different choices using grid search. See (Judd 1993) and (Tauchen and R. Hussey 1991).

4 Parameterization

4.1 Labor Income Process

The labor income process described in section 2 was estimated using data from the Panel Study of Income Dynamics (PSID). The PSID is a longitudinal study of a representative sample of U.S. individuals and family units. The study is conducted at the Survey Research Center, Institute for Social Research, University of Michigan, and has been running since 1968. The following adjustments were made to the initial data set. In order to obtain a random sample, families that were part of the Survey of Economic Opportunities sample were dropped. Only male headed households are considered. This is due to the fact that age income profiles of households headed by a female are potentially. However, the latter subsample is too small to be considered separately. I truncate the problem at retirement following (Gourinchas and Parker 1996), so that households with the head aged over 65 and retirees are eliminated. Non-respondents, students, housewives and families reporting more than 6 children are also dropped from the sample.

In order to estimate labor income variances which do not overstate the real income risk faced by the household, one has to include, in the definition of labor income, ways of self-insuring against shocks. For this reason, labor income is defined to be: total reported labor income plus unemployment compensation, workers compensation, social security, supplemental social security, other welfare, child support and total transfers (mainly help from relatives), all this for both head of household and his spouse, if present. Any observation reporting zero for this broad definition of income was dropped. Real labor income was calculated by deflating labor income using the Consumer Price Index, with 1992 as base year.

Age-profiles tend to differ in shape across education groups, a finding that has already been reported in several papers, including (Attanasio 1995) and (Hubbard, Skinner and Zeldes 1994). For this reason the sample was split in three according to the education of the head of the household: no high school, high school and college graduates. In the few observations where education has changed over the life cycle, I considered the household as a new entity.

One of the advantages of using the PSID is that the same household is followed over time and one can account for many sources of heterogeneity. The sample runs from 1970 to 1992, therefore a household appears at most 23 times. Households with less observations were not removed from the sample, therefore an unbalanced panel is estimated.

I estimate equation (3) using fixed effects. The function $f(t, Z_{it})$ is assumed to be additively separable in t and Z_{it}. t is composed by a set of age dummies, and the vector Z_{it} includes individual characteristics, other than age and education, which potentially have an effect on labor income and therefore have to be controlled for. In this set I include: family size (number of children); marital status; and a family-specific fixed effect. The coefficients on the age dummies are all significant and the results match intuition and stylized facts.

In order to obtain smoothed versions of the above estimates, third-order polynomials were fitted to the age-dummies. The resulting profiles are shown in Figure 1. The income profiles generated are similar to the ones reported in (Gourinchas and Parker 1996), (Attanasio 1995), and (Carroll and Summers 1991). For consumers with no high school degree, earnings are almost flat during their life-cycle, whereas for families with a college graduate head, life-cycle earnings are hump-shaped. These differences in estimated profiles will allow us to study the effects of the shape of labor income profile on consumers' borrowing and defaulting behavior.

Risk is another important element in the characterization of the labor income process. I estimate the variances of both permanent and transitory income shocks by using the same variance decomposition methodology as (Carroll and Samwick 1997). The estimation results are presented in Table 1.

Following (Cocco et al. 1998) I have calculated the retirement income using a replacement ratio α of 60%. Throughout, and otherwise stated the benchmark case is the income profile for high school graduates.

4.2 Interest Rate and Borrowing Contract

The real interest rate r is set equal to the average of the annualized three month Treasury bills rate from 1970 to 1992, the same time period which was used to estimate labor income profiles.

The parameters associated with the borrowing contract (borrowing rate and credit limit) are difficult to parameterize. In recent years there has been a dramatic growth in credit card offers, both in terms of quantity and credit card features. Nowadays it is not uncommon for one household to own more than one credit card. There are credit cards with and without annual fee, with a low introductory rate, that give cashback, air miles, and so on. Obviously, the stylized model does not allow us to deal with this. Perhaps the simplest and most consistent way to parameterize the model is to assume that there is competition in the credit card industry, so that credit card firms on average earn zero abnormal profits. In particular, I set the interest rate premium θ to 5% and let firms set the credit limit so that abnormal profits are on average zero. In that case the corresponding zero profit credit limit is roughly 10,000 USD. Another zero profit combination would be θ equal to 7% and credit limit λ equal to 8,000 USD. Increasing the interest rate premium makes consumers more willing to default. As a result credit card companies have to lower the credit limit in order to avoid losses.

