Cheap Talk with Two Senders and Complementary Information

Andrew Mcgee and Huanxing Yang*
Department of Economics, Ohio State University

January 2009

Abstract

This paper studies a two-sender cheap talk model with two senders having partial and non-overlapping private information. Moreover, two senders' private information is complementary in the sense that the marginal impact of one sender's private information on the receiver's ideal action depends on the other sender's private information. When two senders communicate simultaneously, their information transmission exhibits strategic complementarity: more information transmitted by one sender leads to more information transmitted by the other sender. When two senders have like biases, it is always optimal for the receiver to delegate the decision rights to the sender with a smaller bias. On the other hand, we two senders have opposing biases, simultaneous communication is more likely to dominate delegation.

JEL Classification: D23, D72, D83, L23
Keywords: Cheap talk; Multiple senders; Delegation.

1 Introduction

Decision makers often consult multiple experts for advice, who have expertise but their interests may not be perfectly aligned with the decision makers. This observation motivated a growing literature on cheap talk with two senders (Gilligan and Krehbiel, 1989; Epstein, 1998; Krishna and Morgan, 2001a, KM hereafter). A common feature of these models is that the state of the world is one dimensional and *both* senders (experts) *perfectly* observe the realized state. For example, a CEO has a decision to make regarding the size of a new factory, the optimal size of which depends on the profitability of the new product. For that purpose, the CEO consults one marketing expert and one production expert, both of whom observe the profitability of the new product.

While in some situations it is reasonable to assume that both experts perfectly observe the realized state, more typically each expert might only observe *some aspects of but not the whole* of the realized state. In a modern world emphasizing specialization, experts usually only have expertise in their own fields. In the previous CEO example, the marketing expert may only have knowledge about the demand for the new product, while the production expert may only have knowledge

*We would like to thank Yaron Azrieli for helpful comments and suggestions.

about the cost of production. However, both pieces of information are essential to determine the profitability of the new product and the optimal size of the new factory. Similar examples abound in the real world. For instance, suppose the president is deciding on the size of a stimulus program and he consults one financial expert and one expert on politics, each of whom observes only one aspect of the whole scenario. Alternatively, suppose a military leader is considering how many troops to send to a war; and consults a strategy expert and an intelligence expert, each of whose information alone is not enough to determine the optimal number of troops.

This paper studies a two-sender cheap talk model in which two senders have *partial and non-overlapping* private information regarding the state of the world. To model partial and non-overlapping private information, we assume that the state of the world has two dimensions (aspects, θ_1 and θ_2). Each expert i perfectly observes the realized state in dimension i (θ_i), but does not observe the realized state in dimension j (θ_j). The receiver's ideal decision is a function of the realized states, $y^*(\theta_1, \theta_2)$. We will focus on the case that $\partial^2 y^*(\theta_1, \theta_2)/\partial\theta_1\partial\theta_2 \neq 0$. In other words, to determine the ideal decision the marginal impact of information in dimension i depends on the realized state in dimension j. In this case we say that *two states are complementary.* Our leading example will be $y^*(\theta_1, \theta_2) = \theta_1\theta_2$.[1] This condition has an intuitive interpretation. Consider the CEO example in which θ_1 is the demand size and θ_2 is efficiency or production. Then the condition means that the bigger the market size, the bigger the marginal impact of product efficiency on the optimal size of factory. In other words, there is complementarity between demand size and production efficiency.[2]

As in other cheap talk models, each expert has his own interests, which we model as a bias relative to the ideal decision for the receiver. Adopting the terminology from KM, two experts can have opposing biases in which case one expert wants to pull the decision to the left and the other to the right. Alternatively, two experts can have like biases if both of them want to pull the decision in the same direction but possibly to different degrees.

We first study equilibrium information transmission when two experts send messages simultaneously. Equilibria are shown to be partition equilibrium, where each sender only indicates to which interval the realized state that he observes belongs, as in standard cheap talk models (Crawford and Sobel, 1982, CS hereafter). We focus on the most informative equilibrium. Interestingly, the information transmission of two senders exhibits strategic complementarity: the more information that one sender transmits, the less the incentive the other sender has to distort information toward his own bias, hence the more information he will transmit. Thus a reduction in one sender's bias will not only leads to more information transmitted by himself, but also induce the other agent

[1] If $\partial^2 y^*(\theta_1, \theta_2)/\partial\theta_1\partial\theta_2 = 0$, say $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$, then the cheap talk game with two-senders is qualitatively similar to standard cheap talk game with one sender. See Section 6 for more detailed discussion.

[2] In the military example, θ_1 is the weakness of own army and θ_2 is the strength of the enemy. Then the condition implies that the weaker the own army is, the bigger the marginal impact of the strength of the enemy on the optimal number of the troops.

to transmit more information. Intuitively, when sender i transmits more information, the other sender j faces greater uncertainty regarding the principal's conditional expectation of the other dimension of the state space. Given that the two states are complementary, this magnifies the loss in matching the states if sender i distorts his information, thus sender j has less incentive to distort his information.

Under simultaneous communication, the equilibrium information transmission only depends on the absolute value of two senders' biases. Changing the sign of either sender's bias does not affect the information transmitted in equilibrium. This implies that it does not matter whether two agents have like biases or opposing biases.

We then study the possibility of delegation in which case the receiver delegates his decision rights to one of the senders. The agent to whom the decision rights are delegated first consults the other sender regarding his private information, and then makes a decision. Given two senders, there are two options in delegating decision rights. We show that it is always better for the receiver to delegate decision rights to the expert with a smaller bias in absolute value. Finally, we compare optimal delegation to simultaneous communication. Interestingly, whether delegation is optimal depends on whether two experts have like biases or opposing biases. When two experts have like biases and communication is informative under simultaneous communication, optimal delegation dominates simultaneous communication for the receiver. This result still holds when two experts have opposing biases and the absolute value of the smaller bias is small enough. Simultaneous communication dominates delegation if two experts have opposing biases and the absolute value of the smaller bias is big enough.

The rest of the paper is organized as follows. The next subsection discusses related literature. Section 2 lays out the model. Simultaneous communication is studied in Section 3. In Section 4 we consider delegation and compare it to simultaneous communication. Section 5 discusses possible extensions and the robustness of our results. Section 6 concludes. All the longer proofs can be found in the Appendix.

1.1 Related Literature

Following the original work of CS on cheap talk, there is a growing literature on cheap talk with two senders. Gilligan and Krehbiel (1989) study a model in which two experts with symmetric opposing biases simultaneously communicate (submitting bills) to the decision-making legislature. They show that the restrictive "closed rule," in which amendments to bills are not permitted, is informationally superior to the "open rule" in which bills are freely amendable. Krishna and Morgan (2001b) reexamine the model and derived different results. Epstein (1998) generalizes the model of Gilligan and Krehbiel to the case where two experts have asymmetric opposing biases. KM (2001a) study a more general model with two senders. In their model, two experts can have like or opposing biases and they communicate sequentially. As pointed in the introduction, a common feature of these models is that the state space is one dimensional and both senders perfectly observe the same realized state. In contrast, in our model two senders have partial and non-overlapping private information.

Austen-Smith (1993) considers a two-sender model with two experts imperfectly informed about the state. In his model each expert receives a noisy (binary) signal about the state, which is also binary. This formulation yields results very different from those of standard cheap talk models.[3] In his model, the two experts' signals are correlated and the combination of both still does not fully reveal the realized state. In our model, two experts' signals are independent and the combination of both fully reveal the realized state. Li (2007) studies a model in which two experts perfectly observe the realized state, but each expert's bias is his own private information. Again in his model the number of states and signals is finite.