4.3 Bankruptcy Law and the Costs of Bankruptcy

In order to obtain some guidance for the value of the exemption level e let us take a brief look at bankruptcy law. Individuals who wish to file for bankruptcy under the U.S. Bankruptcy Code have the right to choose between filing under Chapter 7 or Chapter 13. Under Chapter 7, debtors must turn over to the Bankruptcy court all their assets above a fixed exemption level, in turn for which many types of unsecured debt are discharged. Under Chapter 13 debtors do not give up any assets, but must propose a plan to repay a portion of their debts from future income. Under these arrangements individuals have an incentive to choose Chapter 7 whenever their assets are less than the exemption level, since doing so allows them to completely avoid the obligation to repay. See (Domowitz and Sartain 1999) for a model which includes both the decision to file for bankruptcy and the choice between the two chapters. In practice 70% of the households filing for bankruptcy do so under Chapter 7. Therefore I assume that all fillings for bankruptcy are done under Chapter 7. Although bankruptcy is a matter of federal law, individual states are allowed to adopt their own bankruptcy exemptions. (Gropp, Scholz and White 1997) show that differences in state-level bankruptcy exemptions affect the supply and demand for credit. In particular, higher exemption levels redistribute availability of credit from low asset to high asset households. Most states have separate exemptions for equity in the debtor principal residence, equity in motor vehicles, personal property, the cash value of life insurance and IRA Keogh accounts, and a "wildcard" exemption that can be used for any type of property. Since in this model I abstract from housing, durable goods and savings which are illiquid until retirement, I focus on the exemption levels of personal property and "wildcard." See (Pavan 2005) for the effect of bankruptcy exemptions on the accumulation of durable wealth. Although the exemption levels vary widely across states, from zero in Oklahoma to USD 30,000 in Texas, the across states average value for both of these items is USD 2,000, which is the value I set for e.

There are three different types of costs, for the debtor, associated with the process of filing for bankruptcy: the out-of-the pocket costs of lawyer's and court's fees, which are relatively small and therefore ignored in this model; the "stigma effect" s; and the effect on the debtors credit history records. I set the *stigma* parameter value so that in the baseline case the average annual default rate predicted by the model is the same as the one observed in the data. The simulations were done using a population consisting of 25% no highschool, 50% highschool and 25% college graduates. These correspond roughly to the proportions observed in the data. In particular, for the period after the passage of the Bankruptcy Reform act of 1978 and until 1992, the average proportion of households who filed for bankruptcy per year was 0.53%. Data taken from the Administrative Office of the U.S. Courts and the U.S.Census Bureau. This results in a value of 0.0018 for the *stigma* effect s. (Pavan 2005) estimates for the *stigma* effect are considerably higher ranging from 0.0049 to 0.0201.

Another cost borne by the debtor as a consequence of filing for bankruptcy is the damage it causes to his credit history. Credit bureaus are allowed to report consumers' bankruptcy filings up to ten years. This will make access to new credit very difficult, if not impossible, for that period, as is well documented in (Musto 1999). Given that this is a quite long period of time and in order to simplify the solution of the dynamic programing problem, I assume that credit is inaccessible at all after default.

4.4 Other Parameters

Other parameters include preference and time parameters. In order to facilitate comparison with the existing consumption literature I set the discount factor β equal to 0.97 and the coefficient of relative risk aversion γ equal to 3, these are also the benchmark parameters used by (Deaton 1991) and (Carroll 1997).

In our model the main source of uncertainty is labor income uncertainty. Broadly speaking, this is probably the main source of uncertainty faced by individuals during their working lives. However, during retirement uncertainty comes from other sources such as medical expenses and time of death, therefore consumption policy functions during retirement depend on variables not considered in the model. I think that studying consumption behavior during retirement lies beyond the scope of the present paper. Therefore I truncate the problem at retirement age and consider that the salvage value function summarizes all retirement and bequest motives. Note that with this approach I am unable to disentangle the effect of retirement and bequest on life cycle profiles, and refer to both interchangeably. Given this, I analyze individuals during their working lives between age 20 and 65, which gives us a value for T equal to 46. For college graduates, the working live starts at 22 to 65.

Table 2 summarizes the parameters used in the benchmark case.

5 Policy Functions

5.1 Introducing Borrowing and Default in the Consumption Model

The model presented here extends those of (Deaton 1991) and (Carroll 1997) by introducing borrowing and default. The possibility to borrow up to a certain limit, relaxes the credit constrain and makes the consumption function less concave at low levels of wealth (Figure 2). Analytical proof of these properties of the consumption function is given in (Carroll and Kimball 1996). The movements in the consumption function over the life cycle are characterized in (Gourinchas and Parker 2002).

In order to understand the effects of introducing default, we have to keep in mind that the option to discard liabilities at any time in the future in effect provides the consumer with an additional insurance policy. For risk averse consumers, this additional insurance represents a substantial increase in welfare, and consumption will be higher at lower levels of wealth. However, for wealthier households the benefits of being able to file for bankruptcy are smaller. Therefore the value of the option to default decreases and the consumption functions converge as cash on hand increases (see Figure 2). (White 1998) calculates the option value of default for a typical household and shows that it can be very high.

Moreover with the option to default the optimal demand for credit is higher. This is due to the fact that in my model rational consumers act strategically relative to the bankruptcy law, (Fay, Hurst and White 2000) find support for strategic bankruptcy behavior. That is, for low realizations of cash on hand agents borrow all credit available with the intention of defaulting in the near future. This results in a discontinuity in the debt policy function, as illustrated in Figure 3. If we set *stigma* equal to zero there is a further shift in the debt policy function. I will denote this feature of the model as the *strategic borrowing.*

5.2 Asset level which triggers default

In this model the default rule is always such that there is an optimal asset level below which default is triggered. In other words, if in a given period the consumer has a low income realization, outstanding debt is at its limit, and his assets are below this equilibrium trigger level he chooses to default. In order to see this more clearly, Figure 4 plots, as a function of age, the value of assets below which the agent chooses to default. In periods where the cut off level of assets is zero default will never occur.

Assets above the exemption level are seized by credit card firms in the case of default. Therefore agents will never choose to default when assets are very high. If we set the stigma effect to zero, the default trigger assets is around USD 3,000, which means that in case of default consumers have to give up USD 1,000 in order to discharge their liabilities of USD 10,000. This value is constant over the life cycle. However, introducing the stigma effect causes the trigger asset level to change over the life cycle. It is young and older households who choose to default up to higher asset values. That is, over life the default trigger asset value is U-shaped, a pattern inverse to that of labor income. To understand this shape it is important to realize that current income and consumption of young and old households are on average lower than those of middle-aged households, so that the utility function is evaluated at a point where the marginal utility of assets is larger. Because of this the dollar amount that young households are willing to forego to avoid the desutility associated with the stigma effect is smaller, making them more prone to default.

A decrease in the level of outstanding debt results in parallel downward shifts of the asset level that triggers default, making default less likely to occur. (Dunn and Kim 1999) report empirical evidence that supports this result. This is due to the fact that, when the level of outstanding debt is below the maximum there is one additional consideration for the default decision: reneging on the borrowing obligations rules out the possibility of an increase in debt levels in future periods. However, default will still occur for some positive asset values, for the young and old households. This may be surprising as one may have expected that the possibility of increasing debt levels in the future may be most attractive for young liquidity constrained households. However, for this age group borrowing constraints combined with a steep labor income profile effectively lower the discount factor, i.e. make them behave in a more myopic manner. The possibility of increasing debt levels in the future is less relevant for myopic households. When outstanding debt is sufficiently small (e.g. equal to eight thousand dollars), and individuals may increase it by more in the future, even the very young choose not to default for any value of assets.

6 Simulation Results

Using the policy functions described above, I simulate the consumption, borrowing and saving profiles of ten thousand households over the life-cycle. The means of these simulated profiles are presented and discussed next.