Our paper is also related to Alonso et.al (2008), who study strategic communication between a CEO and two division managers. Each manager has private information regarding the local conditions of his own division, and a decision needs to be made for each division. Furthermore, the two decisions of the two divisions need to be coordinated, and each manager has a bias toward maximizing the profit of his own division. They compare two communication modes. In vertical communication (centralization), each manager communicates with the CEO simultaneously, and then the CEO makes decisions for each division. Under horizontal communication (decentralization), the two managers simultaneously communicate with each other and then each manager makes the decision for his own division. They show that even when the need of coordination is large, decentralization can be superior to centralization for the CEO. Our paper is different from theirs in that in our model there is only one decision to make instead of two. In their model the need to communicate results from the need to coordinate two decisions, while in our model the need to communicate arises because the private information of both experts is needed to determine the optimal decision for the receiver. These differences change experts' incentives to communicate.

Battaglini (2002) studies a multidimensional cheap talk model with multiple senders. He concludes that in contrast to one dimensional cheap talk models with one sender, in his setting generically information can be fully revealed in equilibrium communication. Ambrus and Takahashi (2008) provide further conditions under which fully revealing equilibria are possible. Our model differs from multidimensional cheap talk model in two regards. First, in our model each sender only observes the realized state in his own dimension, while in their models each sender observes the realized states in both dimensions. Second, in our model the decision is a one-dimensional variable while in theirs the decision is a two-dimensional vector. Based on these difference, fully revealing equilibria are impossible in our model.

[3] For example, full revelation is possible even with a single agent, which is impossible with a richer signal space.

Another work related to ours is Dessein (2002), who compares delegation to a one-sender cheap talk model, a la CS. Our paper is different in that we compare delegation with cheap talk to a cheap talk model with two senders (see Section 4 for more details). Harris and Raviv (2005) and McGee (2008) study a cheap talk model and the possibility of delegation when both the receiver and sender have private information. In McGee's model, the receiver and the sender's private information exhibits complementarity, a formulation similar to ours in that the two senders' private information are complementary. In both models again there is only one sender.

Since the work of CS, it is well-known that cheap talk models have multiple equilibria. There have been efforts made on equilibrium refinement (Matthews et. al, 1991; Chen et. al, 2008). We will follow a common practice in cheap talk models: whenever there are multiple equilibria, we will focus on the most informative equilibrium since it is Pareto dominant.

2 Model

To formalize the examples mentioned in the introduction, we provide a stylized model that can be applied to a broad range of institutional settings. Consider a decision maker (DM, receiver) who consults two experts (senders, agents) $i = 1, 2$. The DM takes an action $y \in R$, and his utility depends on some underlying states of nature θ_1 and θ_2. Each θ_i is uniformly distributed on $[0, A_i]$ with density $1/A_i$, and θ_1 and θ_2 are independent from each other. Expert i observes only the realized value of θ_i. The DM does not observe the realization of either θ_1 or θ_2, nor does agent i observe the realization of θ_j, $j \neq i$. This captures the fact that each expert's knowledge is specialized within his won field. Note that both experts have private information, yet they are not overlapping in the sense that θ_1 and θ_2 are independent.

Expert i offers advice to the DM by sending message $m_i \in [0, A_i]$. In the basic model we consider the case of simultaneous communication, in which two experts send messages simultaneously. After receiving messages m_1 and m_2, the DM takes an action $y(m_1, m_2)$.

The utility functions are of quadratic loss form. Given realized states θ_1 and θ_2, the ideal action for the DM is $y^*(\theta_1, \theta_2) = \theta_1\theta_2$.Specifically, the utility function for the DM is

$$U^P(y, \theta_1, \theta_2) = -(y - y^*(\theta_1, \theta_2))^2.$$

Note that $\frac{\partial^2 y^*(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} > 0$. As we mentioned in the introduction, this condition implies that two states are complementary: the marginal impact of state θ_i depends on the realized state θ_j. It will be shown that this relationship leads to strategic interactions between two experts' information transmission.

Expert i's ideal action is $y^*(\theta_1, \theta_2) + b_i$, where b_i is expert i's bias (relative to DM). Bias b_i can be positive or negative, which measures to which degree the DM and expert i's interests are divergent. Specifically, the utility function for expert i is

$$U^{A_i}(y, \theta_1, \theta_2, b_i) = -[y - (y^*(\theta_1, \theta_2) + b_i)]^2.$$

The biases are common knowledge. When b_1 and b_2 have the same sign, we say that experts have like biases; otherwise, we say that they have opposing biases. Both experts and the DM are expected utility maximizers.

3 Simultaneous Communication

Under simultaneous communication, a strategy for expert i specifies a message m_i for each θ_i, which denote as communication rule $\mu_i(m_i|\theta_i)$. A strategy for the DM specifies an action y for each message pair (m_1, m_2), which we denote as decision rule $y(m_1, m_2)$. Let belief function $g(\theta_1, \theta_2|m_1, m_2)$ be the DM's posterior beliefs on θ_1 and θ_2 after hearing messages m_1 and m_2. Since θ_1 and θ_2 are independent and expert i only observes θ_i, the belief function can be separated as $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

We focus on Perfect Bayesian Equilibria (PBE) of the communication game, which requires:

(i) Given DM's decision rule $y(m_1, m_2)$ and expert j's communication rule $\mu_j(m_j|\theta_j)$, for each i, expert i's communication rule $\mu_i(m_i|\theta_i)$ is optimal.

(ii) DM's decision rule $y(m_1, m_2)$ is optimal given beliefs $g_1(\theta_1|m_1)$ and $g_2(\theta_2|m_2)$.

(ii) The belief functions $g_i(\theta_i|m_i)$ are derived from agents' communication rules $\mu_i(m_i|\theta_i)$ whenever possible.

We first derive DM's optimal decision rule $\overline{y}(m_1, m_2)$. Given m_1 and m_2, $\overline{y}(m_1, m_2)$ maximizes $-E[(y - \theta_1\theta_2)^2|m_1, m_2]$. Since θ_1 and θ_2 are independent, and $\theta_1|m_1$ and $\theta_2|m_2$ are independent, it can be readily seen that

$$\overline{y}(m_1, m_2) = E[\theta_1|m_1]E[\theta_2|m_2]. \tag{1}$$

As in CS and Alonso *et.al* (2008), all PBE are interval equilibria. Specifically, state space $[0, A_i]$ is partitioned into intervals and expert i only reveal which interval θ_i belongs to.

Lemma 1 *All PBE in the communication game must be interval equilibria.*

Having established all PBE must be interval equilibria, we now characterize them. Let N_i, which is a positive integer, be the number of partitions for agent i, and $(a_{i,0}, a_{i,1}, ..., a_{i,n}, ..., a_{i,N_i})$ be the partition points, with $a_{i,0} = 0$ and $a_{i,N_i} = A_i$. Define random variable $\overline{m}_i$ as the posterior of state θ_i given message m_i, that is, $E[\theta_i|m_i] \equiv \overline{m}_i$. Define $\overline{m}_{i,n}$ as the receiver's posterior expectation of θ_i after receiving a message $m_{i,n} \in (a_{i,n-1}, a_{i,n})$. It follows that $\overline{m}_{i,n} = (a_{i,n-1} + a_{i,n})/2$. In state

$\theta_1 = a_{1,n}$, agent 1 should be indifferent between sending a message that induces a posterior $\overline{m}_{1,n}$ and a posterior $\overline{m}_{1,n+1}$, that is, $E_{\theta_2}[U^{A_1}|\overline{m}_{1,n}, a_{1,n}] = E_{\theta_2}[U^{A_1}|\overline{m}_{1,n+1}, a_{1,n}]$. More explicitly, this indifference condition can be written as

$$E_{\theta_2}[\{\overline{m}_2\frac{a_{1,n}+a_{1,n-1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2] = E_{\theta_2}[\{\overline{m}_2\frac{a_{1,n}+a_{1,n+1}}{2} - (\theta_2 a_{1,n} + b_1)\}^2],$$

which can be simplified as

$$E[\overline{m}_2^2](a_{1,n+1} + 2a_{1,n} + a_{1,n-1}) - 4E[\theta_2\overline{m}_2]a_{1,n} = 4b_1E(\theta_2). \tag{2}$$

In the appendix we show that $E[\theta_2\overline{m}_2] = E[\overline{m}_2^2]$. Then the indifference condition (2) can be further simplified as

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{E(\theta_2)}{E(\overline{m}_2^2)}4b_1. \tag{3}$$

Similarly, the cutoff points $a_{2,n}$ that characterize the interval equilibria of agent 2 satisfy the following indifference condition:

$$(a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{E(\theta_1)}{E(\overline{m}_1^2)}4b_2. \tag{4}$$

Inspecting indifference conditions (3) and (4), we see that there is strategic interaction between two senders' information transmission, as the term $\frac{E(\theta_j)}{E(\overline{m}_j^2)}$ appears in the condition that determines sender i's cutoff points. In the quadratic-uniform case of CS's one-sender model, the indifference condition for cutoff points is that the difference of two adjacent intervals (the incremental step size) is always $4b$. Conditions (3) and (4) show that the strategic interaction between two senders occurs through changing the incremental step sizes: the effective incremental step size now becomes $\frac{E(\theta_j)}{E(\overline{m}_j^2)}4b_i$. Note that

$$E(\overline{m}_j^2) = (E(\overline{m}_j))^2 + var(\overline{m}_j) = (E(\theta_j))^2 + var[E(\theta_j|m_j)].$$

Thus a bigger $E(\overline{m}_j^2)$ (conditional variance of θ_j given m_j) means more information is transmitted by sender j. Then we see that as one agent transmits more information, the effective incremental step size for the other agent decreases, which leads to more information transmitted by the other agent (this will be formalized later). Therefore, two agent's information transmission exhibits strategic complementarity.

Let $\frac{E(\theta_i)}{E(\overline{m}_i^2)} \equiv x_i$. Given $a_{i,0} = 0$, the solution for difference equations (3) and (4) are:

$$a_{i,n} = a_{i,1}n + 2n(n-1)b_ix_j.$$

In the most informative equilibrium, N_i is the largest integer such that

$$2|b_i|x_jN_i(N_i - 1) < A_i. \tag{5}$$

Given N_i and x_j, using the fact $a_{1,N_i} = A_i$ we can solve for $a_{i,1}$. Then

$$a_{i,n} = A_i \frac{n}{N_i} + 2b_j x_i n(n - N_i). \tag{6}$$

Lemma 2 *(i)* $E(\overline{m}_i^2) = \frac{A_i^2}{3} - \frac{A_i^2}{12N_i^2} - \frac{b_i^2 x_j^2 (N_i^2-1)}{3}$*; (ii)* $E(\overline{m}_i^2)$ *is strictly increasing in* N_i*, strictly decreasing in* x_j*, and strictly increasing in* $E(\overline{m}_j^2)$*; (iii)* $\frac{A_i^2}{4} \leq E(\overline{m}_i^2) \leq \frac{A_i^2}{3}$.

Lemma 2 formally shows that two agents' equilibrium information transmission exhibits strategic complementarity. This result comes from the fact that two states are complementary: the ideal decision for DM given realized states is $\theta_1\theta_2$. Intuitively, when agent 2 transmits more information, $\overline{m}_2$ or $E[\theta_2|m_2]$ has a bigger variance. Now for agent 1 overstating (or understating) θ_1 in the direction of b_1 will lead to a bigger loss for agent 1, since it will be magnified by the variance of $\overline{m}_2$. This reduces agent 1's incentive to distort information (effective bias), hence he transmits more information as well. Note that two states are independent ($\frac{\partial^2 y^*(\theta_1,\theta_2)}{\partial\theta_1\partial\theta_2} = 0$), then the strategic interaction between two agents is absent.[4]

For PBE of the overall communication game, a babbling equilibrium always exists, with $N_1 = N_2 = 1$, both agents babbling and the DM ignoring the messages. Thus we do not need to worry about the existence of PBE. Straightforward calculation shows that the ex ante equilibrium payoff of DM U_{ST}^P and that of agent i, $U_{ST}^{A_i}$, are given by:

$$\begin{aligned} U_{ST}^P &= -E[(\overline{m}_1\overline{m}_2 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\overline{m}_1^2)E(\overline{m}_2^2). \\ U_{ST}^{A_i} &= -E(\theta_1^2)E(\theta_2^2) + E(\overline{m}_1^2)E(\overline{m}_2^2) - b_i^2 \end{aligned} \tag{7}$$

Therefore the most informative equilibrium that we will focus on is also Pareto dominant.

Proposition 1 *The most informative equilibrium is characterized by a pair of numbers of partition elements* (N_1^*, N_2^*)*. They satisfy*

$$E(\overline{m}_1^2) = \frac{E(\theta_1)}{x_1} \Leftrightarrow \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2 x_2^2 (N_1^2 - 1)}{3} = \frac{A_1}{2x_1}, \tag{8}$$

$$E(\overline{m}_2^2) = \frac{E(\theta_2)}{x_2} \Leftrightarrow \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{b_2^2 x_1^2 (N_2^2 - 1)}{3} = \frac{A_2}{2x_2}; \tag{9}$$

[4] Suppose $y^*(\theta_1, \theta_2) = \theta_1 + \theta_2$. Then the indifference condition for agent i's partition points $a_{i,n}$ is given by

$$(a_{i,n+1} - a_{i,n}) - (a_{i,n} - a_{i,n-1}) = 4b_i.$$

It is clear that two agents' information transmission is independent from each other's, and the two-sender model collapses to a one sender model.

and are the largest N_1 and N_2 such that

$$2|b_1|x_2N_1(N_1-1) < A_1;\quad 2|b_2|x_1N_2(N_2-1) < A_2. \tag{10}$$

Moreover,

$$\langle -\frac{1}{2}+\frac{1}{2}(1+\frac{A_1A_2}{|b_i|})^{1/2}\rangle \leq N_i^* \leq \langle -\frac{1}{2}+\frac{1}{2}(1+\frac{4A_1A_2}{3|b_i|})^{1/2}\rangle \tag{11}$$

Proof. By (7), in the most informative equilibrium $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$ are maximized. By part (ii) of Lemma 2, $E(\overline{m}_i^2)$ is increasing in N_i. Moreover, $E(\overline{m}_i^2)$ is decreasing in x_j and x_j is decreasing in $E(\overline{m}_j^2)$. Thus $E(\overline{m}_i^2)$ is increasing in N_j as well. Therefore, in the most informative equilibrium N_1 and N_2 should be maximized. But the total length of partitions for each agent $2|b_i|x_jN_i(N_i-1)$ should be less than the length of the support of θ_i, A_i, which gives rise to condition (10). The bounds of N_i^* comes from part (iii) of Lemma 2. The bounds of $E(\overline{m}_i^2)$ implies that $x_i \in [\frac{3}{2A_i}, \frac{2}{A_i}]$. Then (11) follows. ■

Corollary 1 *In the most informative equilibrium, a decrease in b_i results in not only an increase in $E(\overline{m}_i^2)$, but also an increase in $E(\overline{m}_j^2)$.*

Proof. By Proposition 1, a decrease in b_1 will not decrease N_1^* or N_2^*. The rest follows Lemma 2. ■

Corollary 1 can potentially be empirically tested: as one agent is replaced by a new agent whose interest is aligned more with the DM, more information is transmitted by the other agent. This is illustrated in the following example.

Example 1 *Suppose $A_1 = 4$ and $A_2 = 1$. Agent 1 with bias $b_1 = \frac{1}{8}$ and agent 2 with bias $b_2 = \frac{1}{2}$. Under simultaneous communication, $N_1^* = 3$, $N_2^* = 2$, $E(\overline{m}_1^2) =$, $E(\overline{m}_2^2) =$. When b_2 decreases to $\frac{1}{4}$, $N_1^* =$, $N_2^* =$, $E(\overline{m}_1^2) =$, $E(\overline{m}_2^2) =$. Note that $E(\overline{m}_1^2)$ increases as b_2 decreases.*

In KM, equilibrium information transmission depends on whether two agents have opposing or like biases. Specifically, if two agents have like biases, then it is better for the DM consults only one agent. When two agents have opposing biases, generally more information will be transmitted and it is better for the DM to consult both agents. In our model, the following corollary shows that equilibrium information transmission does not depend on whether two agents have opposing or like biases, and it is always better for the DM to consult two agents instead of one.