First I consider the simulated profiles for the baseline model. Figure 5 plots the average profiles for simulated income, consumption and savings. Individuals are able to smooth consumption over the life cycle, to some extent, by borrowing early in life and by saving for retirement. The steepness of consumption profile early in life is due to the restrictive credit limit and the large cost of credit, which makes it less attractive. As consumers start accumulating wealth for retirement the consumption profile becomes less steep. The upward flip in consumption in the last periods of life is due to consumers spending down their precautionary savings as remaining uncertainty becomes small. This results from the assumption made that the time of death is certain. In order to eliminate this unattractive feature of the consumption profile one could extend the model to allow for a probability of death in each year. Figure 6 plots mean credit card debt. In accordance with the consumption and income profiles, agents borrow early in life due to increasing labor income, and later in life to smooth consumption over short periods, possibly due to a bad income shock. The sharp decrease in the average amount borrowed in the third period reflects the unavailability of credit due to a large default rate in the previous year.

6.1 Comparative Statics

I now discuss the simulation results obtained by solving the model with some parameter changed by the same proportion: 50%. The parameters considered are: labor income characteristics; exemption level; credit limit and *sitgma* effect. Table 3 presents default rates and average debt holdings (when borrowing is positive). This exercise allows us to answer the question of what factors affect default rates most.

6.2 Effect of Labor Income Characteristics

Depending on the nature of the income shocks, optimal responses of consumption, borrowing and default will differ. The next section explores these differences.

6.2.1 Transitory Income Shock

In case there is a transitory shock to labor income, households' optimal response is to smooth consumption over that period, and to accommodate that consumption level through saving or borrowing. In the present model credit availability is limited, i.e., in the case of no history of default there is a fixed limit of credit, and in the case of previous default there is no credit available. Therefore and in order to insure against transitory income shocks, consumers would like to maximize credit availability for periods when a negative shock occurs. In order to quantify the importance of this response I have simulated the model increasing the standard deviation of transitory income shock by one half. This results in a decrease in the default rate from 0.5% to 0.25%. In addition, as we can see in Table 3 average borrowing is 78% less, when transitory income shock has higher variability.

From this simulation we learn the following. First, that it is optimal for consumers to rely on credit card debt to smooth consumption over periods of transitory negative income shocks, such as unemployment spells. These results are consistent with empirical evidence in (Sullivan 2005), who finds that households use unsecured credit card debt to smooth consumption during periods of negative transitory income shocks that result from unemployment spells, in particular younger and less educated households. Second, that in periods of greater variability in transitory income, as it is usually the case during times of higher unemployment (assuming that the unemployment spells have no permanent affect on income which is probably the case for most households), our model generates lower default rates. Inversely, in periods of economic expansion, usually characterized by low unemployment rates and low volatility in transitory income shocks, we are likely to see higher default rates.

6.2.2 Permanent Income shock and Persistency

When the labor income shock is permanent it is optimal for households to adjust the consumption level accordingly, as opposed to the transitory case where consumption is held constant and accommodated through borrowing or saving. This implies that credit availability is not as important and, consequently, when we increase the variance of the permanent income shock by on half the default rate increases by almost 100%.

In order to analyze the effect of persistency in labor income shocks on default rates I relax the unit root assumption and allow for a autoregressive parameter value of 0.5 in permanent labor income shocks. This results in a decrease in the default rate by 90%.The intuition behind this is the same as the case for transitory shocks: persistent income shocks are now "more transitory."

6.2.3 Different Labor Income Profiles

When we consider different income profiles, namely through the analysis of consumer behavior for different education groups, the model allows us to draw some interesting implications.

The level of current income affects the relative importance of the desutility caused by social embarrassment of default, *stigma*. If current income is low, and consumption is also low, marginal utiliy of consumption is very high making the cost of *stigma* relatively unimportant. On the contrary if current income is high, marginal utility of consumption is low, and the relative importance of the cost of *stigma* is higher. On the other hand, the steepness of labor income profile affects the desirability of access to credit.

Flat Income Profiles. First let us consider the group with no high school education, for whom labor income earnings are almost flat over the life cycle. Due to the flatness of labor income age-profiles consumers do not wish to borrow early in life and as a result the availability of credit is not very valuable. Moreover, since income and consumption levels are on average lower the relative importance of the stigma effect is also low. These two factors make the cost of defaulting less severe and as a consequence there is more strategic borrowing. In addition, this behaviour is constant through life because the default trigger asset level is constant over all ages. As a result simulated default rates are very high, averaging 1.29%, with most of the borrowing made strategically in order to default in the future.