Corollary 2 *(i) It is always better for the DM to consult two agents instead of one. (ii) Fix all the other parameter values except changing b_i to $-b_i$. Then (N_1^*, N_2^*), $E(\overline{m}_i^2)$ and $E(\overline{m}_2^2)$ remain the same and each players' ex ante equilibrium payoffs are unchanged as well.*

Proof. Suppose the DM only consults one agent, say agent i. Then no information is transmitted by agent j. Thus $E(\overline{m}_j^2)$ reaches the lower bound $\frac{A_j^2}{4}$ and x_j reaches the upper bound $\frac{2}{A_j}$. It follows that the effective incremental step size for agent i is bigger than that of agent i when two agents are consulted. This implies that both $E(\overline{m}_i^2)$ and $E(\overline{m}_1^2)E(\overline{m}_2^2)$ are smaller when only agent i is consulted. This proves part (i).

Observing (8) and (9), we see that, fixing N_i, $E(\overline{m}_i^2)$ remains the same as b_i changes to $-b_i$. This means $E(\overline{m}_j^2)$ remains the same as well. Also note that conditions (10) remains the same when b_i changes sign. Therefore, N_1^* and N_2^* will not change as well. This proves part (ii). ■

The differences of the results comes from the fact that in KM two agents have the same private information and they communicate sequentially, while in our model two agents have partial and non-overlapping private information. When two agents have the same private information, their messages can potentially discipline each other. In our model this strategic effect is absent since agents have non-overlapping private information. Given that two agents' information transmission is strategic complements and they have non-overlapping private information, it is always better for the DM to consult two agents instead of one.

The result that whether two agents have like or opposing biases does not matter in our model is because the interaction between two agents' equilibrium communication occur only through the terms of $E(\overline{m}_1^2)$ and $E(\overline{m}_2^2)$. When b_i changes sign the only change occurred is that the partition of m_i reverse its direction, but still the same amount of information is transmitted in equilibrium. One may still wonder whether two agents have like or opposing biases matters based on the following logic. Suppose two agents initially both have positive biases. Now after agent 1's bias changes to $-b_1$, two agents' bias become further apart. One may think that in the first case agents have less incentive to overstate information, as each agent expects the other to overstate so he needs to overstate less to reach the optimum for himself. And in the second case, agent 2 might have stronger incentive to overstate his own information as he expects agent 1 to understate θ_1. However, this is not what occurred in equilibrium. In equilibrium, the principal is not fooled, which means that each agent cannot successfully understate or overstate their information. Anticipating this, whether the other agent has incentive to understate or overstate information (as long as the absolute value of the bias is the same) will not affect one agent's incentive to misrepresent his own information.

Comparison In some environments, the DM may have the freedom to change the assignment of agents. Suppose two dimensions have different underlying uncertainty and two agents have different biases. Without loss of generality, suppose $A_1 > A_2$ and $|b_1| = r|b|$ and $b_2 = b$ with $r > 1$, that is, θ_1 has a bigger variance and agent 1 has a bigger bias. Now a question naturally arises: to induce more effective overall communication, should the DM assign the agent with a smaller to observe the dimension with more uncertainty or should he assign the agent with a bigger bias to observe the dimension with more uncertainty. We call the first assignment as positive assortative (PA) assignment (assigning agent 2 to observe θ_1 and assigning agent 1 to observe θ_2), and the reverse assignment as negative assortative (NA) assignment.

Proposition 2 *(i) If r is big enough such that agent 1's communication is uninformative under both assignments, then both assignments yield the same ex ante payoff. (ii) if $b \to 0$ but $r|b| > 0$, then both assignments yield the same ex ante payoff.*

Proposition 2 identifies the conditions under which assignments do not matter. Part (i) shows that when one agent's bias is big enough such that his communication is uninformative under both assignments, then assignments do not matter. Part (ii) shows that when one agent's bias is arbitrarily small such that his communication will be fully informative, then assignments do matter as well.

It would be desirable to compare the effectiveness of communication when communications are informative but not perfectly informative in both dimensions under either assignments. However, given that equations (8) and (9) are highly nonlinear, it is difficult to derive any analytical result. Thus we have to resort to numerical simulations. Suppose $A_1 = 1$, $A_2 = 4$, $b_1 = 1/2$ and $b_2 = 1/8$. Under PA assignment, $N_1^* = 2$ and $N_2^* = 3$. Under NA assignment, $N_1'^* = 3$ and $N_2'^* = 2$. The ex ante payoffs are the same under both assignment. We did other numerical examples, and in all the examples the assignments do not matter for the overall information transmission.

4 Delegation

Though the DM has formal authority to make decision, he may find it optimal to delegate the decision rights to one of the agents. Given that there are two agents, two kinds of delegations need to be considered: the decision right is delegated to agent 1 (D1 delegation) or the decision right is delegated to agent 2 (D2 delegation). Under either delegation, the agent delegated with decision rights first consults the other agent, then makes the decision. In the CEO example, if the marketing expert (department manager) is delegated with decision right, he first consult the production expert (department manager) regarding the production efficiency, then combining his own information on market demand makes the decision about the new plant size. We will answer two questions in this section. Which delegation mode is optimal (D1 or D2 delegation)? When does the DM have an incentive to delegate his decision rights?

4.1 Optimal delegation

First consider D1 delegation. In this case agent 2 first sends message m_2 to agent 1, and then agent 1 chooses the decision $y(\theta_1, m_2)$. Agent 1's optimal decision rule can be obtained as follows: $\overline{y}(\theta_1, m_2) = \theta_1 \overline{m}_2 + b_1$. Now the communication game between agent 2 and agent 1 is a one-sender cheap talk game with the receiver having private information, as setting studied by McGee (2008). It can be shown that all PBE are partition equilibria (see McGee for details). Let N_2 be the number of partition elements and $\{a_{2,0}, ..., a_{2,n}, ..., a_{2,N_2}\}$ be the cutoff points. In particular, given $\overline{y}(\theta_1, m_2)$, when $\theta_2 = a_{2,n}$ agent 2 should be indifferent between sending a message immediately to the left of $a_{2,n}$ and a message immediately to the right of $a_{2,n}$. This indifference condition can be explicitly written as:

$$E[\{\theta_1 \frac{a_{2,n} + a_{2,n-1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2] = E[\{\theta_1 \frac{a_{2,n} + a_{2,n+1}}{2} + b_1 - (\theta_1 a_{2,n} + b_2)\}^2]$$

$$\Leftrightarrow \quad (a_{2,n+1} - a_{2,n}) - (a_{2,n} - a_{2,n-1}) = \frac{4E(\theta_1)}{E(\theta_1^2)}(b_1 - b_2) = \frac{3}{2A_1}4(b_1 - b_2)$$

In the most informative equilibrium, the (largest) number of partition elements $N_2^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2} \rangle$. And

$$E(\overline{m}_2^2) = \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^2} - \frac{(b_1 - b_2)^2(\frac{3}{2A_1})^2(N_2^2 - 1)}{3} \tag{12}$$

Note that under D1 delegation, the incremental step size of agent 2's partition equilibrium only depends on the difference in biases ($|b_1 - b_2|$). As a result, N_2^*, $E(\overline{m}_2^2)$ and hence the equilibrium information transmission only depends on the difference of biases between two agents. The principal's equilibrium payoff U_{D1}^P under D1 delegation can be computed as

$$U_{D1}^P = -E[(\theta_1 \overline{m}_2 + b_1 - \theta_1 \theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_1^2)E(\overline{m}_2^2) - b_1^2 \tag{13}$$

Now consider D2 delegation. In this case agent 1 first sends message m_1 to agent 2, and then agent 2 chooses the decision $y(\theta_2, m_1)$. Agent 2's optimal decision rule is given by $\overline{y}(\theta_2, m_1) = \theta_2 \overline{m}_1 + b_2$. Let N_1 be the number of partition elements and $\{a_{1,0}, ..., a_{1,n}, ..., a_{1,N_1}\}$ be the cutoff points of agent 1's interval equilibrium of the communication game. In particular, $a_{1,n}$ is characterized by

$$(a_{1,n+1} - a_{1,n}) - (a_{1,n} - a_{1,n-1}) = \frac{4E(\theta_2)}{E(\theta_2^2)}(b_2 - b_1) = \frac{3}{2A_2}4(b_2 - b_1).$$

In the most informative equilibrium, $N_1^* = \langle -\frac{1}{2} + \frac{1}{2}(1 + \frac{4A_2A_1}{3|b_1-b_2|})^{1/2}\rangle$. And

$$E(\overline{m}_1^2) = \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{(b_1-b_2)^2(\frac{3}{2A_2})^2(N_1^2-1)}{3} \tag{14}$$

Comparing the expressions of N_1^* and N_2^*, we see that the number of partition elements are the same under D1 and D2 delegation. The principal's equilibrium payoff U_{D2}^P under D2 delegation is

$$U_{D2}^P = -E[(\theta_2\overline{m}_1 + b_2 - \theta_1\theta_2)^2] = -E(\theta_1^2)E(\theta_2^2) + E(\theta_2^2)E(\overline{m}_1^2) - b_2^2 \tag{15}$$

Proposition 3 *Between D1 and D2 delegation, it is always optimal for the DM to delegate the decision rights to the agent has a smaller bias (the agent has a smaller $|b_i|$).*

Proof. Without loss of generality, suppose agent 1 has a smaller bias, $|b_1| < |b_2|$. We want to show that D1 delegation is better for the DM. From previous derivations, it is clear that N_1^* under D2 delegation is the same as N_2^* under D1 delegation. Let us call both of them N^*. From (13) and (15),

$$U_{D1}^P - U_{D2}^P = E(\theta_1^2)E(\overline{m}_2^2) - E(\theta_2^2)E(\overline{m}_1^2) + (b_2^2 - b_1^2).$$

By (12) and (14),

$$\begin{aligned} U_{D1}^P - U_{D2}^P &= \frac{A_1^2}{3}[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3(b_1-b_2)^2(N^{*2}-1)}{4A_1^2}] \\ &- \frac{A_2^2}{3}[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3(b_1-b_2)^2(N^{*2}-1)}{4A_2^2}] + (b_2^2 - b_1^2) \\ &= b_2^2 - b_1^2 > 0. \end{aligned}$$

Therefore, D1 delegation yields a higher ex ante payoff. ■

Proposition 3 indicates that the decision rights should be delegated to the agent with a smaller bias. Note that this does not depend on which agent's private information has more underlying uncertainty. Intuitively, since equilibrium information transmission only depends on the difference of biases $|b_1 - b_2|$, under either delegation the agent with the decision right always ends up (after communication) with the same amount of information to utilize ($E(\theta_1^2)E(\overline{m}_2^2)$ under D1 delegation equals to $E(\theta_2^2)E(\overline{m}_1^2)$ under D2 delegation). Now it is clear that decision rights should be delegated to the agent with a smaller bias to minimize the loss of control.

4.2 Compare delegation and simultaneous communication

Now without loss of generality, suppose agent 1 has smaller bias, $|b_1| < |b_2|$. Thus it is optimal to delegate decision rights to agent 1. We are interested in identifying the conditions under which the principal has incentives to delegate instead of retaining decision right and carrying out simultaneous cheap talk.

Proposition 4 *When two agents have like biases and informative communication is feasible for agent 1 under simultaneous communication, then the DM prefers D1 delegation to simultaneous cheap talk.*

Proposition 4 is related to Dessein (2002), who shows that in a one-sender cheap talk model the principal prefers delegation whenever cheap talk is informative. Our result shows that a modified result holds in a two-sender model with complementary information. In our setting, compared to simultaneous communication, delegation leads to four changes. First, delegation means loss of control, which is measured by b_1^2. Second, delegation always leads to more information utilized in dimension θ_1, $E(\theta_1^2) \geq E(\overline{m}_1^2)$. Third, due to strategic complementarity, $E(\theta_1^2) \geq E(\overline{m}_1^2)$ means $x_{2D} \leq x_2$, implying more information will be transmitted by agent 2 under delegation. Finally, under delegation the effective bias for agent 2 is changed from $|b_2|$ to $|b_2 - b_1|$, which will affect agent 2's equilibrium information transmission as well. The last three effects measure potential informational gain under delegation relative to simultaneous communication.[5] While the second and third effects always favor delegation, the fourth effect depends on whether two agents have like or opposing biases.

When two agents have like biases, optimal delegation always leads to a smaller effective bias in communication, $|b_2 - b_1| < |b_2|$. Thus both the third and fourth effects work in the same direction: more information is transmitted by agent 2 under delegation. This additional informational gain, which is absent in Dessein's (2002) one-sender cheap talk model, would make the overall informational gain under delegation even bigger. Thus delegation is preferred by the DM.

Example 2 *Consider example 1: $A_1 = 4$, $A_2 = 1$, $b_1 = \frac{1}{8}$ and $b_2 = \frac{1}{2}$. Under simultaneous communication, $N_1^* = 3$ and $N_2^* = 2$. Under D1 delegation, $x_2 = 3/8$ and the effective bias is $|b_2 - b_1| = 3/8$. By inequality $N_{2D}(N_{2D} - 1) < \frac{A_1 A_2}{3|b_2 - b_1|}$, $N_{2D}^* = 2$. The difference between the principal's ex ante payoffs can be expressed as*

$$U_{D1}^P - U_{ST}^P = (\frac{16}{3})(\frac{1199}{4096}) - \frac{1}{1.8300242447368062 * 0.3963815134045646} - \frac{1}{64} = 0.167.$$

Delegation dominates simultaneous communication.

[5] Dessein (2002) compares the information gain in the second effect to the loss of control in the first effect.

The condition that two agents have like biases is not necessary for delegation to dominate simultaneous communication. This is because even if two agents have opposing biases so that fourth effect works against delegation, effects two and three can still outweigh effect four, meaning delegation leads to informational gain. The following example illustrates this.

Example 3 *$A_1 = 4$, $A_2 = 1$, $b_1 = \frac{1}{8}$ and $b_2 = -\frac{1}{4}$. Under D1 delegation, $|b_2 - b_1| = \frac{3}{8}$, $A_1 = 4$, $A_2 = 1$, $b_1 = \frac{1}{8}$ and $b_2 = -\frac{1}{4}$. Because $|b_2 - b_1| = \frac{3}{8}$, hence U^P_{D1} is the same as in the previous example. Under simultaneous communication, $N^*_1 =$, $N^*_2 =, x_1 = 0.3943559607401785$, and $x_2 = 1.651362875813561$.*

$$U^P_{D1} - U^P_{ST} = (\frac{16}{3})(\frac{1199}{4096}) - \frac{1}{0.3943559607401785 * 1.651362875813561} - \frac{1}{64} = 0.0100048$$

Again delegation dominates simultaneous communication.

Actually, the proof of Proposition 4 indicates that as long as $E(\overline{m}^2_{2D}) \geq E(\overline{m}^2_2)$, that is, more information is transmitted by agent 2 under D1 delegation than under simultaneous communication, the principal always prefers delegation. Thus we have the following corollary.

Corollary 3 *Suppose two agents have opposing biases. As long as $|b_1|/|b_2|$ is small enough such that $E(\overline{m}^2_{2D}) \geq E(\overline{m}^2_2)$ and informative communication is feasible for agent 1 under simultaneous communication, D1 delegation always leads to higher ex ante payoff for the principal.*

Of course, when two agents have opposing biases and $|b_1|$ is big enough, then the fourth effect might outweigh the second and third effect, leading to small information gain or even information loss under D1 delegation relative to simultaneous communication. In this case, simultaneous communication might dominate delegation.