Steep Income Profiles. On the other side if we consider households with college education, the results described above are reversed. More precisely, labor income is extremely steep and peaks late in life. Hence, credit availability is crucial for these consumers. Also, income is on average higher making the stigma effect dominate over any incentive to default. As a result, the trigger value of assets for which consumers would choose to default is zero for almost all years, which means default will almost never occur. By the same reason, there is no strategic borrowing.

6.3 Effect of Asset Exemption

Given the present debate on the adequacy of the current bankruptcy law it is important to determine how a change in the exemption level would affect consumer's behavior. With this in mind I have solved the model for a lower asset exemption level, equal to USD 1,000. In this case strategic borrowing is lower and the resulting simulated default rate is only 0.35%.

Another interesting feature of the model is that it predicts that consumers simultaneously hold assets and debt. (Bertraud and Haliassos 2001) term this as "Puzzle of Debt Revolvers" and (Lehnert and Maki 2002) call it "Borrowing to Save." In my model, as in Lehnert and Maki's model, this is due to the exemption level on assets, which households are allowed to hold in case they file for bankruptcy. As can be seen from Table 3, the average amount of savings, for those individuals who are borrowing, is higher for cases where the probability of default is higher.

6.4 Effect of Credit Limit and *Stigma*

From the results in Table 3 we see that *stigma* effect and credit limits have a substantial effect on default rates. A fifty percent decrease/increase in these parameter values increases default rates by three fold and two fold respectively. Thus changes in any of these parameter values have a large impact on average default rates. The importance of the *stigma* effect is consistent with the empirical findings of (Gross and Souleles 2002a). Using a new and comprehensive data set, that includes information held by several credit card companies on their costumers, the authors were able to control for risk-composition and other economic fundamentals, and to investigate the role of the stigma effect in explaining bankruptcy and default. They found robust evidence that the decline in the social embarrassment from defaulting has had an important role in explaining the recent growth in personal bankruptcy. The effect of the credit limit on debt levels is also large in my model, and once again this is consistent with empirical findings by (Gross and Souleles 2002b). They find that an increase in the credit limit generates an immediate and significant rise in debt even for individuals starting below the credit limit. More over, in my model this increase in debt levels also implies a significant increase in default rates.

7 Validating the Model's Predictions with Data

Before concluding, let us first compare quantitatively some of the model's predictions with real world facts. From the previous section we have learned that the model predicts the following: (i) default decreases with education level; (ii) default is negatively correlated with expectations about unemployment; and (iii) exemptions levels drive consumers to simultaneously hold financial assets and credit card debt.

In order to assess the validity of the first result I have used PSID data for year 1996, which was the only wave collecting data on personal bankruptcy filings. The survey asked households wether they had ever filed for bankruptcy and if yes, in which year. The number of filings was also asked, although there are very few observations with more than filing. Since in my model it is not possible to file for bankruptcy more than once, I have dropped these observations. In order to be consistent with the model I have also dropped households with head aged less than 20 and over 65. Using this sample of 6400 observations, I have estimated a probit model of the bankruptcy decision on the education level, (I have created three classes of education level: no high school, high school and college graduates, according to the reported years of education) controlling also for age (where age is the age of household's head at the time of bankruptcy filing) and income. It is not possible to control for the amount of debt because debt levels are only reported in the case of bankruptcy, creating a perfect collinearity problem. The results are reported in Table 4, together with the results of the same estimation but using a sample generated by the model. The coefficient on education level is negative and statistically significant in both samples. However, the effect of education on bankruptcy is higher in the model. It is interesting to note that the coefficient on age is exactly the same in the model as in the data, showing that the probability to default is decreasing with age.

In order to investigate the second result highlighted above, I have used aggregate data on the unemployment rate (collected from the Bureau of Labor Statistics) and bankruptcy rate (calculated by dividing the number of annual non-business fillings reported by the Administrative Office of the Courts and the number of households reported by the Census) for the period from 1980 to 2002. For this period the correlation between the two variables is -0.7 (see Figure 7). Moreover, the correlation of bankruptcy rate and unemployment lagged one period was -0.82. These values show supporting evidence for the result that higher unemployment expectation reduce the probability to default, because consumers become more cautious and borrow less. The correlation between unemployment rate lagged one period and the household debt-service burden is -0.11 for consumer credit and -0.57 if mortgage is also included. Estimate of the ratio of debt payments to disposable personal income, taken from The Federal Reserve Board.