Example 4 *$A_1 = 4$, $A_2 = 1$, $b_1 = \frac{1}{8}$ and $b_2 = -\frac{1}{2}$. By Corollary 2, under simultaneous communication the principal's expected utility is still the same as in example 1. Under D1 delegation, it can be verified that $N^*_{2D} = 2$ again. Because $|b_2 - b_1| = \frac{5}{8}$, $E(\bar{m}^2_{2D}) = \frac{1055}{4096}$. Thus*

$$U^P_{D1} - U^P_{ST} = (\frac{16}{3})(\frac{1055}{4096}) - \frac{1}{1.8300242447368062 * 0.3963815134045646} - \frac{1}{64} = -0.0205.$$

simultaneous communication dominates delegation.

The empirical implication for Proposition 2 and Corollary 3 is that when two agents have like biases or opposing biases but the smaller bias is small, then the decision right is more likely to be delegated to the agent with a smaller bias. In that case, we may observe one agent (functional department) consults another functionally parallel and complementary agent (functional department), and then makes a decision for the DM (CEO). On the other hand, when two agents have opposing biases and the smaller bias is big, then the DM is more likely to retain the decision rights and consults both agents.

5 Discussions

5.1 Sequential Talk

One agent sends message early, and his message becomes public. Then the other agent sends message. Suppose agent 1 sends message first. Agent 1's communication rule is still $\mu_1(m_1|\theta_1)$, but agent 2's communication rule now becomes $\mu_2(m_2|\theta_2, m_1)$.

We work by backward induction. Given m_1 and m_2, the principal's optimal decision rule is $\overline{y} = E[\theta_1|m_1]E[\theta_2|m_2(m_1)]$. Now given m_1, consider agent 2's problem. His equilibrium communication rule is of partition form. In general, cutoff $a_{2,n}$ satisfies the following indifference condition:

$$\begin{aligned} E[\{\overline{m}_1\frac{a_{2,n}+a_{2,n-1}}{2}-(\theta_1 a_{2,n}+b_2)\}^2|m_1] &= E[\{\overline{m}_1\frac{a_{2,n}+a_{2,n+1}}{2}-(\theta_1 a_{2,n}+b_2)\}^2|m_1] \\ &\Leftrightarrow (a_{2,n+1}-a_{2,n})-(a_{2,n}-a_{2,n-1})=\frac{4b_2}{\overline{m}_1} \end{aligned}$$

As we can see, agent 2's communication does depend on m_1.

Suppose agent 1's equilibrium communication rule is of partition form (needs to be checked). Cutoff $a_{1,n}$ satisfies the following indifference condition:

$$\begin{aligned} & E[\{E[\theta_2|m_2,\mu_2(\frac{a_{1,n}+a_{1,n-1}}{2})]\frac{a_{1,n}+a_{1,n-1}}{2}-(\theta_2 a_{1,n}+b_1)\}^2] \\ = \; & E[\{E[\theta_2|m_2,\mu_2(\frac{a_{1,n}+a_{1,n+1}}{2})]\frac{a_{1,n}+a_{1,n+1}}{2}-(\theta_2 a_{1,n}+b_1)\}^2] \end{aligned}$$

which can only be simplified as:

$$\begin{aligned} & E[\overline{m}_2^2(a_{1,n},a_{1,n+1})][(a_{1,n}+a_{1,n+1})^2-4(a_{1,n}+a_{1,n+1})a_{1,n}]- \\ & E[\overline{m}_2^2(a_{1,n-1},a_{1,n})][(a_{1,n}+a_{1,n-1})^2-4(a_{1,n}+a_{1,n-1})a_{1,n}] \\ = \; & 4E(\theta_2)b_1[(a_{1,n}+a_{1,n+1})-(a_{1,n}+a_{1,n-1})] \end{aligned}$$

Since $E[\overline{m}_2^2(a_{1,n},a_{1,n+1})] \neq E[\overline{m}_2^2(a_{1,n-1},a_{1,n})]$, this equation is highly nonlinear.

Compare the equilibrium outcomes under sequential and simultaneous communication. Within sequential talk, if we have asymmetric signals structure and asymmetric agents, decide the optimal order of cheap talk.

5.2 Robustness

Our leading example, $y^*(\theta_1, \theta_2) = \theta_1\theta_2$, allows us to derive the exact partitions used in communication in equilibrium. Of interest is the extent to which our results generalize to more flexible functional forms of the optimal decision function $y^*(\theta_1, \theta_2)$. To address this question, consider the conditions under which PBE of the communication game will be interval equilibria. As in the proof for lemma 1, suppose agent 2 employs a communication rule $\mu_2(\cdot)$ and that the principal induces a posterior belief υ_1 of θ_1. The expected utility of agent 1 is

$$E_{\theta_2}\left[U^{A_1}|\upsilon_1, \theta_1\right] = -E_{\theta_2}\left[\left\{y^*(\upsilon_1, E\left[\theta_2|\mu_2(\cdot)\right]) - y^*(\theta_1, \theta_2) - b_1^2\right\}\right] \tag{16}$$

Given that the utility functions are quadratic loss functions, necessary conditions for agents 1's equilibrium communication rule to be of the interval form are $\frac{\partial^2}{\partial\theta_1\partial\upsilon_1}\left[U^{A_1}|\upsilon_1, \theta_1\right] > 0$ and $\frac{\partial^2}{\partial\theta_1\partial\upsilon_1}\left[U^{A_1}|\upsilon_1, \theta_1\right] < 0$. For general functional forms of the optimal decision functions $y^*(\theta_1, \theta_2)$ these conditions are only satisfied when $\frac{\partial y^*(\theta_1,\theta_2)}{\partial\theta_1}$ and $\frac{\partial y^*(\theta_1,\theta_2)}{\partial\theta_2}$ are single-signed and $\frac{\partial^2 y^*(\theta_1,\theta_2)}{\partial\theta_i\partial\theta_i} = 0$ for $i = 1, 2$. The only function form satisfying these conditions for θ_1 and θ_2 is our leading example, $y^*(\theta_1, \theta_2) = \theta_1\theta_2$. While this implies that our results are not robust to other functional forms for the optimal decision function when the utilities are given by quadratic loss functions, it remains to be seen whether other forms of the optimal decision function exhibit strategic complementarities between agent 1's information and agent 2's information for other utility functions.

The fact that our results apply only to our leading example, however, is not as limiting as one might expect. In our leading example, a higher value of θ_1 means that the optimal decision is more sensitive to small changes in agent 2's information. After a simple rescaling in which $\theta_1 = \frac{1}{\theta_1}$, though, the optimal decision will will be more sensitive to small changes in agent 2's information for small values of θ_1. If θ_1 is distributed uniformly over $[a_1, A_1]$, then $\frac{1}{\theta_1}$ is also distributed uniformly over $[\frac{1}{A_1}, \frac{1}{a_1}]$ because of the one-to-one mapping between θ_1 and $\frac{1}{\theta_1}$. As such, we can simply define $\theta_1' = \frac{1}{\theta_1}$ and our analytical results follow as before.

6 Conclusions

We examine two-sender cheap talk model in which two senders have partial and non-overlapping information regarding the state of the world. Moreover, the marginal impact of one sender's private information on the receiver's ideal action depends on the other sender's private information.

Under simultaneous communication, we show that information transmission displays strategic complementarities in that more informative communication from one expert induces more informative communication from the other. Interestingly, the informativeness of communication from both senders in equilibrium does not depend on whether they have like or opposing biases, but only depends on the magnitudes of these biases. When the principal can assign the experts to the different dimensions of the state space, we show that under a broad range of circumstances this assignment will not affect the principal's expected utility.

We then study delegation when the decision rights are delegated to one of the two experts. We show that the principal always prefers to delegate decision rights to the expert with the a smaller bias. Comparing delegation to simultaneous communication, we demonstrate that when two experts have like biases, delegation is always superior for the principal whenever informative communication between the principal and experts is possible. On the other hand, simultaneous communication dominates delegation when the experts have opposing biases and the smaller bias is big enough.

Unlike other models of strategic communication with multiple experts in which two experts observe basically the same realized state of the world, our model highlights how the nature of the relationship between the private information of the two experts influences their communication to the principal. How much information one expert transmits depends on how much information the other experts will tranmit. When the experts are uncertain about each other's information, the contingent nature of their own information curtails their desires to misrepresent their own information The additional incentives for informative communication arising from the relationship between the experts' information widen the scope for delegation of the decision rights to the less-biased expert.

References

[1] Alonso, R., W. Dessein and N. Matouschek. "When does Coordination Requires Centralization?" *American Economic Review* 98(1), 2008, 145-179.

[2] Ambrus, A. and S. Takahashi. "Multi-sender Cheap Talk with Restrictive State Space," *Theoretical Economics* 3(1), 2008, 1-27.

[3] Austen-Smith, D.. "Interested Experts and Policy Advice: Multiple Referrals under Open Rule," *Games and Economic Behavior* 5, 1993, 1-43.

[4] Battaglini, M.. "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica* 70(4), 2002, 1379-1401.

[5] Chen, Y., N. Kartik and J. Sobel. "Selecting Cheap-Talk Equilibria," *Econometrica*, 2008.

[6] Crawford, V. and J. Sobel. "Strategic Information Transmission," *Econometrica* 50(6), 1982, 1431-1451.

[7] Dessein, W.. "Authority and Communication in Organizations," *Review of Economic Studies* 69, 2002, 811-838.

[8] Epstein, D.. "Partisan and Bipartisan Signaling in Congress," *Journal of Law, Economics, and Organization* 14(2), 1998, 183-204.

[9] Gilligan, T. and K. Krehbiel. "Asymmetric Information and Legislative Rules with a Heterogeneous Committee," *American Journal of Political Science*, 1989, 459-490.

[10] Harris , M. and A. Raviv. "Allocation of Decision-making Authority," *Riview of Finance* 9, 2005, 353-383.

[11] Krishna, V. and J. Morgan. "A Model of Expertise," *Quarterly Journal of Economics* 116, 2001a, 747-775.

[12] Krishna, V. and J. Morgan. "Asymmetric Information and Legislative Rules," *American Political Science Review* 95, 2001b, 435-452.

[13] Li, M. "Combining Expert Opinions," working paper, 2007, Concordia University.

[14] Matthews, S., M. Okuno-Fujiwara and A. Postlewaite. "Refining Cheap Talk Equilibria," *Journal of Economic Theory*, 1991, 247-273.

[15] McGee, A. "Strategic Communication with an Informed Principal and Contingent Information," working paper, 2008, Ohio State University.

Appendix

Proof of Lemma 1:

Proof. First note that in any PBE of the communication game, the optimal decision given beliefs satisfy (1). We first show that given any communication rule for agent 2, $\mu_2(\cdot)$, agent 1's optimal communication rule is of interval form. Suppose the DM holds a posterior belief v_1 of θ_1. Then the expected utility of agent 1 is

$$E_{\theta_2}[U^{A_1}|v_1, \theta_1] = -E_{\theta_2}[\{v_1 E[\theta_2|\mu_2(\cdot)] - \theta_1\theta_2 - b_1\}^2]. \tag{17}$$

It is readily seen that $\frac{\partial^2}{\partial\theta_1\partial v_1}[U^{A_1}|v_1, \theta_1] > 0$ and $\frac{\partial^2}{\partial\theta_1^2}[U^{A_1}|v_1, \theta_1] < 0$. This implies that for any two different posterior beliefs of the DM, say $\underline{v}_1 < \overline{v}_1$, there is at most one type of agent 1 that is indifferent between both. Now suppose that contrary to interval equilibria, there are two states $\underline{\theta}_1 < \overline{\theta}_1$ such that $E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] \geq E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$ and $E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] > E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1]$. But then $E_{\theta_2}[U^{A_1}|\overline{v}_1, \overline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \overline{\theta}_1] < E_{\theta_2}[U^{A_1}|\overline{v}_1, \underline{\theta}_1] - E_{\theta_2}[U^{A_1}|\underline{v}_1, \underline{\theta}_1]$, which contradicts $\frac{\partial^2}{\partial\theta_1 \partial v_1}[U^{A_1}|v_1, \theta_1] > 0$.

The same argument can be applied to agent 2 given any communication rule $\mu_1(\cdot)$ for agent 1. Therefore, all PBE of the communication game must be interval equilibria. ■

Claim: $E[\theta_i \overline{m}_i] = E[\overline{m}_i^2]$.

Proof. Note that $\overline{m}_i = E[\theta_i|m_i]$ and m_i is coarser than of θ_i. Therefore,

$$E[\theta_i \overline{m}_i] = E[\theta_i E[\theta_i|m_i]] = E\{E[\theta_i E[\theta_i|m_i]]|m_i\} = E\{E[\theta_i|m_i]E[\theta_i|m_i]\} = E[\overline{m}_i^2].$$

■

Proof of Lemma 2.

Proof. We prove the results for $E(\overline{m}_1^2)$. The results for $E(\overline{m}_2^2)$ can be proved similarly. From (6), we have

$$\begin{aligned} a_{1,n} - a_{1,n-1} &= \frac{A_1}{N_1} + 2b_1 x_2 (2n - N - 1), \\ a_{1,n} + a_{1,n-1} &= \frac{A_1}{N_1}(2n-1) + 2b_1 x_2 [2n^2 - (2n-1)(N+1)]. \end{aligned}$$

By definition,

$$\begin{aligned} E(\overline{m}_1^2) &= \sum_{n=1}^{N_1} \int_{a_{1,n-1}}^{a_{1,n}} \frac{1}{A_1} \frac{(a_{1,n} + a_{1,n-1})^2}{4} = \frac{1}{4A_1} \sum_{n=1}^{N_1} (a_{1,n} - a_{1,n-1})(a_{1,n} + a_{1,n-1})^2 \\ &= \frac{A_1^2}{3} - \frac{A_1^2}{12N_1^2} - \frac{b_1^2 x_2^2 (N_1^2 - 1)}{3}. \end{aligned}$$

This proves part (i). To show part (ii), consider the difference of $E(\overline{m}_1^2)$ when N_1 decreases to $N_1 - 1$:

$$\begin{aligned} E(\overline{m}_1^2)(N_1) - E(\overline{m}_1^2)(N_1 - 1) &= \frac{A_1^2}{12}[\frac{1}{(N_1-1)^2} - \frac{1}{N_1^2}] - \frac{b_1^2 x_2^2}{3}[N_1^2 - (N_1-1)^2] \\ &\propto A_1^2 - 4b_1^2 x_2^2 N_1^2 (N_1 - 1)^2 > 0 \end{aligned}$$

The last inequality follows from $a_{1,1} > 0$, which implies that $A_1 > 2b_1 x_2 N_1 (N_1 - 1)$. Thus $E(\overline{m}_1^2)$ is strictly increasing in N_1. x_2 affects $E(\overline{m}_1^2)$ in two ways. First, a decrease in x_2 directly increases $E(\overline{m}_1^2)$. Second, by (5) a decrease in x_2 leads to a weakly larger N_1, which increases $E(\overline{m}_1^2)$ as well. Therefore, $E(\overline{m}_1^2)$ is strictly decreasing in x_2. Since x_2 is decreasing in $E(\overline{m}_2^2)$, it follows that $E(\overline{m}_1^2)$ is strictly increasing in $E(\overline{m}_2^2)$.