I provide evidence for the result that exemption levels affect consumers' portfolio decisions using the Survey of Consumer Finances (SCF). Every three years, the Federal Reserve conducts a survey obtaing detailed information for a random sample of households, on their assets, liabilities, income, and use of financial institutions and instruments such as credit cards. I have used data from the 1998 wave and merged it with the exemption levels on personal property, by region. For households holding positive credit card balances I have regressed liquid financial wealth (which could be used to pay off the credit card balances) on the exemption level, controlling for age, education, income and credit card balance. The estimation results are shown on Table 5. The coefficient on the exemption level is positive and significant, suggesting that consumers hold higher savings in regions where they are allowed to keep more of them in the case they file for bankruptcy. This evidence is also confirmed by (Lehnert and Maki 2002). They use the SCF in conjunction with other data sets to find supporting evidence that households are more likely to hold simultaneously low return assets and owe high-cost debt in states with higher exemption levels.

8 Concluding Remarks

I have solved an empirically parameterized model of optimal consumption, allowing for borrowing and default. I have carried out sensitivity analysis to quantify the effects of the model parameters on default rates. The model makes several important contributions to the literature on personal bankruptcy. From the sensitivity analysis results we learn that both the *stigma* effect and the credit limit have large impact on default rates. We also learn that the default rate is decreasing in the education level, and that households with less education are more likely to borrow strategically. The model generates predictions that are consistent with empirical facts: (i) simultaneous holdings of credit card debt and liquid assets; (ii) higher default rates when transitory income uncertainty is higher; (iii) use of credit card debt to smooth consumption over periods of temporary reduced income.

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Table 1: Variance Decomposition Estimates. This table shows estimated variances (standard errors) for the transitory income shock ε_{it} and for the permanent income shock μ_t, using household-level income data from the PSID, for the period of 1970 to 1992.

	No High School	High School	College Graduates
Transitory income shock $\sigma^2_{\varepsilon_{it}}$	0.057 (0.0145)	0.038 (0.0123)	0.024 (0.0119)
Permanent income shock $\sigma^2_{\mu_t}$	0.033 (0.0096)	0.027 (0.0071)	0.043 (0.0098)

Table 2: Baseline Parameters.

Description	Parameter Value
Retirement age (T)	65
Discount factor (β)	0.97
Risk aversion (γ)	3
Stigma effect (s)	0.0018
Interest rate premium (θ)	0.05
Credit limit (λ) in USD	10,000
Exemption level (e) in USD	2,000
Education Level	High School Graduate

Table 3: This table shows annual default rates and average debt holdings (for those with positive debt), for different parameterizations.

Note: Values of debt are in thousand USD.

	Baseline	$Stigma = 0.0009$	$\lambda = 15,000$	$e = 1.0$
Default rate	0.005	0.013	0.011	0.0035
Debt (B)	4.89	4.32	6.22	4.46
Elasticity		3.2	2.4	-0.6

	Baseline	$\sigma_{\varepsilon_{it}}$ =0.284	σ_{μ_t} =0.24	$persistency = 0.5$	$NoHiSch$	$ColGrad$
Default rate	0.005	0.0025	0.0095	0.00065	0.0129	0.00003
Debt (B)	4.89	3.79	4.08	5.17	4.12	5.62
Elasticity		-1.0	1.8	-1.74		

Table 4: Probit Regression Results. Bankruptcy decision was regressed on age, education, income and constant, using PSID data (1996 wave) and data simulated by the model. Coefficients and corresponding standard deviations are shown.

Independent Variable	PSID		Model	
Bankruptcy Decision	Coefficient	(Std. Dev.)	Coefficient	(Std. Dev.)
Age	-0.03	0.003	-0.03	0.002
Education	-0.10	0.05	-0.30	0.04
Income	-0.001	0.001	-0.04	0.004
Constant	-0.10	0.128	-0.27	0.09

Table 5: Survey Linear Regression. Liquid financial assets were regressed on state exemption levels, controlling for age, education, income and credit card balances. A subsample from the SCF of 1998 was used. Only households with positive credit card balances were considered.

Independent Variable		
Liquid Assets	Coefficient	(Std. Dev.)
Exemption	7.34	4.05
Age	1497.4	215.8
Education	-2982.42	4438.4
Income	2.35	0.27
Credit Card Balance	-0.93	0.68
Constant	-136194	13925.35