To prove part (iii), note that $E(\overline{m}_1^2) = (E(\theta_1))^2 + var[E(\theta_1|m_1)]$. Since the conditional variance $var[E(\theta_1|m_1)] \in [0, var(\theta_1)]$, $(E(\theta_1))^2 \leq E(\overline{m}_1^2) \leq E(\theta_1^2)$. And part (iii) immediately follows. ■

Proof of Proposition 2:

Proof. (i) One sufficient condition for agent 1's communication uninformative under both assignments is $6r|b| \geq A_1 A_2$. To see this, consider NA assignment. N_1^* is the largest N_1 such that $2r|b|x_2 N_1(N_1-1) < A_1$. Given $x_2 \geq \frac{3}{2A_2}$, $2r|b|x_2 N_1(N_1-1) \geq 3r|b|N_1(N_1-1)/A_2$. Now if $N_1^* \geq 2$, then $3r|b|N_1(N_1-1)/A_2 \geq 6r|b|/A_2$, which by the condition $6r|b| \geq A_1 A_2$ is bigger than A_1. This contradicts $2r|b|x_2 N_1^*(N_1^*-1) < A_1$. Therefore, $N_1^* = 1$. By a similar argument, one can show that under PA assignment $N_2'^* = 1$.

Now consider NA assignment. Given $N_1^* = 1$, $E(\overline{m}_1^2) = \frac{A_1^2}{4}$ and $x_1 = \frac{2}{A_1}$. Then N_2^* is the largest integer such that $4|b|N_2(N_2-1) < A_1 A_2$. Under PA assignment, $N_2'^* = 1$, $E(\overline{m}_2'^2) = \frac{A_2^2}{4}$, $x_2' = \frac{2}{A_2}$. Then $N_1'^*$ is the largest integer such that $4|b|N_1'(N_1'-1) < A_1 A_2$. It is readily seen that $N_1'^* = N_2^* \equiv N^*$. Now, the difference of ex ante payoffs between NA assignment and PA assignment becomes

$$\begin{aligned} E(\overline{m}_1^2)E(\overline{m}_2^2) - E(\overline{m}_1'^2)E(\overline{m}_2'^2) &= \frac{A_1^2}{4}[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{4b^2(N^{*2}-1)}{3A_1^2}] \\ -\frac{A_2^2}{4}[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{4b^2(N^{*2}-1)}{3A_2^2}] &= 0. \end{aligned}$$

Therefore, both assignments lead to the same ex ante payoff.

(ii) When $b \to 0$, under NA assignment $E(\overline{m}_2^2) = \frac{A_2^2}{3}$ and $x_1 = \frac{3}{2A_2}$. Moreover, N_1^* is the largest integer such that $4r|b|N_1(N_1-1) < A_1 A_2$. Similarly, under PA assignment $E(\overline{m}_1'^2) = \frac{A_1^2}{3}$ and $x_2' = \frac{3}{2A_1}$. Moreover, $N_2'^*$ is the largest integer such that $4r|b|N_2'(N_2'-1) < A_1 A_2$. It is readily seen that $N_2'^* = N_1^* \equiv N^*$. Now, the difference of ex ante payoffs between NA assignment and PA assignment becomes

$$\begin{aligned} E(\overline{m}_1^2)E(\overline{m}_2^2) - E(\overline{m}_1'^2)E(\overline{m}_2'^2) &= \frac{A_2^2}{3}[\frac{A_1^2}{3} - \frac{A_1^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2}-1)}{4A_2^2}] \\ -\frac{A_1^2}{3}[\frac{A_2^2}{3} - \frac{A_2^2}{12N^{*2}} - \frac{3r^2b^2(N^{*2}-1)}{4A_1^2}] &= 0. \end{aligned}$$

Thus under both assignments the ex ante payoff is the same. ■

Proof of Proposition 4:

Proof. We first show that $E(\overline{m}_2^2)$ under simultaneous communication is smaller than $E(\overline{m}_{2D}^2)$ under D1 delegation. By previous results, under simultaneous communication N_2^* is the largest integer such that $2|b_2|\frac{A_1}{2E(\overline{m}_1^2)}N_2(N_2-1) < A_2$. Under D1 delegation, N_{2D}^* is the largest integer such that $2|b_2 - b_1|\frac{A_1}{2E(\theta_1^2)}N_2(N_2-1) < A_2$. Since b_1 and b_2 have the same sign and $|b_1| < |b_2|$, $|b_2 - b_1| < |b_2|$. Moreover, $E(\theta_1^2) \geq E(\overline{m}_1^2)$. Therefore, $N_{2D}^* \geq N_2^*$. Comparing $E(\overline{m}_2^2)$ and $E(\overline{m}_{2D}^2)$,

$$\begin{aligned} E(\overline{m}_2^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_2^{*2}} - \frac{b_2^2 A_1^2 (N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2}, \\ E(\overline{m}_{2D}^2) &= \frac{A_2^2}{3} - \frac{A_2^2}{12N_{2D}^{*2}} - \frac{(b_2-b_1)^2 A_1^2 (N_{2D}^{*2}-1)}{12[E(\theta_1^2)]^2}, \\ E(\overline{m}_{2D}^2) - E(\overline{m}_2^2) &\geq \frac{b_2^2 A_1^2 (N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2} - \frac{(b_2-b_1)^2 A_1^2 (N_2^{*2}-1)}{12[E(\overline{m}_1^2)]^2} \geq 0; \end{aligned}$$

where the first inequality holds because $E(\overline{m}_{2D}^2)$ is increasing in N_{2D}^* and $N_{2D}^* \geq N_2^*$, and the second inequality follows the fact that $|b_2 - b_1| < |b_2|$ and $E(\theta_1^2) \geq E(\overline{m}_1^2)$.

The difference of the DM's ex ante payoffs can be expressed as

$$\begin{aligned} U_{D1}^P - U_{ST}^P &= E(\theta_1^2)E(\overline{m}_{2D}^2) - E(\overline{m}_1^2)E(\overline{m}_2^2) - b_1^2 \\ &\geq E(\overline{m}_2^2)[E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2)]. \end{aligned}$$

Now $U_{D1}^P - U_{ST}^P > 0$ is equivalent to the term in the bracket is strictly greater than 0. More explicitly,

$$\begin{aligned} E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2) &= \frac{A_1^2}{12N_1^{*2}} + \frac{b_1^2 A_2^2 (N_1^{*2}-1)}{12[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \\ \geq \frac{b_1^2 A_2^2 (N_1^{*2}-1)}{12[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} &\geq \frac{b_1^2}{E(\overline{m}_2^2)}(\frac{N_1^{*2}-1}{4} - 1), \end{aligned}$$

where the second inequality follows that fact that $E(\overline{m}_2^2) \leq \frac{A_2^2}{3}$. From the above expressions it is evident that $U_{D1}^P - U_{ST}^P > 0$ if $N_1^* \geq 3$.

Now consider the case that $N_1^* = 2$. Note that under simultaneous communication, the partition $a_{1,1} > 0$ satisfies

$$a_{1,1} + a_{1,1} + \frac{2b_1 A_2}{E(\overline{m}_2^2)} = A_1.$$

Given that $a_{1,1} > 0$, we have $\frac{2|b_1|A_2}{E(\overline{m}_2^2)} < A_1$. Now

$$\begin{aligned} E(\theta_1^2) - \frac{b_1^2}{E(\overline{m}_2^2)} - E(\overline{m}_1^2) &= \frac{A_1^2}{48} + \frac{b_1^2 A_2^2}{4[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \\ > \frac{A_2^2 b_1^2}{12E(\overline{m}_2^2)} + \frac{b_1^2 A_2^2}{4[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} &= \frac{b_1^2 A_2^2}{3[E(\overline{m}_2^2)]^2} - \frac{b_1^2}{E(\overline{m}_2^2)} \geq \frac{b_1^2}{E(\overline{m}_2^2)}(1-1) = 0, \end{aligned}$$

where the first inequality follows $\frac{2|b_1|A_2}{E(\overline{m}_2^2)} < A_1$ and the second one is by the fact $E(\overline{m}_2^2) \leq \frac{A_2^2}{3}$. Thus $U_{D1}^P - U_{ST}^P > 0$ if $N_1^* = 2$. ■